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MPQ-Report
110

MPQ 110

February 1986

Max-Planck-Institut für Quantenoptik
8046 GARCHING bei MÜNCHEN, Bundesrepublik Deutschland

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A computer code for one-dimensional multigroup radiation hydrodynamics

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Abstract

This report describes the basic physical equations as well as a computer code for the simulation of radiation hydrodynamics. The hydrodynamic equations are combined with a multigroup method for the radiation transport. The code, written in standard FORTRAN-77, is characterized by one-dimensional planar geometry with multilayer structure. A time-splitting scheme has been adopted with implicit finite-difference formulation (including hydrodynamics) in Lagrangean coordinates. Tabulated equations of state and opacities are used.

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This work was supported in part by the Bundesministerium für Forschung und Technologie and by the Commission of the European Communities in the framework of the Euratom/IPP Association.

1. Introduction

At the extreme thermodynamic conditions found in inertial confinement fusion (ICF) and related experiments, energy transport by thermal radiation plays an important role [1,2]. The radiation field is strongly coupled to the hydrodynamic motion through emission and absorption phenomena, both essentially frequency dependent. In order to achieve a thorough understanding of the processes going on in such situations a numerical simulation code has been developed. This report describes the physical model and the numerical methods used, and gives also a detailed description of the internal functioning of the code.

The code is called **MULTI** (MULTIgroup radiation transport in MULTIlayer foils) and solves the one-dimensional planar hydrodynamic equations coupled to the radiation transfer equation.

The equations used in the code are the first terms in the expansion of the radiation hydrodynamic equations in the small parameter v/c (characteristic velocity / light velocity). This means that relativistic effects as well as the time derivatives in the transfer equation are not taken into account. Obviously, this imposes limits on the area of applicability of the code. However, this is not a major shortcoming because in a wide range of situations the matter velocity is really well below the light velocity.

The frequency and angle variables in the equation of transfer are handled by a multigroup method which amounts to a discretization of these variables [3,4]. In this respect, the present treatment goes beyond the usual 'grey' approximation [5,6].

In addition, the code can use the grey description of the radiation, as a particular case in which the number of groups is one. This ability has been intensively used to test the code. The results were compared with that of the grey-approximation code **MINIRA** [7]. They agree within an error of one percent.

The electronic heat conduction is treated in a rather rough way (classical Spitzer's conductivity without a flux limit) [8]. This can be justified because this is not the main concern of the code. However, further code development in this direction is planned for the future.

The present version of the code includes laser absorption by anomalous mechanisms and inverse Bremsstrahlung absorption [9]. Other forms of energy deposition can be easily implemented by changing the appropriate routines.

The properties of the matter are given through tabulated equations of state (usually taken from the SESAME library [10]) and tabulated opacities.

A time-splitting scheme is used; the physical phenomena are treated successively during the time step. The numerical stability is guaranteed by using a fully implicit method in every substep [11]. Although a time-splitting scheme has only a first order accuracy, several advantages justify its use, namely

i) Since the hydrodynamic equations are solved implicitly, the Courant limit in the time step usually found in the explicit schemes [12] , can be exceeded.

ii) The necessary numerical work increases only linearly with the number of photon-groups (in simultaneous resolution schemes this increase occurs quadratically).

iii) A modular design can be easily implemented.

The code is written in standard FORTRAN-77 and it consists of 33 program units with a total of 1304 statements. LINPACK library routines [13] have been applied to solve the linear system of equations. The vectorization capability of the CRAY-1 computer has been intensively taken into account. The only non-standard features are the call to the system routines **DNAME0** and **ERREXIT**, these can be easily replaced by others or simply eliminated.

2. Physical Model

2.1 Radiation Transfer Equation

The equation of transfer, also called the transport equation, is the mathematical statement of the conservation of photons. In quite general form it is given by

$$(\frac{1}{c}\partial_t + \vec{n}.\nabla)I(\vec{r},\vec{n},\nu,t) = \eta(\vec{r},\vec{n},\nu,t) - \chi(\vec{r},\vec{n},\nu,t)I(\vec{r},\vec{n},\nu,t) \qquad (2.1)$$

$I(\vec{r},\vec{n},\nu,t)$ is the specific intensity of radiation of frequency ν at a position $\vec{r}$

travelling in direction $\vec{n}$ at time t, η is the total emissivity and χ is the total opacity. The term on the right hand side is the effective rate of energy emission (emission minus absorption) by the matter per unit of volume, frequency and solid angle. The relationship between the photon momentum $\vec{p}$ and its energy e is given by: $\vec{p} = (e/c)\vec{n}$. Consequently the specific rate of momentum emission is $((\eta - \chi I)/c)\,\vec{n}$. The total emission rates per unit volume of energy Q and momentum $\vec{R}$ are obtained by integrating over all frequencies and directions

$$Q = \int_0^\infty \int_{4\pi} (\eta - \chi I)\, d\vec{n}\, d\nu \qquad (2.2)$$

$$\vec{R} = \int_0^\infty \int_{4\pi} \frac{(\eta - \chi I)}{c}\, \vec{n}\, d\vec{n}\, d\nu \qquad (2.3)$$

Under the assumptions that scattering is not important and Kirchhoff's law is valid the emissivity may be written as

$$\eta(\vec{r},\vec{n},\nu,t) = \chi(\vec{r},\vec{n},\nu,t) I_p(T(\vec{r},t),\nu) \qquad (2.4)$$

Where T is the matter temperature and I_p is the Planckian specific intensity of radiation in LTE (Local thermodynamic equilibrium) with matter

$$I_p(T,\nu) = \frac{2h\nu^3}{c^2}(e^{\frac{h\nu}{kT}} - 1)^{-1} \qquad (2.5)$$

The velocity of the matter is assumed to be small in comparison with the light velocity. Consequently the opacity can be considered as isotropic (this is equivalent to neglecting the Doppler effect). In addition the opacity is assumed to depend only on the frequency and the thermodynamic properties of the matter: temperature T and density ρ. This is obviously correct for thermodynamic equilibrium, and is the simpler choice for more complicated situations.

$$\chi(\vec{r},\vec{n},\nu,t) = \chi(T,\rho,\nu,N) \qquad (2.6)$$

$N(\vec{r},t)$ represents the matter composition i.e. a number associated to a specific material. The order of magnitude of the time derivative in (1) is I_c/t_c whereas the convective derivative is of the order I_c/ℓ_c, with I_c, t_c and ℓ_c being characteristic values of radiation specific intensity, time and length respectively. The assumption

of small enough matter velocity ($\ell_c/t_c \ll c$) implies then that the time derivative can be dropped in (1). With all those simplifications the equation of transfer, in unidimensional planar geometry reads

$$\mu\,\partial_x I(x,\mu,\nu,t) = \chi(T,\rho,\nu,N)(I_p(T,\nu) - I(x,\mu,\nu,t)) \tag{2.7}$$

Where μ is the cosine of the angle between the photon direction and the x axis, and T, ρ and N are functions of x and t. Analogously the relations (2) and (3) take the form

$$Q(x,t) = \int_0^\infty \chi(T,\rho,\nu,N)[4\pi I_p(T,\nu) - 2\pi \int_{-1}^{1} I(x,\mu,\nu,t)\,d\mu]\,d\nu \tag{2.8}$$

$$R_x(x,t) = \frac{2\pi}{c}\int_0^\infty [\chi(T,\rho,\nu,t)\int_{-1}^{1} I(x,\mu,\nu,t)\,\mu\,d\mu]\,d\nu \tag{2.9}$$

$$R_y(x,t) = R_z(x,t) = 0 \tag{2.10}$$

2.2 Hydrodynamic Equations

The fluid motion is governed by a set of three equations that state the conservation of mass, momentum and internal energy

$$D_t\,\rho = -\rho\nabla.\vec{v} \tag{2.11}$$

$$\rho D_t\,\vec{v} = -\nabla P - \vec{R} \tag{2.12}$$

$$\rho D_t\,e = -P\nabla.\vec{v} - \nabla.\vec{q} - Q + S \tag{2.13}$$

The main variables matter density $\rho(\vec{r},t)$, velocity $\vec{v}(\vec{r},t)$, specific internal energy $e(\vec{r},t)$ and pressure $P(\vec{r},t)$ are considered here to be functions of coordinate and time. D_t is the time derivative in a frame moving with the fluid velocity: ($D_t \equiv \partial_t + \vec{v}.\nabla$). $\vec{R}$ and Q are the radiated momentum and energy per unit volume, respectivelly. $\vec{q}$ is the thermal flux and S includes other energy sources like laser or ion beam energy deposition. The Eulerian fluid equations for the one-dimensional

planar case can be obtained easily from the system (11–13), but instead it proves convenient to use the Lagrangean formulation. The Lagrangean coordinate is defined here by

$$m(x,t) = \int_{-\infty}^{x} \rho(x',t)\,dx' \tag{2.14}$$

m is in fact the total mass per unit area at the left of the considered point. The system (11–13) becomes in planar geometry

$$\partial_t\rho = -\rho^2\partial_m v \tag{2.15}$$

$$\partial_t v = -\partial_m(P_{eq} + P_{vis}) \tag{2.16}$$

$$\partial_t e = -(P_{eq} + P_{vis})\partial_m v - \partial_m q - \frac{Q}{\rho} + \frac{S}{\rho} \tag{2.17}$$

P_{vis} is an artificial viscous pressure that must be included in order to achieve numerical stability and

$$P_{eq} = P_{eq}(\rho,e,N) \tag{2.18}$$

is the matter pressure that is assumed to depend only on the thermodynamic variables. Diffusive mixing of different materials is assumed to be not important, consequently the matter composition N depends only on m.

Although the temperature does not enter explicitly in the fluid equation, it is needed in the radiation transfer and thermal flux equations. In analogy to the pressure the temperature is given by

$$T = T_{eq}(\rho,e,N) \tag{2.19}$$

It must be noticed that the radiation momentum emission has been dropped in (16). This is consistent with the terms which are also dropped in the radiation transfer equation. The equations used in the code are, in fact, the first order equations in a hierarchy of equations obtained by developing in powers of the small factor v_c/c, where v_c is the characteristic velocity ($\equiv \ell_c/t_c$).

