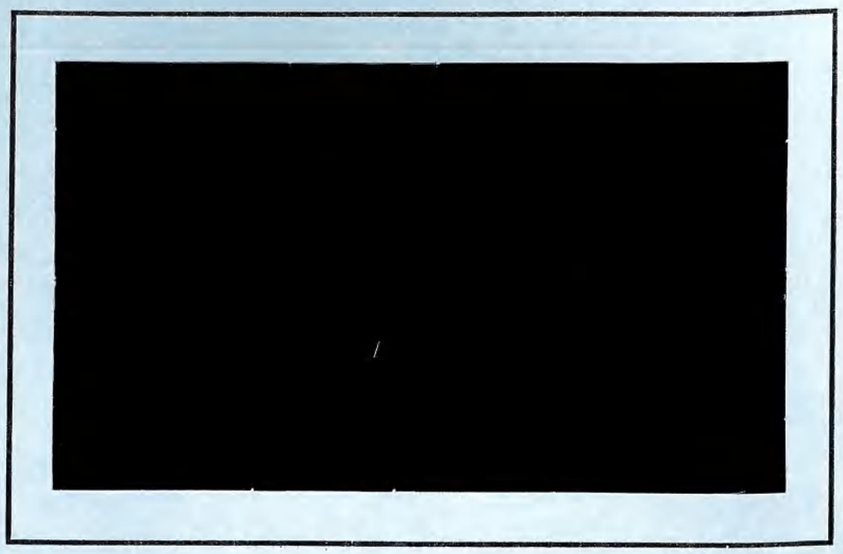
Max Planck-Institut
für Quantenoptik
BIBLIOTHEK
D-8046 Garching

Handbibliothek
MPQ

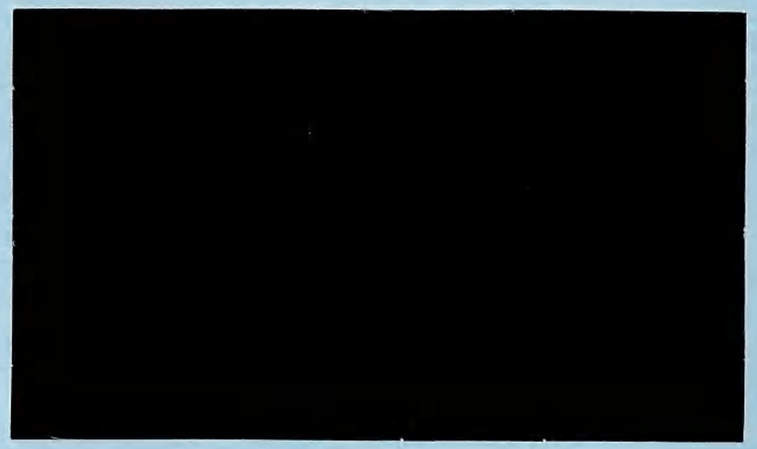


MPQ-Report
47

MPQ

MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK

8046 GARCHING BEI MÜNCHEN



MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK

Effect of Trapping on Breaking of Large-amplitude Plasma Oscillations

M. Kono* and P. Mulser**

MPQ 47 September 1981

Max Planck-Institut
für Quantenoptik
BIBLIOTHEK
D-8046 Garching

* Permanent address: Research Institute for Applied Mechanics,
Kyushu University, Hakozaki, 812, Japan

** Institut für Angewandte Physik,
Technische Hochschule Darmstadt,
D-6100 Darmstadt, FRG

D-8046 Garching bei München

Effect of Trapping on Breaking of Large-amplitude Plasma Oscillations

Abstract

The effect of trapping on the wave-breaking amplitude of Langmuir oscillations is discussed in a homogeneous system on the basis of a phenomenological discription of trapped particles. The significant effect of trapping is to steepen the wave profile and to enhance the wave-breaking amplitude of the elctric field.