2.3 Heat Flux

Most of the enery transported by heat flux is carried away by the electrons. In the quasi-equilibrium limit (small temperature gradients) the heat flux is proportional to the temperature gradient according to Spitzer's formula

$$q = -\bar{K}\, T^{5/2}\, \partial_x T \tag{2.20}$$

where $\bar{K}$ is given by

$$\bar{K} = \frac{10.16\, \epsilon\delta_t\, k^{7/2}}{\sqrt{m_e}\, e^4\, Z_i\, \log\Lambda} \tag{2.21}$$

with

$$\epsilon\delta_t \cong 0.95\, \frac{Z_i + 0.24}{1 + 0.24 Z_i} \tag{2.22}$$

being k Boltzmann's constant, m_e and e the electronic mass and charge, Z_i the effective ion number and $\log\Lambda$ Coulomb's logarithm. The last two quantities are assumed to be constants. This version of the thermal heat flux is rather poor. Nevertheless the program is written in such a way that it can be easily modified in order to include a Z_i dependence on the thermodynamic variables, a maximum value for the thermal flux (flux-limiter) and so on.

2.4 Laser Deposition

The laser beam is assumed to come from the right ($x \to \infty$). Electromagnetic waves propagate in a plasma only when the density is below the critical value . If this density is reached at some point they are reflected. Thus, in general, there are two laser beams, one incident and one reflected whose respective intensities $I_+(x,t)$ and $I_-(x,t)$ are governed by the equations

$$\partial_x I_+ = \kappa I_+ \tag{2.23}$$

$$\partial_x I_- = -\kappa I_- \tag{2.24}$$

where κ is the attenuation coefficient. If inverse Bremsstrahlung is assumed to be the attenuation mechanism, this coefficient is given by

$$\kappa = C \frac{1}{T^{3/2}} \left(\frac{\rho}{\rho_c}\right)^2 \frac{1}{\sqrt{1 - \frac{\rho}{\rho_c}}} \tag{2.25}$$

The critical density ρ_c is determined by $\rho_c = n_c m_i / Z_i$. Here m_i and Z_i are the ion mass and charge number, respectively and n_c the electron number density given by the condition that the plasma frequency equals the laser frequency ν_L

$$n_c = \frac{\pi\, \nu_L^2\, m_e}{e^2} \tag{2.26}$$

On the other hand, the constant C is given by

$$C = \frac{16\pi\, Z_i\, n_c^2\, e^6\, \log\Lambda}{3\, c^2\, \nu_L^2\, (2\pi\, m_e\, k)^{3/2}} \tag{2.27}$$

It must be noticed that the equations (23, 24) become singular when $\rho = \rho_c$.

The incident laser intensity $I_+(\infty, t)$ is assumed to be known and consequently equation (23) can be integrated from $x = \infty$ to the point x_c where $\rho = \rho_c$. In this way the value $I_+(x_c, t)$ can be achieved. A given fraction αI_+ is absorbed there and the rest is reflected ($I_-(x_c, t) = (1 - \alpha) I_+(x_c, t)$). Starting from this value equation (24) can be integrated from x_c to ∞.

In the case that the density is everywhere below the critical, (23) can be integrated from ∞ to $-\infty$ and (24) implies $I_-(x,t) = 0$ because $I_-(\infty, t) = 0$ (there is no laser incident on the left hand side)

Once I_+ and I_- are known the deposition term is given by

$$S = \partial_x I_+ - \partial_x I_- \tag{2.28}$$

The code includes also the possibility that the laser incidents from the left hand side; the discussion is analogous.

2.5 Other Energy Sources

There is the possibility to modify the code to include other forms of energy deposition. Only the routines **QUELLE** and **QUELIN** must be changed.

2.6 Boundary Conditions

The preceding equations must be complemented by appropriate boundary conditions. In planar geometry the problem is confined between two limiting planes, at the left and at the right sides, whose position change with time. However, their Lagrangean mass coordinates, denoted by m_L and m_R, respectively, are constant. ($m_L = 0$ because there is no mass to the left of the left boundary, and m_R ,the total mass per unit area, is constant if the mass is conserved).

First the specific radiation intensity $I(m, \mu, \nu, t)$ will be considered. For positive μ this function represents the intensity of radiation travelling from left to right. Consequently the natural boundary conditions must be , in this case, imposed on the left boundary m_L. Once these are known, (7) can be integrated and the values at the right boundary determined. Conversely, the boundary conditions for I with negative μ must by imposed at m_R. Some of the possible combinations at the left hand side are (for $\mu > 0$)

No incident radiation

$$I(m_L, \mu, \nu, t) = 0 \tag{2.29}$$

Incident Planckian radiation at temperature T_L

$$I(m_L, \mu, \nu, t) = I_p(T_L, \nu) \tag{2.30}$$

Specular reflection

$$I(m_L, \mu, \nu, t) = I(m_L, -\mu, \nu, t) \tag{2.31}$$

Partial specular reflection

$$I(m_L, \mu, \nu, t) = \alpha_L \, I(m_L, -\mu, \nu, t) \quad (0 < \alpha_L < 1) \tag{2.32}$$

The third condition can be used in the case of a symmetric 'Hohlraum', that is two symmetric layers, each of them receiving the radiation emitted by the other. The program actually carries out only one layer computations, the plane of symmetry is modeled as a mirror. The fourth condition takes into account some possible losses in the 'Hohlraum', i.e. non-planar effects, holes in the layer, and so on.

It must be noted that the second and third condition are particular cases of the fourth one for $\alpha_L = 0$ and $\alpha_L = 1$, respectively. That is actually the condition implemented in the code. The boundary conditions on the right hand side are completely analogous. The corresponding 'reflection factor' is denoted by α_R.

The boundary conditions for the hydrodynamics take the usual form. At a free surface the pressure must be zero, while the density, specific internal energy and velocity can take in principle arbitrary values (In the case of a gas this condition implies also zero density). At a rigid wall the velocity must be zero whereas all the other variables can take arbitrary values. The input variable **IFLAG2** controls these conditions. It must be set to 1 to have free left and right surfaces and 0 for rigid walls.

In addition a condition for the thermal flux is needed. It is assumed always to have isolated boundaries $q(m_L, t) = q(m_R, t) = 0$.

Although the program manages composed layers there are no explicit boundary conditions at the interfaces. Instead a matter composition function $N(m)$ (usually taking integer values) is given, which enters as a parameter in the opacity and equation of state.

3. Multigroup Radiation Model

3.1 Introduction

The radiation transfer equation can be rewritten as

$$\mu \, \partial_m I(m, \mu, \nu, t) = \kappa(T, \rho, \nu, N)(I_p(T, \nu) - I(m, \mu, \nu, t)) \tag{3.1}$$

Now the radiation intensity is considered to depend on the Lagrangean coordinate. The quantity κ ($\equiv \chi/\rho$) is the opacity expressed in units of surface per mass. The above equation is obviously very complicated; the specific intensity depends on four variables. Careless discretization can easily lead to an enormous amount of computational work or a substantial loss of accuracy. The approach adopted here carries out the discretization in two steps. In the first the equation (1) is replaced by its integrals over the variables ν and μ in a finite number of domains called 'groups'. This procedure leads to a finite number of differential equations

involving a finite number of variables depending only on m and t. In the second step, discussed in the next sections, this set of equations together with the fluid, thermal flux and laser equations is discretized in a computational mesh in the m, t space, generating finite difference equations.

3.2 Group Definition

Let us consider the set of possible pairs of values (ν, μ) of frequency and cosine $(0 < \nu < \infty, -1 < \mu < 1)$. This set can be partitioned in a finite number **NG** of subsets that will be called here 'groups' (although properly speaking a 'group' is formed by the photons whose frequency and cosine belong to such a subset). The groups considered here are defined by two boundary frequencies $(\nu_a^k < \nu_b^k)$ and two boundary cosines $(0 < \mu_a^k < \mu_b^k)$, where the superindex k stands for the number of the group. The group k is thus the set of photons whose frequency verifies $\nu_a^k < \nu < \nu_b^k$ and whose cosine verifies $\mu_a^k < \mu < \mu_b^k$ (photons travelling to the right) or $-\mu_b^k < \mu < -\mu_a^k$ (travelling to the left). It is assumed, of course, that the groups do not overlap and that they cover the whole ν, μ space.

3.3 Group Equations

It proves convenient to introduce first a few definitions. If f is an arbitrary function of the variables m, ν, μ and t, it can be transformed into a function of m and t only through the use of the integral operators $\mathbf{L}_+^k$, $\mathbf{L}_-^k$ and $\mathbf{L}^k$ defined by

$$\mathbf{L}_+^k(f) = 2\pi \int_{\mu_a^k}^{\mu_b^k} \int_{\nu_a^k}^{\nu_b^k} f(m,\nu,\mu,t)\, d\nu\, d\mu \tag{3.2}$$

$$\mathbf{L}_-^k(f) = 2\pi \int_{-\mu_b^k}^{-\mu_a^k} \int_{\nu_a^k}^{\nu_b^k} f(m,\nu,\mu,t)\, d\nu\, d\mu \tag{3.3}$$

$$\mathbf{L}^k(f) = \mathbf{L}_+^k(f) + \mathbf{L}_-^k(f) \tag{3.4}$$

The last of these operators is the integration over all the frequencies and angles in the the group k. The density of energy U_k and the energy flux S_k of the photons belonging to the group k can be expressed as

$$U_k = \frac{1}{c}\mathbf{L}^k(I) \tag{3.5}$$

$$S_k = \mathbf{L}^k(\mu I) \tag{3.6}$$

Analogously the rate of energy emission per volume by the photons of this group is given by

$$Q_k = \mathbf{L}^k(\chi(I_p - I)) \tag{3.7}$$

and consequently the total rate is computed adding all the groups

$$Q = \sum_{1 \le k \le \mathbf{NG}} Q_k \tag{3.8}$$

This expression coincides with the equation (2.8). The following properties of these operators will be needed below: if g is some function of m, ν and t but not dependent on μ, it is verified that

$$\mathbf{L}_+^k(g) = \mathbf{L}_-^k(g) = \frac{1}{2}\mathbf{L}^k(g) \tag{3.9}$$

$$\mathbf{L}_+^k(\mu g) = -\mathbf{L}_-^k(\mu g) = \frac{1}{4}(\mu_a^k + \mu_b^k)\mathbf{L}^k(g) \tag{3.10}$$

$$\mathbf{L}_+^k(\mu^2 g) = \mathbf{L}_-^k(\mu^2 g) = \frac{1}{6}(\mu_a^{k\,2} + \mu_a^k \mu_b^k + \mu_b^{k\,2})\mathbf{L}^k(g) \tag{3.11}$$

In order to obtain the group-k equations, equation (1) is integrated separately over the photons travelling to the rigth and over the photons travelling to the left. This is done by applying the operators $\mathbf{L}_+^k$ and $\mathbf{L}_-^k$, respectively

$$\partial_m \mathbf{L}_+^k(\mu I) = \mathbf{L}_+^k(\kappa I_p) - \mathbf{L}_+^k(\kappa I) \tag{3.12}$$

$$\partial_m \mathbf{L}_-^k(\mu I) = \mathbf{L}_-^k(\kappa I_p) - \mathbf{L}_-^k(\kappa I) \tag{3.13}$$

In order to make these equations useful, it is necessary to make some reasonable assumptions about the form of the function I. Between the wide range of possible choices, I is selected such that one is able, with a reduced number of groups, to manage situations near the thermodynamical equilibrium with a reasonable

accuracy. In these situations $I \simeq I_p$ and applying perturbation methods to (1) results in

$$I = I_p - \mu \frac{I'_p}{\kappa} \partial_m T + \cdots \tag{3.14}$$

Where I_p and I'_p ($\equiv \partial I_p / \partial T$) are functions of ν and T only. This equation suggests to assume for the group k the following form of I

$$I(m, \mu, \nu, t) = \alpha_k(m,t) I_p(T,\nu) + \mu \beta_k(m,t) \frac{I'_p(T,\nu)}{\kappa(T,\rho,\nu,N)} \tag{3.15}$$

α_k and β_k being the functions that describe completely the group. If there is local quasi-equilibrium they take the values 1 and $-\partial_m T$, respectively; this makes it possible to manage correctly situations with one group only. On the other hand, when the system is far away from the equilibrium, the present assumption is not worse as any other;in this case the accuracy can be reached only by taking a large number of groups. Using the properties (9–11) the energy density and flux expressions take now the form

$$U_k = \frac{1}{c} \alpha_k \mathbf{L}^k(I_p) \tag{3.16}$$

$$S_k = \frac{\beta_k}{3} (\mu_a^{k\,2} + \mu_a^k \mu_b^k + \mu_b^{k\,2}) \mathbf{L}^k(I'_p/\kappa) \tag{3.17}$$

Whereas the equations (12, 13) can be rewritten as

$$\partial_m \left(\pm \frac{\mu_a^k + \mu_b^k}{4} \alpha_k \mathbf{L}^k(I_p) + \frac{\mu_a^{k\,2} + \mu_a^k \mu_b^k + \mu_b^{k\,2}}{6} \beta_k \mathbf{L}^k(I'_p/\kappa) \right)$$

$$= \frac{1}{2} \mathbf{L}^k(\kappa I_p) - \frac{\alpha_k}{2} \mathbf{L}^k(\kappa I_p) \mp \frac{\mu_a^k + \mu_b^k}{4} \beta_k \mathbf{L}^k(I'_p) \tag{3.18}$$

Now these equations are linearly combined adding and substracting each other; after some straightforward algebra this results in

$$\partial_m S_k(m,t) = c\, \kappa_P^k(\rho,T,N)(U_{pk}(T) - U_k(m,t)) \tag{3.19}$$

$$g_k^2\, c\, \partial_m U_k(m,t) = -\kappa_R^k(\rho,T,N) S_k(m,t) \tag{3.20}$$

Where U_{pk} is the energy density that would occur if the specific intensity was given by a Planckian function at the matter temperature

$$U_{pk}(T) = \frac{4\pi(\mu_b^k - \mu_a^k)}{c} \int_{\nu_a^k}^{\nu_b^k} I_p(T,\nu)\, d\nu \tag{3.21}$$

The values κ_P^k and κ_R^k are usually named Planck and Rosseland mean opacities and are defined by

$$\kappa_P^k = \int_{\nu_a^k}^{\nu_b^k} \kappa I_p\, d\nu \Big/ \int_{\nu_a^k}^{\nu_b^k} I_p\, d\nu \tag{3.22}$$

$$\kappa_R^k = \int_{\nu_a^k}^{\nu_b^k} I'_p\, d\nu \Big/ \int_{\nu_a^k}^{\nu_b^k} \frac{I'_p}{\kappa}\, d\nu \tag{3.23}$$

The constant g_k^2 plays the role of the Eddington factor for the group and is given by

$$g_k = \sqrt{\frac{\mu_a^{k\,2} + \mu_a^k \mu_b^k + \mu_b^{k\,2}}{3}} \tag{3.24}$$

Finally, equation (7) can be rewritten as

$$Q_k(m,t) = c\, \rho(m,t)\, \kappa_P^k(\rho,T,N)\, (U_{pk}(T) - U_k(m,t)) \tag{3.25}$$

3.4 BOUNDARY CONDITIONS

As had been pointed out, every group is composed of photons traveling in two directions. Let us consider the partial energy densities U_k^+ and U_k^- of the photons traveling to the right and to the left, respectively, and also the corresponding energy fluxes S_k^+ and S_k^-.They are given by

$$U_k^\pm = \frac{1}{c} \mathbf{L}_\pm^k(I) = \frac{1}{2} U_k \pm \frac{\mu_a^k + \mu_b^k}{4 c g_k^2} S_k \tag{3.26}$$

$$S_k^\pm = \mathbf{L}_\pm^k(I) = \frac{1}{2} S_k \pm \frac{\mu_a^k + \mu_b^k}{4} c U_k \tag{3.27}$$

The boundary conditions (2.32) can be written in terms of energy fluxes instead of specific intensities. Multiplying by μ and carrying out the integration over frequencies between ν_a^k and ν_b^k and cosines between μ_a^k and μ_b^k results in

$$S_k^+(m_L,t) = -\alpha_L S_k^-(m_L,t) \tag{3.28}$$

Taking into account the previous expressions it becomes

$$S_k(m_L,t) = -\frac{\mu_a^k+\mu_b^k}{2}\left(\frac{1-\alpha_L}{1+\alpha_L}\right) c\, U_k(m_R,t) \tag{3.29}$$

Analogously, the corresponding condition at the right boundary is

$$S_k(m_R,t) = \frac{\mu_a^k+\mu_b^k}{2}\left(\frac{1-\alpha_R}{1+\alpha_R}\right) c\, U_k(m_R,t) \tag{3.30}$$

3.5 Alternative Model

Besides the above outlined model, it is interesting to consider other possible assumptions for the specific intensity. One simple choice is, for the group k, given by

$$I_k(m,\mu,\nu,t) = \alpha_k^+(m,t)\, I_p(T,\nu), \qquad (\mu>0) \tag{3.31}$$

$$I_k(m,\mu,\nu,t) = \alpha_k^-(m,t)\, I_p(T,\nu), \qquad (\mu<0) \tag{3.32}$$

Proceeding analogously as in the previous sections the equations for the groups become

$$\partial_m S_k(m,t) = c\,\kappa_P^k(\rho,T,N)(U_{pk}(T) - U_k(m,t)) \tag{3.33}$$

$$g_k'^{\,2}\, c\,\partial_m U_k(m,t) = -\kappa_P^k(\rho,T,N) S_k(m,t) \tag{3.34}$$

where the Eddington factor $g_k'^{\,2}$ is given now by

$$g_k' = \frac{\mu_a^k+\mu_b^k}{2} \tag{3.35}$$

On the other hand, the boundary conditions are the same, provided that g_k' is used instead g_k. These equations can be compared with the system (19, 20). There are two differences, namely: the appearance of the Planck opacity instead of the Rosseland opacity and the different expression for the factor g_k' in (34). Nevertheless, both descriptions are equivalent. In fact, they coincide when the size of the groups is made arbitrarily small. That is: $\kappa_R^k \to \kappa_P^k$ and $g_k' \to g_k$ when $\nu_a^k \to \nu_b^k$ and $\mu_a^k \to \mu_b^k$. The advantage of the previous model had been already pointed out. On the other hand, the alternative model has an interesting property: the equations (33–34) can be linearly combined giving

$$\partial_m(S_k + g_k' c U_k) = c\kappa_P^k U_{pk} - \frac{\kappa_P^k}{g_k'}(S_k + g_k' c U_k) \tag{3.36}$$

while the boundary condition in m_L implies $S_k + g_k' c U_k > 0$ there, because α_L must be less than unity. The above equation implies that $(S_k + g_k' c U_k) > 0$ everywhere (otherwise, starting from positive values of $S_k + g_k' c U_k$ at the left hand side implies that as soon as this quantity changes sign the slope needs to be negative, in contradiction to the above equation). Consequently $S_k < g_k' c U_k$. Analogously the condition $S_k > -g_k' c U_k$ is obtained. Thus the alternative model supplies a natural 'flux-limit' without the need of ad-hoc assumptions. The program is written in such a way that both models can be easily used; instead of computing g_k, the program reads it as a basic parameter of the group, whereas both Planck and Rosseland opacities can be forced to be equal loading the same tables.

3.6 Specific Intensity

Although the equations actually solved by the code have been already given in the previous sections, it is interesting to write down expressions for the specific intensity of the radiation that could be used to display the results. This is done assuming two constant intensities $\langle I_k^+\rangle$ and $\langle I_k^-\rangle$ for the photons traveling in two directions in every group. They are defined by the condition that by integrating over their respective half-groups the energy densities U_k^+ and U_k^- are obtained. This gives

$$\langle I_k^\pm\rangle = \frac{c\, U_k^\pm}{2\pi(\mu_b^k-\mu_a^k)(\nu_b^k-\nu_a^k)} \tag{3.37}$$

Obviously, these expressions make only sense when the number of groups is large enough.

4. Spatial Discretization

4.1 Introduction

Once the radiation transfer equation has been replaced by a finite set of group equations, the only independent variables are the mass coordinate m and the time t. The discretization of the system is carried out in two steps. First the spatial operators are replaced by finite-difference operators; the equations thus become a system of ordinary differential equations with the time as independent variable. In the second step this system is again discretized in time; a set of algebraic equations are obtained. This section is devoted to the first step.

In terms of the Lagrangean variable m, the matter (and thus the problem) is confined between the boundaries m_L and m_R. Let us divide this interval in N subintervals, called 'cells', not necessarily equal. Each of them will be referenced by the index i, that increases from left to right; $i = 1$ corresponds to the leftmost cell and $i = \mathrm{N}$ to the rightmost. The thickness of the subinterval itself is denoted by Δm_i. The cut points will be called 'interfaces'. Obviously there are $\mathrm{N}+1$ interfaces; their index i varies from 1 (left boundary) to $\mathrm{N}+1$ (right boundary). With this notation the cell i lies between the interfaces i and $i+1$ while the interface i $(2 \le i \le \mathrm{N})$ lies between the cells $i-1$ and i. The advantage of this notation is that it corresponds directly to the index on the storage arrays used by the program.