§ 1 Introduction

The nonlinear evolution of an electron plasma oscillation driven in a plasma is a current topic, particularly in connection with resonant absorption. The crucial feature of wave propagation with a large amplitude is the existence of a critical amplitude above which wave-breaking occurs.

Wave-breaking as overtaking of the characteristics of a hyperbolic system has been widely discussed in various branches of physics. In a cold plasma, where oscillations can be analyzed exactly, Dawson[1] and Koch and Albritton[2] have calculated the breaking amplitudes for the spatially uniform and non-uniform cases, respectively. The effect of finite temperature on the corresponding amplitudes has been discussed by Coffey[3] and Kruer[4].

In a cold plasma an important role is played by the acceleration of fluid elements, which causes deformation of the wave profile. An increase in amplitude eventually leads to an onset of the reflection of the fluid elements by the wave potential at which the density diverges. In a warm plasma, on the other hand, the wave amplitude is limited in such a way that the velocity of fluid elements is bounded by the thermal velocity. In other words, wave-breaking occurs when the density reaches the critical value at which the response of the density to the potential perturbation changes from rarefactive to compressive. The critical density remains finite, being expressed in terms of the ratio of the phase velocity to the thermal velocity, and becomes infinite in a cold plasma limit.

It is obvious, however, that for a large amplitude wave a quite appreciable number of particles are driven into resonant interaction with the wave. In a homogeneous plasma these particles are regarded as well-defined trapped particles, provided that the mixing due to the difference in bounce frequencies is assumed to be negligible. In this case, when the amplitude of the wave potential increases, an increase in number of the trapped particles has a tendency to restore charge balance in the region where ions are

relatively rich. Trapping, then, first deforms the wave profile steepened and eventually causes a totally electron rich response to the potential, leading to breakdown of the wave. In an inhomogeneous plasma, however, trapping is no longer well-defined because trapped particles can escape from the resonant region within a finite time with a net increase in particle kinetic energy, which may lead to catastrophic damping of the wave.

§ 2 Equations of Motion

This paper is concerned with the effect of well-defined trapping on the breaking amplitude of a stationary wave. For this purpose we use a phenomenological distribution function for trapped electrons as follows:

$$f_t(v,x) = \frac{n_{to}}{2\,(T_e/m)^{1/2}} \quad \text{for } |v| < U(\phi)$$
$$U(\phi) = \left\{ \frac{2e}{m} (\phi(x) - \phi_m) \right\}^{1/2}, \tag{1}$$

where ϕ_m is the minimum value of the wave potential. Untrapped particles may be described by a fluid approximation. The basic equations for our system then are

$$\frac{d}{dx} nu = 0,$$
$$\frac{d}{dx} \left\{ \frac{u^2}{2} - \frac{e}{m}\phi + \frac{T_e}{m} \frac{\gamma}{\gamma-1} \left(\frac{n}{n_0}\right)^{\gamma-1} \right\} = 0, \tag{2}$$
$$\frac{d^2}{dx^2}\phi = 4\pi e \left\{ n + \int f_t(v,x)\,dv - n_0 \right\},$$

where ions are assumed to be an immobile background though it is straight forward to take into account an ion density modification through the ponderomotive force of the electron plasma oscillations. For a Langmuir plasma = 3. The boundary conditions at $x = x_0$ are

$$n = n_0, \quad u = -\lambda,$$
$$\phi = \phi_0 \quad \frac{d\phi}{dx} =: -E_0, \tag{3}$$
$$n_t = n_{to} \left\{ e(\phi_0 - \phi_m)/T_e \right\}^{1/2},$$

where λ is the phase velocity. Equations (2) are now rewritten

$$\psi - \psi_0 = \frac{1}{2} \left(\frac{\beta}{\rho^2} - 3 \right)(1 - \rho^2), \tag{4}$$

$$\frac{d^2}{d\xi^2} = \rho - 1 + \rho_{to} (\psi - \psi_m)^{1/2}, \tag{5}$$

where we have used the following normalizations:

$$\rho = \frac{n}{n_0}, \quad \rho_{to} = \frac{n_{to}}{n_0}, \quad \psi = \frac{e\phi}{T_e}, \quad \xi = \frac{x}{\lambda_D}$$

and ß = λ^2/(Te/m). An integral of eq. (5) is readily obtained:

$$\frac{1}{2} \left(\frac{d\psi}{d\xi} \right)^2 = G(\psi, \psi_m) = \beta \left(\frac{1}{\rho} - 1 \right) + \rho^3 - 1 - (\psi - \psi_0)$$
$$+ \frac{2}{3} \rho_{to} \left\{ (\psi - \psi_m)^{3/2} - (\psi_0 - \psi_m)^{3/2} \right\} + \frac{1}{2} W_0, \tag{6}$$

where $W_0 = E_0^2/4\pi n_0 Te$.

The potential structure is calculated knowing the distribution of zeros of the function G with respect to ψ . It is worthwhile to note that the density of the untrapped electrons is a double-valued function of ψ , which is easily seen from eq. (4). The schematic structure of the potential dependence of the density is shown in Fig. 1. If the phase velocity is smaller than the thermal velocity (i.e. ß < 3) the upper branch (a dotted line in Fig. 1) should be used, indicating that the thermal pressure causes electrons to show a static response to the potential perturbation. On the other hand, when the phase velocity is larger than the thermal velocity, the electrons show a dynamic response to the potential. The lower branch (a solid line in Fig. 1) should be used in our case.

These two different manners of response merge at the critical amplitude of the potential ψ_* :

$$\psi_* - \psi_0 = -\frac{1}{2}\beta\left\{1-\left(\frac{3}{\beta}\right)^{1/2}\right\}^2. \tag{7}$$

The corresponding density is

$$\rho_* = \left(\frac{\beta}{3}\right)^{1/4}. \tag{8}$$

It is this existence of a critical amplitude that causes wave-breaking when trapped particles are not taken into account.

§ 3 Wave-breaking without trapping

In this section we briefly discuss wave-breaking for the case of no trapping ($\rho_{te} = 0$). Noting the relation $dG/d\psi = \rho - 1$, we easily get a qualitative potential structure from eq. (6). From Fig. 1, in which ρ and G are depicted, the amplitude of the wave potential is determined by $\psi_M - \psi_m$, where $G(\psi_m) = G(\psi_M) = 0$. The amplitude of the electric field is, on the other hand, obtained at $\psi = \psi_0$ where $(dG/d\psi)_{\psi_0} = 0$:

$$\varepsilon^2(\rho=1) = 3\rho_*^4 f(\rho_m), \tag{9}$$

where $\rho_m = \rho(\psi_m)$ and

$$f(x) = (1-x^2)\left(\frac{1}{x^2} - \frac{1}{3\rho_*^4} - \frac{2x}{3\rho_*^4}\right). \tag{10}$$

The function $f(x)$ is concave in the upward direction with the minimum at $x = 1$, since $3\rho_*^4 = \beta > 3$ is assumed.

Wave-breaking apparently occurs at $\rho_m = \rho_*$, where the electric field amplitude is at its maximum:

$$\varepsilon^2_{max} = 3\rho_*^4 f(\rho_*) = (\rho_* - 1)^2(3\rho_*^2 - 2\rho_* - 1). \tag{11}$$

Obviously, as the temperature increases, ρ_* decreases up to unity

where the wave cannot persist. This confirms the result of Coffey and Kruer. Equation (11) shows that wave-breaking occurs when in a cold plasma limit the oscillation velocity of an electron equals the phase velocity, and that the breaking amplitude is reduced due to the finite temperature effect.