Here, as customary, the quantities that imply a flux (velocity, laser intensity, heat flux and radiation flux) are assumed to be known on the interfaces whereas all the others are assumed to be known in the cells. They are usually named interface-centered and cell-centered quantities, respectively. The finite diference equations are obtained in a rather straightforward way; only special refinements are needed for the laser deposition.

Although the equations are coupled, they are presented separately.

4.2 Fluid Equations

The equations for mass, momentum and internal energy conservation take the form

$$\frac{d\rho_i}{dt} = -\rho_i^2 \frac{v_{i+1} - v_i}{\Delta m_i}, \quad (i = 1, \ldots, \mathrm{N}) \tag{4.1}$$

$$\frac{dv_i}{dt} = -\frac{P_i - P_{i-1}}{\frac{1}{2}(\Delta m_i + \Delta m_{i-1})}, \quad (i = 2, \ldots, \mathrm{N}) \tag{4.2}$$

$$\frac{de_i}{dt} = -P_i \frac{v_{i+1} - v_i}{\Delta m_i} - \frac{q_{i+1} - q_i}{\Delta m_i} - \left(\frac{Q}{\rho}\right)_i + \left(\frac{S}{\rho}\right)_i, \quad (i = 1, \ldots, \mathrm{N}) \tag{4.3}$$

In addition, two equations are needed for the velocities at the boundaries. For free surfaces they read

$$\frac{dv_1}{dt} = -\frac{2P_1}{\Delta m_1}, \quad \frac{dv_{\mathrm{N}+1}}{dt} = \frac{2P_{\mathrm{N}+1}}{\Delta m_{\mathrm{N}+1}} \tag{4.4}$$

On the contrary, for rigid walls

$$\frac{dv_1}{dt} = \frac{dv_{\mathrm{N}+1}}{dt} = 0 \tag{4.5}$$

The pressure is given by the static term plus the artificial viscosity term

$$P_i = P_{eq,i} + a_v^2 \, \rho_i \min(0, v_{i+1} - v_i)^2, \quad (i = 1, \ldots, \mathrm{N}) \tag{4.6}$$

Where a_v is a dimensionless parameter that represents the number of cells needed to describe numerically the shock waves; typically $a_v = 2$. The thermal flux q_i, specific radiated power $(Q/\rho)_i$ and specific laser power deposition $(S/\rho)_i$ are given in the following sections.

4.3 Thermal Flux

It takes the form

$$q_i = -\frac{2}{7} \bar{K} \, (\rho_i + \rho_{i-1}) \left(\frac{T_i^{7/2} - T_{i-1}^{7/2}}{\Delta m_i + \Delta m_{i-1}} \right), \quad (i = 2, \ldots, \mathrm{N}) \tag{4.7}$$

While the boundary conditions read

$$q_1 = q_{\mathrm{N}+1} = 0 \tag{4.8}$$

4.4 Group Equations

The specific energy radiated by the matter is given by

$$\left(\frac{Q}{\rho}\right)_i = \sum_{1\le k\le \mathbf{NG}} \left(\frac{Q_k}{\rho}\right)_i, \quad (i = 1,\ldots,\mathbf{N}) \tag{4.9}$$

Where the contribution of the group k is

$$\left(\frac{Q_k}{\rho}\right)_i = c\,\kappa^k_{P,i}\,(U_{pk,i} - U_{k,i}), \quad (i = 1,\ldots,\mathbf{N}) \tag{4.10}$$

Being $U_{pk,i} = U_{pk}(T_i)$ given by (3.21). On the other hand the equations (3.19,3.20) for the group k take the form

$$\frac{S_{k,i+1} - S_{k,i}}{\Delta m_i} = c\,\kappa^k_{P,i}\,(U_{pk,i} - U_{k,i}), \quad (i = 1,\ldots,\mathbf{N}) \tag{4.11}$$

$$c\,g_k^2\,\frac{U_{k,i} - U_{k,i-1}}{\frac{1}{2}(\Delta m_i + \Delta m_{i-1})} = -\left(\frac{2}{1/\kappa^k_{R,i} + 1/\kappa^k_{R,i-1}}\right) S_{k,i}, \quad (i = 2,\ldots,\mathbf{N}) \tag{4.12}$$

and the boundary conditions (3.29,3.30) can be written as

$$S_{k,1} = -\frac{\mu^k_a + \mu^k_b}{2}\left(\frac{1-\alpha_L}{1+\alpha_L}\right) cU_{kL} \tag{4.13}$$

$$S_{k,\mathbf{N}+1} = \frac{\mu^k_a + \mu^k_b}{2}\left(\frac{1-\alpha_R}{1+\alpha_R}\right) cU_{kR} \tag{4.14}$$

The values of U_{kL} and U_{kR} at the boundaries must be computed extrapolating the values in the adjoint cells (U_k is a cell centered quantity)

$$U_{kL} = \left(1 + \frac{\Delta m_1}{\Delta m_1 + \Delta m_2}\right) U_{k,1} + \left(-\frac{\Delta m_1}{\Delta m_1 + \Delta m_2}\right) U_{k,2} \tag{4.15}$$

$$U_{kR} = \left(1 + \frac{\Delta m_{\mathbf{N}}}{\Delta m_{\mathbf{N}} + \Delta m_{\mathbf{N}-1}}\right) U_{k,\mathbf{N}} + \left(-\frac{\Delta m_{\mathbf{N}}}{\Delta m_{\mathbf{N}} + \Delta m_{\mathbf{N}-1}}\right) U_{k,\mathbf{N}-1} \tag{4.16}$$

It must be noted that in (12) the Rosseland opacity in the interfaces is obtained as the inverse of the average of the inverses in the adjoint cells. This procedure gives smaller values than the direct average, especially with large gradients. Thus allowing for large fluxes makes it possible to smooth such gradients.

4.5 Laser Equation

The specific laser energy deposition in the cell i must be computed cautiously; the laser equations are strongly nonlinear. The procedure adopted here computes the energy deposited in one cell integrating the equation (2.28) between cell boundaries; the specific deposition is given by this quantity divided by the thickness of the cell

$$\left(\frac{S}{\rho}\right)_i = \frac{1}{\rho_i \Delta x_i}\int_{x_i}^{x_{i+1}} S\,dx = \frac{I_{+,i+1} - I_{+,i} - I_{-,i+1} + I_{-,i}}{\Delta m_i} \tag{4.17}$$

The laser intensities $I_{+,i}$ and $I_{-,i}$ are computed in the interfaces between cells as follows. $I_{+,N+1}$ is the known incident laser intensity while the other values $I_{+,i}$ are computed applying successively the formula

$$I_{+,i} = I_{+,i+1}\,exp\left(-\frac{C}{T_i^{3/2}}\int_{x_i}^{x_{i+1}} \frac{(\rho/\rho_c)^2\,dx}{\sqrt{1-\rho/\rho_c}}\right) \tag{4.18}$$

Where the temperature T_i is assumed constant througth the cell. If ρ is assumed to vary linearly between the values at the interfaces, the integral can be carried out analytically. The needed interface-centered values of ρ are assumed to be the mean between cell centered-values, except the first and the last ones that are taken equal to zero. This method can be applied only if the density at x_i is higher than critical. Otherwise the incident intensity at the critical point can be determined by the similar expression

$$I^{cr}_{+} = I_{+,i_{cr}+1}\,exp\left(-\frac{C}{T_i^{3/2}}\int_{x_{cr}}^{x_{i_{cr}+1}} \frac{(\rho/\rho_c)^2\,dx}{\sqrt{1-\rho/\rho_c}}\right) \tag{4.19}$$

being i_{cr} the cell where the critical density is located. Once I^{cr}_{+} is known the reflected intensity is given by $I^{cr}_{-} = (1-\alpha)I^{cr}_{+}$. Then, observing that (2.23,2.24) implies $I_+ I_- = Constant$, the values of the reflected intensity are easilly computed by

$$I_{-,i} = \frac{I^{cr2}_{+}(1-\alpha)}{I_{+,i}} \tag{4.20}$$

Finally the energy deposition corresponding to i_{cr} is divided between this cell and its neighbours. This smoothing is needed in order to prevent strong numerical noise when the critical point jumps from one cell to another.

4.6 MATTER EQUATIONS

The above equations must be completed by the equation of state and opacities of the matter given by the relations

$$P_{eq,i} = P_{eq}(\rho_i, e_i, N_i), \quad (i = 1, \ldots, \mathbf{N}) \tag{4.21}$$

$$T_i = T(\rho_i, e_i, N_i), \quad (i = 1, \ldots, \mathbf{N}) \tag{4.22}$$

$$\kappa^k_{P,i} = \kappa^k_P(\rho_i, T_i, N_i), \quad (i = 1, \ldots, \mathbf{N}, k = 1, \ldots, \mathbf{NG}) \tag{4.23}$$

$$\kappa^k_{R,i} = \kappa^k_R(\rho_i, T_i, N_i), \quad (i = 1, \ldots, \mathbf{N}, k = 1, \ldots, \mathbf{NG}) \tag{4.24}$$

Where the composition N_i is fixed. These relations are implemented in the standard version of the code interpolating between tabulated values. Nevertheless it is possible, by changing the appropriate routines, to use analytic expressions.

5. Temporal Discretization

5.1 TIME SPLITTING

Once the equations have been discretized in space, the physical system is represented by a finite set of variables which are continuous in time but defined only on a finite number of points (computational mesh). From the mathematical point of view, the equations consist of the rate equations for the variables ρ_i, v_i and e_i joined to a set of algebraic equations for q_i (Thermal flux equation), $(Q_k/\rho)_i$,$S_{k,i}$ and $U_{k,i}$ (Group equations), $(S/\rho)_i$, $I_{+,i}$ and $I_{-,i}$ (Laser equations) and $P_{eq,i}$, T_i, $\kappa^k_{P,i}$ and $\kappa^k_{R,i}$ (Matter properties). The system can be thought of as a set of ordinary differential equations depending on the main variables ρ_i, v_i and e_i whose right hand side depend only on these variables, since all the other variables can be written in terms of the main variables solving the algebraic equations. Thus, denoting by X the vector whose $3\mathbf{N}+1$ components are the functions ρ_i, v_i and e_i, the system can be represented schematically by

$$\frac{dX}{dt} = f(X) \tag{5.1}$$

This system would be solved, in principle, by standard methods. For example, denoting by X^n the representation of X at time t^n ($\equiv n\Delta t$), it is clear that the recurrence relation

$$\frac{X^{n+1} - X^n}{\Delta t} = f(X^n) \tag{5.2}$$

supplies the required solution, provided that Δt is small enough. In general, this explicit scheme needs a prohibitively small value for the time step Δt in order to be numerically stable. This makes it useless in practice. The numerically stable implicit scheme

$$\frac{X^{n+1} - X^n}{\Delta t} = (1-\theta) f(X^n) + \theta f(X^{n+1}), \quad (\frac{1}{2} < \theta \leq 1) \tag{5.3}$$

is unfortunately very complicated, it involves the simultaneous solution of a set of $3\mathbf{N}+1$ non-linear equations.

Thus it becomes necessary to use a different approach which is able to achieve the necessary stability with a reasonable amount of computational work. The procedure adopted here is the so-called 'time-splitting'. Before continuing, it proves convenient to explain briefly the basis of this method. Let us consider the equation

$$\frac{dX}{dt} = f_1(X) + f_2(X) \tag{5.4}$$

and the two substeps integration method given by

$$\frac{X^* - X^n}{\Delta t} = f_1(X^*) \tag{5.5}$$

$$\frac{X^{n+1} - X^*}{\Delta t} = f_2(X^{n+1}) \tag{5.6}$$

Where X^* is some intermediate value. If the functions f_1 and f_2 have some reasonable mathematical properties (continous derivatives) , it is straighforward (but rather cumbersome), applying Taylor's series, to show that

$$\frac{X^{n+1} - X^n}{\Delta t} = f_1(X^n) + f_2(X^n) + O(\Delta t) \tag{5.7}$$

The notation $O(\Delta t)$ stands for terms that verify $|O(\Delta t)| < Constant \times \Delta t$ for $\Delta t < \Delta t_0$. This expression implies the so-called 'consitency' of the method. If, in

addition, the method is stable, this leads to the appropriate solutions. If both (5–6) are stable, it is reasonable to think that the two substeps method is also stable; this occurs in practice. The global method has only a first order accuracy (the error is of order $O(\Delta t)$), but this is scarcely a trouble; the main (and unavoidable) sources of error had been made in the modelization of the physics. The extension to more than two terms on the right of (4) is straightforward.

Now coming back to the physical equations, it is clear that the different terms can be grouped in the following way

	f_H (Hydrodynamics)	f_T (Thermal flux)	f_R^1 (Group-1)	$\cdots$	$f_R^{\mathbf{NG}}$ (group-NG)	f_L (Laser)
$\partial_t \rho =$	$-\rho^2 \partial_m v$	$+0$	$+0$	$+\cdots+$	$+0$	$+0$
$\partial_t v =$	$-\partial_m (P_{eq} + P_{vis})$	$+0$	$+0$	$+\cdots+$	$+0$	$+0$
$\partial_t e =$	$-(P_{eq} + P_{vis}) \partial_m v$	$-\partial_m q$	$-Q_1/\rho$	$-\cdots-$	$-Q_{\mathbf{NG}}/\rho$	$+\mathrm{S}/\rho$

And thus equation (1) can be written as

$$\frac{dX}{dt} = f_H + f_T + f_R^1 + f_R^2 + \cdots + f_R^{\mathbf{NG}} + f_L \tag{5.8}$$

Before applying the time-splitting method to this equation, two points must be taken into account:

i) The laser energy deposition has usually a very sharp profile. This fact would produce strong oscillations in the temperature through the time step; first the deposition of energy produces a very hot spot that is cooled immediately by the heat and radiation transport terms. This phenomena can be the source of important errors. It can be avoided by splitting the energy deposition in $\mathbf{NG}+1$ pieces, each of them will be solved together with one of the transport terms. This is made using weight factors η_k $(k = 1, 2, \ldots, \mathbf{NG})$ for the radiation transport,

and η_0 for the heat flux transport. The appropriate values for these factors will be discussed at the end of this section.

ii) Often the structures related to the hydrodynamics (i.e. shock waves) move faster than the structures related to the heat or radiation transport (i.e. thermal waves). Consequently, the maximum bound for the time step would be related to the fluid motion, usually leading to quite strong restrictions. This can be overcome using 'subcycling' for the hydrodynamics; i.e. the fluid equations are advanced **NS** times during one time step. This is implemented in the code by dividing the term f_H into **NS** pieces and placing them between the radiation transport terms. Although not strictly necessary, the same is done with the thermal heating.

The equation(8) now takes the form

$$\frac{dX}{dt} = g_H + g_R^1 + g_R^{\mathbf{NS}+1} + g_R^{2\mathbf{NS}+1} + \cdots + g_R^{\mathbf{NG}-\mathbf{NS}+1} + g_T$$

$$+ g_H + g_R^2 + g_R^{\mathbf{NS}+2} + g_R^{2\mathbf{NS}+2} + \cdots + g_R^{\mathbf{NG}-\mathbf{NS}+2} + g_T$$

.....

$$+ g_H + g_R^{\mathbf{NS}} + g_R^{2\mathbf{NS}} + g_R^{3\mathbf{NS}} + \cdots + g_R^{\mathbf{NG}} + g_T \tag{5.9}$$

Where

$$g_H = \frac{f_H}{\mathbf{NS}} \tag{5.10}$$

$$g_R^k = f_R^k + \eta_k f_L \tag{5.11}$$

$$g_T = \frac{f_T + \eta_0 f_L}{\mathbf{NS}} \tag{5.12}$$

Now the time-splitting method can be directly applied to this equation; the $\mathbf{NG}+2\mathbf{NS}$ terms are added in succesive substeps, in the order given above. However it proves convenient to introduce still some minor approximations that, without spoiling the accuracy, reduce the computational work:

i) The laser deposition profile is rather insensitive to the exact temperature and density profiles; thus the f_L term will be computed only after every hydrodynamic substep and used without changes until the next hydrodynamic substep (It will be computed only **NS** times instead of **NG** + **NS**).

ii) The equations of state will be evaluated only before every hydrodynamic substep and approximated by the linearized formulas

$$P_{eq}(\rho, e) = P_{eq}(\rho_0, e_0) + \partial_\rho P_{eq0}(\rho_0, e_0)(\rho - \rho_0) + \partial_e P_{eq0}(\rho_0, e_0)(e - e_0) \quad (5.13)$$

$$T(\rho, e) = T(\rho_0, e_0) + \partial_\rho T_0(\rho_0, e_0)(\rho - \rho_0) + \partial_e T_0(\rho_0, e_0)(e - e_0) \quad (5.14)$$

until the next hydrodynamic substep.

iii) Although the substeps involved by the time-splitting are given fully implicitly (the operators at the right hand sides of (5,6) are evaluated at the new values of the solution), in order to guarantee numerical stability, the values of the transport coeficients κ_P^k and κ_R^k are computed with the old values of the termodynamical variables. This does not affect stability.

Details on every of the substeps are given in the following sections.

5.2 Hydrodynamical Substep

As had been pointed out, the fluid state is advanced **NS** times every time step. The fluid variables before and after the advancing will be denoted by the superindexes O and N, respectively. The substep is then given by

$$\frac{X^N - X^O}{\Delta t} = g_H(X^N) \equiv \frac{f_H(X^N)}{\mathbf{NS}} \quad (5.15)$$

This equation is nonlinear and in principle it would be necessary to use some iteration procedure. However, this is not actually carried out; instead the equation is linearized expanding the right hand side into Taylor's series

$$\frac{X^N - X^O}{\Delta t'} = f_H(X^O) + \frac{\partial f_H}{\partial X}(X^O).(X^N - X^O) \quad (5.16)$$

Where $\Delta t' = \Delta t/\mathbf{NS}$. This equation is now linear in the unknown value X^N, $\partial f_H/\partial X$ being a 7-diagonal matrix. Standard library routines are employed to solve this system. The dropped terms in the Taylor series are of the order $O(\Delta t^2)$ and thus do not spoil the overall accuracy of the method (of order $O(\Delta t)$).

5.3 Heat Flux Substep

All that is said for the hydrodynamics can be directly applied to the heat flux. However, in this case, the density and velocity are not advanced (its derivatives in f_T are zero), consequently the resulting system is only tridiagonal.

5.4 Group Substep

The equation for a group can be written as

$$\frac{X^N - X^O}{\Delta t} = g_R^k(X^N) \quad (5.17)$$

On the right hand side there is only one nonlinear term, namely $U_{pk}(T_i^N)$. This term will be linearized in the following way

$$U_{pk}(T_i^N) \simeq U_{pk}(T_i^*) + \frac{\partial U_{pk}}{\partial T}(T_i^*)(T_i^N - T_i^*) \quad (5.18)$$

being T_i^* a reference value. The resulting equation is a tridiagonal system that can be easily solved once T_i^* is given. The code uses an iterative procedure; for the first iteration it takes $T_i^* = T_i^O$ and a provisional T_i^N result, for the second iteration it takes $T_i^* = T_i^N$ and so on. The number of iterations is given by the input variable **NITER**. Usually the results are reasonable good with one iteration only.

5.5 Laser Deposition Partition

In this section the algorithm used by the program to avoid strong oscillations in the temperature during one time step will be explained. This problem can be especially serious in the cell i_{max} where the laser deposition is at its maximum.

Let us consider first the thermal flux substep. It is given by

$$\left(\frac{\partial e}{\partial T}\right)_i \frac{T_i^N - T_i^O}{\Delta t} = \eta_0 \left(\frac{S}{\rho}\right)_i + \textbf{Transport terms} \quad (5.19)$$

If the time step is small enough, the transport terms are not affected by the deposition. Thus the temperature increment in i_{max} can be rewritten as

$$\Delta T_0 = \eta_0\, a + b_0 \quad (5.20)$$

Where $a = (S/\rho)_{i_{max}} (\partial T/\partial e)_{i_{max}} \Delta t$, while b_0 does not depend on η_0. Analogous relations can be obtained for the group substeps

$$\Delta T_k = \eta_k \, a + b_k, \qquad (k = 1, 2, \ldots, \mathbf{NG}) \tag{5.21}$$

the value a being common to all the transport processes. The global increment of the temperature at i_{max} is obviously

$$\Delta_T T = \sum_{0 \le k \le \mathbf{NG}} \Delta T_k \tag{5.22}$$

The factors η_k can be freely chosen. If the values

$$\eta_k = \frac{1}{a} \left(\frac{\Delta_T T}{\mathbf{NG} + 1} - b_k \right) \tag{5.23}$$

are selected, the temperature increments in all the transport processes are the same, supplying the desired smoothing through the step.