§ 4 Effect of Electron Trapping on Wave breaking

Since electrons are trapped in a potential hump where ions are relatively rich, pulling the hump down, there is a critical number of trapped particles above which the wave cannot persist. This is seen by analyzing the behavior of local charge density Q( ) which is defined as

$$Q(\rho, \rho_m) = \rho - 1 + \rho_{te}\left(\frac{3}{2}\right)^{1/2}\left\{\left(\frac{\rho_*^4}{\rho^2\rho_m^2} - 1\right)(\rho_m^2 - \rho^2)\right\}^{1/2}, \tag{12}$$

where $\rho \lesssim \rho_m$. Figure 2(a) shows the density dependence of Q, indicating a possibility for G to have such structures as depicted in Fig. 2(b). It is clear from Fig. 2 that the conditions for waves to propagate are

$$G(\psi_m, \psi_m) = G(\psi_M, \psi_m) = 0, \tag{13}$$

$$G(\psi^*, \psi_m) < 0, \tag{14}$$

$$\psi_* < \psi_m \leq \psi_0, \tag{15}$$

where ψ_M is the maximum value of the potential, and ψ^* is determined by following realtions

$$\frac{d}{d\psi} G(\psi, \psi_m)\Big|_{\psi^*} = 0 \quad \text{and} \quad \frac{d^2}{d\psi^2} G(\psi, \psi_m)\Big|_{\psi^*} > 0. \tag{16}$$

The second inequality of relations (15) is necessary for untrapped particles to sustain the wave.

There are two possible ways for breaking to occur. Wave-breaking is dominated by the thermal effect when ψ_m reaches ψ_* before the relation (14) breaks down. On the other hand trapping is essential to wave-breaking when ψ_M reaches ψ^* under the conditions (15). In this case the critical state is characterized by propagation of a negative pulse. The simplest way to see which effect is more decisive is to examine conditions given as

$$G(\psi^*, \psi_*) < 0, \tag{17}$$

where ψ^* is a solution of eq. (16) when $\psi_m = \psi_*$.

Wave-breaking is ruled by the thermal effect only when the inequality (17) holds.

When $\rho_m = \rho_*$, eq. (16) is reduced to a simple form

$$\rho^*(1-\rho^*) = \left(\frac{3}{2}\right)^{1/2} \rho_{to}(\rho_*^2 - \rho^{*2}), \tag{18}$$

which is readily solved under the second condition of eq. (16) to give

$$\rho^* = \frac{1}{2\{1-(3/2)^{1/2}\rho_{to}\}}\left\{1-\left(1+6\rho_{to}\rho_*^2(\rho_{to}-(2/3)^{1/2})^{1/2}\right)^{1/2}\right\}, \tag{19}$$

where a constraint on ρ_{to} and ρ_* is needed in order for eq. (18) to have a solution:

$$\rho_{to} < \frac{1}{6^{1/2}}\left\{1-(1-1/\rho_*^3)^{1/2}\right\}. \tag{20}$$

When relation (20) violates, the local charge Q is always positive. Inequality relation (17) now reads

$$f(\rho_*) - f(\rho^*) + \frac{2}{3}\left(\frac{1-\rho^*}{\rho_{to}}\right)^3 \rho_{to} < 0. \tag{21}$$

Conditions (20) and (21) are fulfilled in the region surrounded by the solid line and two axies in Fig. 3. In the region (b) ψ_M reaches ψ^* faster than ψ_m reaches ψ_*, meaning that wave-

breaking is ruled by trapping. The dashed curve is a boundary of the region defined by relation (20). In the region (c) the local charge is always positive, caused by the accumulation of trapped particles. It is interesting to note that even at $\beta = 3$ there can exist a wave in the form of a positive pulse. This makes a striking contrast to the case of no trapping where waves cannot persist at $\beta = 3$. Fig. 3 also shows that wave-breaking is nothing but total trapping in a cold plasma limit ($\beta \to \infty$).