The program initializes first the values of the factors to

$$\eta_k = 0, \qquad (k = 0) \tag{5.24}$$

$$\eta_k = 1/\mathbf{NG}, \qquad (k = 1, 2, \ldots, \mathbf{NG}) \tag{5.25}$$

These values are applied in the first time step. During this step the values of ΔT_k are stored and the values of b_k computed by (21). Finally, the formula (23) provides the most appropriate values that will be used in the next time step. This process is carried out in successive time steps. If the input parameter **IWCTRL** (normally set to 1) is set to 0, the initial values of η_k are always used.

6. Program Description

6.1 General

The program is written (as much as possible) in a modular way; only four routines have more than 56 source lines. Some parts of the program (modules) are completely interchangeable. For example, the routines **QUELIN**, **QUELLE** and **LASER3**, that compute the laser power deposition, can be substituted by other routines with the same name and arguments, but with completely different physics, without the need of additional changes in other program units. The modules are the following:

QUELIN-QUELLE-LASER3

FLUID-HYDRO

EOSM-EOSIN1-EOSIN2-EOSBIN-EOSBIN

WFIN-WFLUSS

OPA-OPAIN-OPABIN

All the common blocks and parameters used in one of these modules are not referenced in other program units.

The program input is done through FORTRAN units 12 to 19. Every unit has assigned a conceptually different sort of data (i.e. on 16 are given the laser characteristics). In some units the read process is carried out until the 'end-of-file' is reached.

Matrices are used at different places of the program. In general they have a banded structure and thus can be stored in condensed format. Every diagonal of the 'mathematical' matrix is stored in one row of the 'FORTRAN' matrix. This format is required by the library routines that perform operations over these matrices.

The program uses the c.g.s. system of units, with the exception of the temperature which is given in electron volts. However, for compatibility with other programs, the equation of state tables must be supplied in SESAME units.

In this section the main program (**MULTI**) and some auxiliary routines for input and initialization (**INITVR,RDGDT,WCTRL,ZONING**) will be described. The different tasks carried out are the following

i) The program reads from the FORTRAN unit 12 a series of parameters that control the subsequent operations. These parameters must be given in a NAMELIST block with the name **INPUT** as follows

PARAMETER	DESCRIPTION
IFLAG1	=1 Allows hydrodynamic motion.
	=0 Inhibits motion.
IFLAG2	=1 Free surfaces for hydrodynamics.
	=0 Rigid walls.
IFLAG3	=1 Energy deposition by laser allowed.
	=0 Laser off.
IFLAG4	=1 Thermal flux allowed.
	=0 No energy transported by thermal conduction.
IFLAG5	=1 Radiation transport enabled.
	=0 No radiation transport.
NITER	Number of iterations in the group equations.
NSPLIT	(**NS**) Number of hydrodynamic substeps in a time step (≥ 1).
ALPHAR	(α_R) Fraction of the radiation reflected at the right boundary; $\alpha_R = 1$ for a Hohlraum and $\alpha_R = 0$ for a foil.
ALPHAL	(α_L) The same for the left boundary.
TIMEX	Total simulated time (in seconds).
DT	(Δt) Integration time step.
DTO	Time interval between output dumps on disk.
IABORT	The job ends after this time step number.
TOUT(1)	If different from zero the program generates a

.....	complete output at these selected times.
TOUT(5)	
ISOUT(1)	If different from zero the program generates a
.....	complete output at these selected time step
ISOUT(5)	numbers.
IWCTRL	=1 Allows automatic adjustment of the partition of the laser deposition between the different transport mechanisms.
	=0 This energy partition is fixed.

Then the program prints out its version number (the actual is 1.4) followed by an echo of all the above parameters.

ii) The initialization routines for the equation of state (**EOSIN1,EOSIN2**), opacity (**OPAIN**), energy deposition (**QUELIN**) and thermal flux (**WFIN**) are called. They are appropriately described in the corresponding sections of this chapter.

iii) The routine **INITVR** reads its input data from FORTRAN unit 14, initializates Δm_i and the main variables ρ_i, v_i and e_i, and stores in the common block **COMLDT** (Layer Definition Table) the composition of the foil N_i.

The program manages multilayer foils; for every individual layer an input line with the following format is read.

FORMAT	VARIABLE	DESCRIPTION
A1	–	A blank character.
E8.0	**I1**	First computational cell of the layer.
E8.0	**I2**	Last computational cell of the layer.
E8.0	**MAT**	Number of the layer material (1,2,3 or 4).
E8.0	**THICK**	Thickness (in cm).
E8.0	**R0**	Initial density (in $g\ cm^{-3}$).
E8.0	**E0**	Initial specific internal energy (in $cm^2\ s^{-2}$) .
E8.0	**ZONPAR**	Parameter for zoning (see below).

If the first field is not a blank character, the line is interpreted as a comment.

For every layer the values of Δm_i are obtained through the use of the routine **ZONING**. The parameter **ZONPAR** allows different sizes of the cells, namely

$$\Delta m_{i+1} = \mathbf{ZONPAR} \times \Delta m_i$$

Thus, if its value is unity the mesh is uniform; if it is greater/less than one, a finer zoning occurs at the left/right. In addition, negative values give finer zoning at both sides ($|\mathbf{ZONPAR}| > 1$) or in the center ($|\mathbf{ZONPAR}| < 1$). Finally, the echo of the read data and the initial values are printed out.

iv) The routine **RDGDT** reads the group descriptions from the FORTRAN unit 17, and stores this information in the common **COMGDT** (Groups Definition Table). For every group an input line with the following format is read

FORMAT	VARIABLE	DESCRIPTION
A1	–	A blank character.
E8.0	ν_a^k	Frequency interval for the group
E8.0	ν_b^k	$(0 < \nu_a^k < \nu_b^k < \infty)$.
E8.0	g_k	Factor for the group.
E8.0	μ_a^k	Cosine interval for the group
E8.0	μ_b^k	$(0 \leq \mu_a^k < \mu_b^k \leq 1)$.
E4.0	**P1**	Numbers of the Planck opacity tables
E4.0	**P2**	that must be used for this group on
E4.0	**P3**	the material numbers 1,2,3 or 4,
E4.0	**P4**	respectively.
E4.0	**R1**	Numbers of the Rosseland opacity tables
E4.0	**R2**	that must be used for this group on
E4.0	**R3**	the material numbers 1,2,3 or 4,
E4.0	**R4**	respectively.

If the first field is not a blank character, the line is interpreted as a comment. The routine prints out an echo of the data read.

v) The routine **SOUTIN** reads from FORTRAN unit 18 information about

what variables must be dumped on disk for later post-processing. A detailed description of the output format will be given in the following section.

vi) In the following, the program performs a series of initialization tasks. The initial partition of the deposition of energy η_k as well as the mass coordinates m_i are computed, the job sequence name is obtained through a call to the system routine **DNAME0**, and the first records of the output file are written on disk (see next section).

vii) Now the integration step is prepared. The internal flags **IPRT1**, **IPRT2**, **IPRT3**, and **IPRT4**, used as print control in the integration routine, are appropriately set. If the time step number exceeds **IABORT** the system routine **ERREXIT** is called, thus stopping both the program and the whole run. Finally the time step is set to its nominal value.

viii) The integration routine **SCHRITT** is called at this point. The main variables are passed through the set **TIME1,R1,V1** and **E1**, and the advanced variables obtained in the set **TIME2,R2,V2** and **E2** together with the completion flag **ICOMP**.

If the integration step has not been successfull (**ICOMP** $\neq 1$) a message is issued, the time step is halved an a new attempt is done. This will be repeated if necessary.

Otherwise, if the time step has been successful, the main variables are updated with the new ones.

ix) If the flag **IWCTRL** is set to one, the routine **WCTRL** adjusts the partition of the deposition of energy η_k to the most appropriate values in order to avoid strong oscillations in the temperature through the integration step. It must be warned that if one of the transport mechanisms is missing (the corresponding flag is zero), this adjustment is a bit different as described in the previous chapter.

x) At selected times the variables are output on disk.

xi) If the actual time is less than the total simulation time a new step begins.

xii) Otherwise the program stops after printing a final message with the number of time steps carried out (**ISTEP**) and the number of calls to the time step routine (**ITRY**). Obviously the difference between both quantities is the number

of time steps which failed.

6.3 Output on Disk/Tape

The present program produces a very large amount of data. A complete printout becomes unconvenient for the user and unpractical. Although the program can produce, at selected times, a rather complete printout, this is intended to use mainly for test purposes. The main output takes place on disk or tape, and from there can be read by other specialized programs that generate partial printout or graphics.

It must be noted that the radiation variables constitute the largest part of the mentioned data. Thus a previous selection must be done. The energy flux $S_{k,i}$, energy density $U_{k,i}$ and the 'representative' specific intensities $\langle I_k^+ \rangle_i$ and $\langle I_k^- \rangle_i$ are given, in a specific instant, for the group numbers from 1 to **NG** and for the interface numbers from 1 to $\mathbf{N}+1$ (although U_k is a cell centered quantity, the program supplies an interface centered version by interpolation and extrapolation). The user can make two different types of selection between these variables:

i) Selected output at specified coordinate (SOC). One of the above variables is selected at a specified interface i_S. The **NG** values are stored in the array **SOC**.

ii) Selected output at specified frequency (SOF). One of the variables is selected for the groups from k_{s1} to k_{s2} and added for all groups in this range. The $\mathbf{N}+1$ values are stored in the array **SOF**.

More than one selection of every type (to a maximum of 10) is possible; the results are stored successively in the corresponding array (i.e. in the first case, the first selection is stored between elements 1 and **NG** of the array **SOC**, the second between elements $\mathbf{NG}+1$ and **2NG** and so on).

The subroutine **SOUTIN** reads from FORTRAN unit 18 a line for every selection, the format is

FORMAT	VALUE(S)	DESCRIPTION
A1	'Δ'	A blank character.
A4	'$U\Delta\Delta\Delta$'	U_k selected.
	'$S\Delta\Delta\Delta$'	S_k selected.
	'$FPLS$'	$\langle I_k^+ \rangle$ selected.
	'$FMNS$'	$\langle I_k^- \rangle$ selected.
5X	'$\Delta FOR\Delta$'	Unused.
A5	'POINT'	Selected output at specified coordinate.
	'GROUP'	Selected output at specified frequency.
1X	'Δ'	Unused.
I3	**I1**	i_s or k_{s1}
4X	'$\Delta TO\Delta$'	Unused.
I3	**I2**	k_{s2} or unused.

If the first field is not a blank character, the line is interpreted as a comment. The auxiliary arrays **CMI** and **CMC** contain the mass coordinates at the interfaces and in the center of the cells, respectively, while **FREC** contains the medium frequencies of the groups. In addition the arrays **SOCFRE** and **SOFCMI** contain the frequencies and mass coordinates corresponding to the values stored in **SOC** and **SOF**, respectively (There is a one to one correspondence between these arrays).

The structure of the output file had been designed in order to make easy a posterior search of the required data. Therefore, besides the numerical data, suitable information over the contents are stored (directory). The file is composed of 'sections', and these are composed of binary records (not formated). The contents of a section is:

1st record: the length in computer words (integer value) of the subsequent records.

2nd record: a list of names (character values) corresponding, one to one, with the variables stored in the data records. These names are ten character long (one word in the Cray-1 computer), and can be different from the FORTRAN

symbolic names used in the program. If the associate data is an array, the name appears as many times as elements of the array are stored.

3rd record: a list of integer values that specifies whether the associated variables are scalars (value zero) or an array (index of the correspondig elements). Nevertheless these values are scarcely used by the post-processor programs and thus can be taken to store other informations.

4th record: a list with the data (real values).

Successive records: equal to the 4th but at different times.

The first three records are called the heading of the section. The specific implementation of this structure in the program is done using files composed by two sections:

1st section: contains the quantities that do not depend on time: **CMC, CMI, SOCFRE, SOFCMI** and, in addition, the program name **.PROGRAM**, version number **.VERSION** and job sequence name **.NAME**. There is only one data record.

2nd section: contains, for every stored time step, a set of values of **TIME, R** (density), **T** (temperature), **P** (pressure), **SOC** and **SOF**.

This configuration can be easily changed. New variables can be included without the need of carrying out changes in the post-processors; they read their data only after a search in the headings (if some requested data are not found, a message is issued).

6.4 Integration Step

In this section the integration step routine **SCHRITT** and the auxiliary routine **DUMPE** will be described. Instead of giving a sequential description of the steps carried out, it proves convenient to describe separately every functional task.

Obviously the most important task is to integrate the equations. As said in the previous chapters, this is done through a time-splitting method. The physical phenomena are applied in successive substeps in the order given in (5.9). For every main variable there are three FORTRAN variables (i.e. for the density ρ_i they are **R1, R2** and **R3**). The first is used as input and is not modified by the routine.

The second is initially set equal to the first, it is advanced in every substep and finally contains the results. The third is only used as intermediate storage.

The subroutine contains two **DO** loops. The external one (**DO 23**) goes over **NS** times (one for every hydrodynamic substep). The internal one (**DO 11**) goes over **NG/NS** times for every time in the external loop; hence a total of **NG** times (once for each group). The tasks carried out in the external loop are:

i) The equation of state, in the form (5.13,5.14), is computed through a call to the routine **EOS**.

ii) The hydrodynamic substep is carried out by the routine **HYDRO**.

iii) The power deposition profile is computed by the routine **QUELLE**.

iv) The loop for the groups is done. The variables are advanced by the routine **GROUP**.

v) The heat flux substep is carried out by the routine **WFLUSS**.

With the appropriate flags one or more of these processes can be switched off. In the case that neither group transport nor heat conduction are present, an additional step is needed:

vi) The power deposition is directly dumped on the matter.

The index of the cell where the power deposition is maximum (**ICTRL**) is determined after **iii**. The temperature increments in this cell through the heat flux substep, and the group substeps are stored in **WBW**(scalar) and **WB**(array), respectively, for posterior use in the control of the partition of the power deposition (see section 5.5). The other quantities related to this process a,η_0 and η_k are passed through the variables **WC**,**WAW** and **WA**, respectively.

The validity of the intermediate results is often checked; if a negative temperature or density is found the complexion variable **ICOMP** takes a negative value and a return occurs. The error origin is coded in **ICOMP** in the format: **-ggssee**, where **ee** is the error type, **ss** the number of the subcycle where occurred and **gg** the group (if any) concerned.

There are many points where the variables and intermediate results are printed. The flags **IPRT1** and **IPRT2** enable this printout in the ranges of the external

loop and internal loop, respectively. In addition the flags **IPRT3** and **IPRT4** are passed to the equation of state and opacity routines, respectively, enabling the printout of their results (for test purposes). Most of this output is generated by the auxiliary routine **DUMPE**.

The routine **GROUP** delivers the radiation variables in the array **SPECTR**, and from there they are selected and put into the arrays **SOC** and **SOF** as described in the previous section. This process take place in the inner loop. The requested options are described in the tables **KSOC** and **KSOF**.

6.5 Hydrodynamics

The hydrodynamic substep is carried out by the routines **HYDRO** called by **SCHRITT**, and **FLUID** called by **HYDRO**. The fluid variables are mixed in a unique vector with $3\mathbf{N}+1$ elements

$$\{X\} = \{v_1, \rho_1, e_1, v_2, \rho_2, e_2, v_3, \ldots, v_{\mathbf{N}}, \rho_{\mathbf{N}}, e_{\mathbf{N}}, v_{\mathbf{N}+1}\}$$

in terms of which the substep equation (5.15) takes the form

$$\frac{1}{\Delta t'}\{X^N - X^O\} = \{B\} + [A]\{X^N - X^O\} \tag{6.1}$$

The superindices N and O refer to the values after and before the substep, respectively. The vector B and the matrix A are only functions on X^O, and are computed by the routine **FLUID**. The vector is stored in the array **X** (that, for computational needs, will also contain later the results), while the diagonals of A are stored in the last seven rows of the array **A** (the three first are required for storage of intermediate results). Once the system (1) is written in standard format, the library routines **SGBFA** (factorization of a general banded matrix) and **SGBSL** (solve a general banded linear system) are called and the value of $\{X^N - X^O\}$ stored in **X**. Between both calls a check is made for the singularity of the system. In the case of rigid wall boundary conditions, the system (1) is truncated excluding the first and the last equations. These are substituted by $v_1 = 0$ and $v_{\mathbf{N}+1} = 0$. Finally the new values of the variables are obtained adding the increments to the original values.

Concerning **FLUID**, it must be noticed that it was written as a general subprogram. Thus, instead of a unique matrix **A**, it manages the submatrices

A1, A2, B1, ..., that, mixed togethe , give this matrix. Such matrices have a sparse structure; the passed parameters **N1** and and **N2** are the distance, in computer storage units, between consecutive elements on the same diagonal and consecutive elements on the same column, respectively.

6.6 Heat Flux

The routine **WFIN**, called by the main program, reads its input from FORTRAN unit 19, from a NAMELIST block with name **THERM**, and computes the conduction coefficient. The only variable to be read is the effective ion charge number **Z**. The value of Coulomb's logarithm is taken equal to 10. Finally, the values of **Z** and the conductivity coefficient are printed out.

The heat flux substep is carried out by the routine **WFLUSS**. The implicit system of equations is first written in form of a matrix and then solved by the library routines **SGBFA** and **SGBSL**. The conductivity coefficient is passed through a common block.

6.7 Group Substep

The group substep is controlled by the routine **GROUP** that performs some trivial computations and calls the auxiliary routines **OPACIT**, **RFE**, **REE**, **MEE**, **SOLVE3**, **MAT1** and **PLANCK**. In addition, the last one calls the routine **PDSTRB**.

First the routine **OPACIT** supplies both Planck (**OP**), and Rosseland (**OR**) opacities from the values of density and temperature at the beginning of the substep.

The equation (5.15) together with the boundary conditions (4.13-4.16) can be written in the form

$$\{S_k\} = [\mathbf{AS}]\{U_k\} + \{\mathbf{BS}\} \tag{6.2}$$

The non-squared matrix **AS** is stored in usual format (the diagonals stored in rows), while the the first and last elements of **BS** are **BS1** and **BS2**, respectively (all the others are zero). These values are evaluated through a call to the routine **RFE**.

As follows, the routine **PLANCK** delivers the energy density corresponding to a Planck distribution as well as their temporal derivative, computed at some temperature T_i^*, in the arrays **UP** and **UPD**, respectively. For a different temperature, the energy density is approximated by

$$\{U_{pk}\} = \{\mathbf{UP}\} + [\mathbf{UPD}]\{T_i - T_i^*\} \tag{6.3}$$

The temperature T_i^* ($\equiv$ **TTRY**) is initially set to the temperature T_i^O.

Now the equation for radiation energy (4.11) can be written, using the relations (2–3), in the form

$$[\mathbf{A11}]\{U_k\} + [\mathbf{A12}]\{T^N - T^O\} = \{\mathbf{B1}\} \tag{6.4}$$

The tridiagonal matrix **A11** and the diagonal one **A12** as well as the vector **B1** are computed by the routine **REE**.

Analogously, the routine **MEE** supplies the matter energy equation (5.17) in the form

$$[\mathbf{A21}]\{U_k\} + [\mathbf{A22}]\{T^N - T^O\} = \{\mathbf{B2}\} \tag{6.5}$$

Here, both **A21** and **A22** are diagonal matrices. This coupled system of equations is solved by the routine **SOLVE3**. It reduces first the number of equations by substituting in (4) $T_i^N - T_i^O$ from (5), and then calls the library routines **SGBFA** and **SGBSL** to solve the system.

At this point a new iteration can be required; **TTRY** will be set to the newly computed temperature T_i^N and the process is carried out again. The number of such iterations is given by the input parameter **NITER**.

Once U_k is known, the values of S_k are computed using (2). The routine **MAT1** performs such a matrix-vector product. Finally, the values of U_k at the interfaces as well as $\langle I_k^+ \rangle$ and $\langle I_k^- \rangle$ are computed.

The **PLANCK** routine uses the function

$$f(x) = \frac{15}{\pi^4} \int_0^x x'^3 \, (e^{x'} - 1)^{-1} \, dx'$$

which is supplied by the routine **PDSTRB** by interpolation in a table. This table is computed by the same routine when it is called the first time.

6.8 Laser Deposition

The main program calls first the routine **QUELIN** which reads from FORTRAN unit 16 the laser characteristics. These data must be given in a NAMELIST block with the name **PULSE**, of the following contents

PARAMETER	DESCRIPTION
IDIR	=1 Laser incidents from right.
	=-1 Laser incidents from left.
PIMAX	Maximum laser intensity ($g\ s^{-3}$).
PITIME	Pulse duration FWHM (*seconds*).
WL	Laser wavelength (cm).
DELTA	Fraction of the incident power dumped at the critical density.
Z	Effective ion charge.
ZMOL	Atomic number.

The laser pulse is assumed to have a sine square shape. All these data are echoed in the printer.