The amplitude of the electric field is given as

$$\begin{aligned} E^2 &= 3\rho_*^4 f(\rho_m) + h(\hat{\rho}, \rho_*) \\ &= 3\rho_*^4\{f(\rho_m) - f(\hat{\rho})\} + \frac{4}{3}\left(\frac{1-\hat{\rho}}{\rho_{to}}\right)^3 \rho_{to}, \end{aligned} \tag{22}$$

where $\hat{\rho} = \rho(\hat{\psi})$ and $\hat{\psi}$ is determined through

$$\frac{d}{d\psi} G(\psi, \psi_m)/\hat{\psi} = 0 \quad \text{and} \quad \frac{d^2}{d\psi^2} G(\psi, \psi_m)/\hat{\psi} < 0. \tag{23}$$

With the aid of the second condition of relations (23) it is easily shown that $dh(\hat{\rho})/d\hat{\rho} < 0$ for $0 < \hat{\rho} < 1$, implying that $h(\hat{\rho}) > 0$ for $0 < \hat{\rho} < 1$, because $h(\hat{\rho}=1) = 0$. We have therefore

$$E^2 > 3\rho_*^4 f(\rho_m). \tag{24}$$

Comparing this with eq. (9), we may conclude that the electric field amplitude is always larger than that obtained for the case of no trapping, if the boundary conditions remain the same. This is interpreted as follows. An increase in amplitude of the wave potential causes the accumulation of the trapped electrons which, in turn, distorts the wave profile in such a way that the potential hump is thrashed down and its trough is deepened, resulting in steepening of the potential profile. The electric field amplitude is, thus, enhanced.

§ 5 Discussion

The effect of trapped particles on wave-breaking of a stationary Langmuir oscillation is analized in a homogeneous plasma, where trapping is well-defined so long as mixing of bouncing motions can be neglected. A phenomenplogical model is used for the trapped particles, whose distribution function in velocity space is assumed to be constant in a range of trapping. The significant effect of trapping is to make the wave profile steep and to enhance the breaking amplitude of the electric field. In a course of analysis, φ_t and ß are regarded as given parameters, though they are, in reality, related to each other. Primitive linear theory yields a relation as $\varphi_t = \exp(-ß/2)$, which might be drastically modified for a large amplitude wave because of the exponential dependence of the distribution function on velocity. Although we need a kinetic treatment for trapped particles, our model may be considered to describe qualitative characteristics of phenomena involving trapped particles.

So far we have considered the wave propagation in a homogeneous system. In an inhomogeneous system wave-particle resonant interactions may again play a crucial role. In this case trapping is no longer well-defined since trapped particles can escape from a trapped region. They can carry a large part of the field energy away, resulting in catastrophic damping of the wave. This could be responsible for the hot electron generation associated with resonant absorption, which will be reported separately.

Acknowledgement:

The authors would like to thank Dr. S. Witkowski for encouragement and his continuous interest in this work.

References

(1) J.M. Dawson, Phys. Rev. Letters 113, 383 (1959)

(2) P. Koch and J. Albritton, Phys. Rev. Letters 32, 1420 (1974)

(3) T.P. Coffey, Phys. Fluids 14, 1402 (1971)

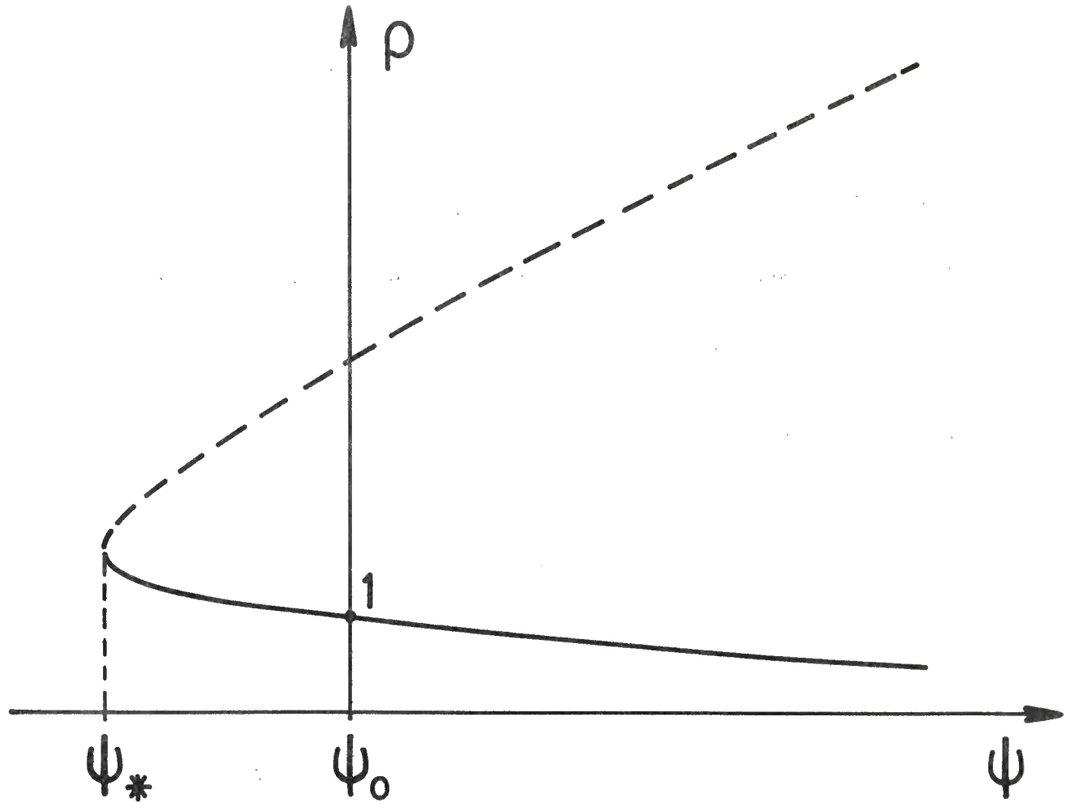
(4) W.L. Kruer, Phys. Fluids 22, 111 (1979)

Figure captions

Fig. 1 Electron density φ and function G versus Ψ for the case of $ß > 3$ and no trapping.

Fig. 2 Local charge Q and function G versus φ and Ψ, respectively.

Fig. 3 Region in parameter space (φ_t , ß) where wave-breaking is dominated by the thermal effect (a) and trapping (b) when $\varphi_m = \varphi_*$ and (c). Conditions (20) and (21) are fulfilled in the region (b), while in the region c the local charge density is always positive.



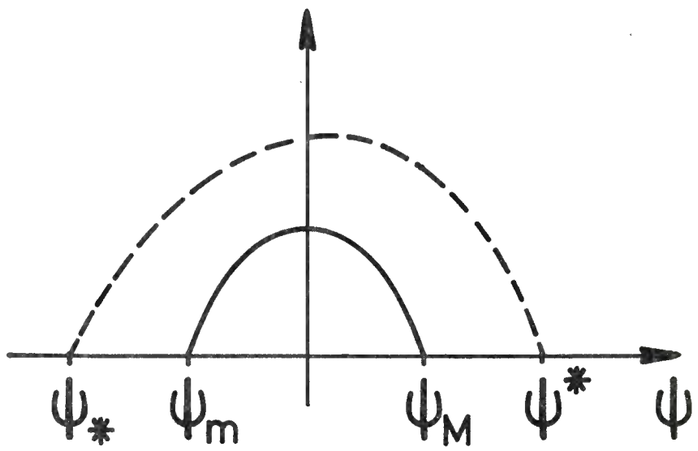
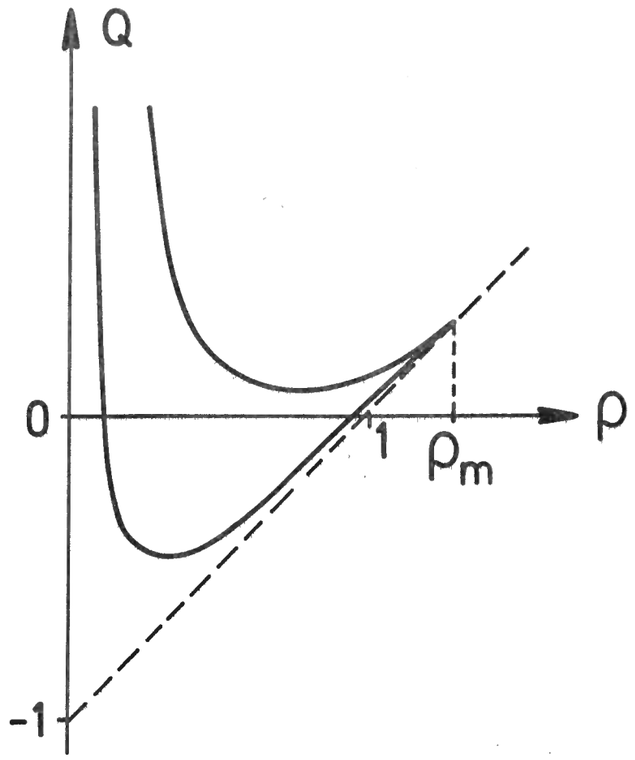
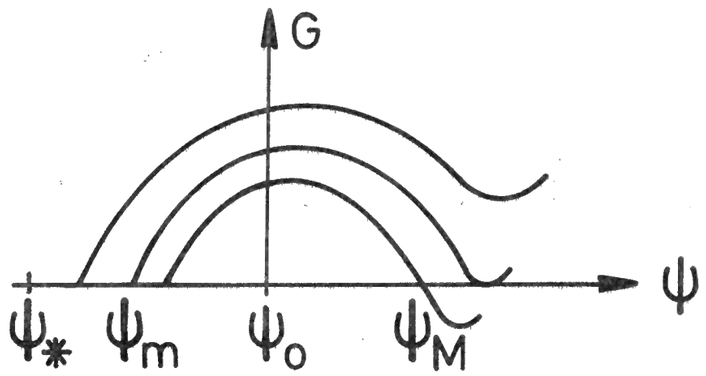


Fig. 1



a)



b)

Fig. 2

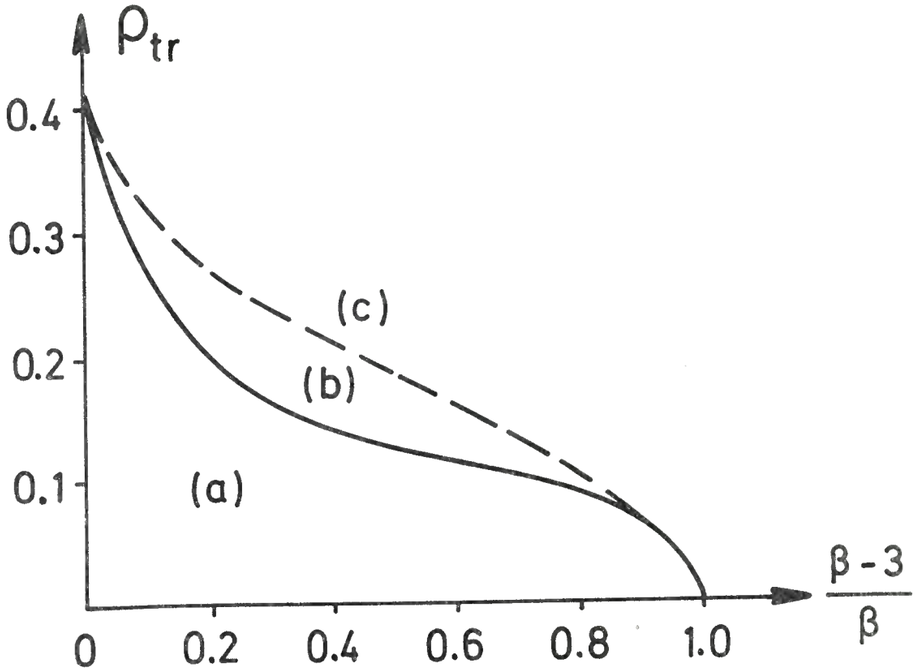


Fig. 3