The routine **QUELLE** supplies correct arguments to the routine **LASER3** which really computes the deposition. The procedure described in (4.5) is exactly followed, thus it is not worthwhile to give more details here.

6.9 Equation of State

The routine **EOSIN1** is called from the main program. It reads, from FORTRAN unit 13, a series of equation of state tables. The format of one of these tables consists of a string of real numbers, four numbers packed in one line in fields of 15 characters (format 4E15.0). In case that the total number is not an even multiple of four, the last line is left partially empty. The meaning of every item is given by

ITEM	DESCRIPTION
MID	Material identification.
R0	Normal density (unused).
NR	Number of tabulated densities.
NE	Number of tabulated energies.
R(I),I=1,NR	Tabulated densities ($g\ cm^{-3}$).
DE(J),J=1,NE	Tabulated energies (difference to the cold energy values) ($Mbar \times\ cm^3\ g^{-1}$).
E0(I),I=1,NR	Values of the cold energy corresponding to the tabulated densities ($Mbar \times\ cm^3\ g^{-1}$).
(P(I,J),I=1,NR),J=1,NE	Pressure at density **R(I)** and at specific internal energy **DE(J)** + **E0(I)** ($Mbar$).
(T(I,J),I=1,NR),J=1,NE	Temperature at density **R(I)** and at specific internal energy **DE(J)** + **E0(I)** ($Kelvin$).

If more than one material is required, the corresponding tables are loaded successively up to a maximum of four. The materials will be referenced everywhere by their order in this loading. The routine prints out the identification and number of memory words needed for every material and the total number of materials loaded.

Afterwards, the routine **EOSIN2**, called by the main program, converts these tables to the system of units of the program. In addition, the negative pressures are set to zero.

The routine **EOS**, called by the routine **SCHRITT**, supplies the values of the pressure **P**, temperature **T** and its derivatives with respect to density and energy **DPDR**, **DPDE**, **DTDR** and **DTDE**. The values in all computational cells are requested at the same time. This routine uses the information stored in the 'Layer Description Table' (**COMLDT**) in order to generate a call to the routine **EOSM** for every layer present. This routine performs really the interpolation using the auxiliary routines **EOSLIN** and **EOSBIN** for linear interpolation in a 1-dimensional array and for bilinear interpolation in a 2-dimensional array,

respectively.

Seting the flag **IPRT** to one, the routine **EOS** generates a listing with all the interpolated values. This is intended to be a diagnostic test of the integrity of the tables.

6.10 OPACITIES

The routine **OPAIN** is called from the main program. It reads, from the FORTRAN unit 15, a series of opacity tables. The format of one of these tables consist of a string of real numbers, four numbers packed in one line in fields of 15 characters (format 4E15.0). In the case that the total number is not an even multiple of four, the last line is left partially empty. The meaning of every item is given by

ITEM	DESCRIPTION
MID	Material identification.
TYP	Data type (unused).
NR	Number of tabulated densities.
NT	Number of tabulated temperatures.
LR(I),I=1,NR	Decimal logarithm of the tabulated densities ($g\ cm^{-3}$).
LT(J),J=1,NT	Decimal logarithm of the tabulated temperatures (eV).
(LO(I,J),I=1,NR),J=1,NT	Decimal logarithm of the opacity corresponding to **LR(I)** and **LT(J)** ($cm^2\ gr^{-1}$).

If more than one opacity is required, the corresponding tables are loaded successively up to a maximum of 100. The opacities will be referenced everywhere by their order in this loading. The routine prints out the identification and number of memory words needed for every table and the total number of tables loaded.

The routine **OPACIT**, called by the routine **GROUP**, supplies the values of the Planck (**OP**) and Rosseland (**OR**) opacities. The values in all computational cells are requested at the same time. This routine uses the information stored in

the 'Layer Description Table' (**COMLDT**) and in the 'Groups Description Table' (**COMGDT**) in order to generate two calls to the routine **OPA** for every layer present. This routine really performs the interpolation using the auxiliary routine **OPABIN** for bilinear interpolation in a 2-dimensional array. In the case that both Planck and Rosseland opacities are the same, only one call is needed for every layer.

Setting the flag **IPRT** to one, the routine **OPACIT** generates a listing with all the interpolated values. This is intended to be a diagnostic test of the integrity of the tables.

7. Output of a typical run

The output consists of the repetition of the input parameters, a printout of the variables at selected times or step numbers, a series of diagnostic messages and a binary output on disk.

A composed target is considered: an aluminium layer of $1\mu m$ thickness covered by 0.05 μm of gold. A laser pulse of $10^{13}\,W/cm^2$ intensity, $300\,ps$ (FWHM) duration, and $1.3\,\mu m$ wavelength is incident from the right.

SESAME EOS, Planck and Rosseland opacities are used for the aluminium. For gold , SESAME EOS together with the following opacities are used:

Frenquecies	Opacity(cm^2/g)
$< 200\,eV$	$2.500 \times 10^6 \times T^{-1}$
$200\,eV - 400\,eV$	$1.250 \times 10^6 \times T^{-1}$
$400\,eV - 600\,eV$	$0.625 \times 10^6 \times T^{-1}$
$600\,eV - 800\,eV$	$2.500 \times 10^6 \times T^{-1}$
$800\,eV - 1200\,eV$	$0.625 \times 10^6 \times T^{-1}$
$> 1200\,eV$	$0.250 \times 10^6 \times T^{-1}$

where T is given in eV. The number of groups is 8. The total CPU time needed by the CRAY-1 computer is 16.2 seconds. This value can be considerably reduced using subcycling. The output file has been post-processed by the program **P3D**, and a series of plots are included: $T(m,t)$, $\rho(m,t)$, $P(m,t)$, $\langle I_k^+ \rangle$ (at the right boundary, as a function of time and frequency), $\sum_{1\le k\le \mathbf{NG}} U_k(m,t)$, and $\sum_{1\le k\le \mathbf{NG}} S_k(m,t)$.

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```
PROGRAM MULTI     VERSION 1.4

IFLAG1=1  (ENABLE HYDRODYNAMICS)
IFLAG2=1  (FREE BOUNDARIES)
IFLAG3=1  (ENABLE ENERGY DEPOSITION)
IFLAG4=1  (ENABLE THERMAL FLUX)
IFLAG5=1  (ENABLE RADIATION TRANSPORT)
NITER =1  (NUMBER OF ITERATIONS IN RADIATION)
NSPLIT=1  (NUMBER OF SUBCICLES)
ALPHAR=0.   (IF ISOLATED =1)
ALPHAL=0.   (IF ISOLATED =1)
TIMEX =6.E-10 (RUNING TIME)
DT    =1.E-12 (TIME STEP)
DTO   =4.E-12 (OUTPUT INTERVAL)
IABORT=1000000 (ABORT AT THIS STEP)
TOUT  =3.E-10,  4*0. (DETAILED OUTPUT AT THESE TIMES)
ISOUT =5*0 (DETAILED OUTPUT AT THESE STEPS)
IWCTRL=1  (FEEDBACK FOR THE ENERGY PARTITION)


EOSIN1-1 READ EOS TABLES FROM UNIT 13
EOSIN1-2 MATERIAL 3712 NEEDS 9125 STORAGE UNITS
EOSIN1-2 MATERIAL 2700 NEEDS 4871 STORAGE UNITS
EOSIN1-3 2 MATERIALS LOADED
```

INITVR-1 INPUT DATA

* MULTILAYER DEFINITION

*. I1	.. I2	.. MAT	..THICK	.. R0	.. E0	..ZONPAR.
1.	30.	1.	1.0E-4	2.7	1.E5	0.9
31.	50.	2.	5.0E-6	19.3	3.E8	1.0

INITVR-4 RESULTS

I	DM(I)	R(I)	V(I)	E(I)
1	0.28195228E-04	2.7000000	0.00000000	100000.00
2	0.25375706E-04	2.7000000	0.00000000	100000.00
3	0.22838135E-04	2.7000000	0.00000000	100000.00
4	0.20554321E-04	2.7000000	0.00000000	100000.00
5	0.18498889E-04	2.7000000	0.00000000	100000.00
6	0.16649000E-04	2.7000000	0.00000000	100000.00
7	0.14984100E-04	2.7000000	0.00000000	100000.00
8	0.13485690E-04	2.7000000	0.00000000	100000.00
9	0.12137121E-04	2.7000000	0.00000000	100000.00
10	0.10923409E-04	2.7000000	0.00000000	100000.00
11	0.98310683E-05	2.7000000	0.00000000	100000.00
12	0.88479614E-05	2.7000000	0.00000000	100000.00
13	0.79631653E-05	2.7000000	0.00000000	100000.00
14	0.71668488E-05	2.7000000	0.00000000	100000.00
15	0.64501639E-05	2.7000000	0.00000000	100000.00
16	0.58051475E-05	2.7000000	0.00000000	100000.00
17	0.52246327E-05	2.7000000	0.00000000	100000.00
18	0.47021695E-05	2.7000000	0.00000000	100000.00
19	0.42319525E-05	2.7000000	0.00000000	100000.00
20	0.38087573E-05	2.7000000	0.00000000	100000.00
21	0.34278815E-05	2.7000000	0.00000000	100000.00
22	0.30850934E-05	2.7000000	0.00000000	100000.00
23	0.27765840E-05	2.7000000	0.00000000	100000.00
24	0.24989256E-05	2.7000000	0.00000000	100000.00
25	0.22490331E-05	2.7000000	0.00000000	100000.00
26	0.20241298E-05	2.7000000	0.00000000	100000.00
27	0.18217168E-05	2.7000000	0.00000000	100000.00
28	0.16395451E-05	2.7000000	0.00000000	100000.00
29	0.14755906E-05	2.7000000	0.00000000	100000.00
30	0.13280315E-05	2.7000000	0.00000000	100000.00
31	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
32	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
33	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
34	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
35	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
36	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
37	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
38	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
39	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
40	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
41	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
42	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
43	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
44	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
45	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
46	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
47	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
48	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
49	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
50	0.48250000E-05	19.300000	0.00000000	0.30000000E+09
51			0.00000000	

OPAIN-1 READ OPACITY DATA FROM UNIT 15
OPAIN-2 TABLE 13710 NEEDS 1505 STORAGE UNITS
OPAIN-2 TABLE 13710 NEEDS 1505 STORAGE UNITS
OPAIN-2 TABLE 1 NEEDS 10 STORAGE UNITS
OPAIN-2 TABLE 0 NEEDS 10 STORAGE UNITS
OPAIN-2 TABLE 0 NEEDS 10 STORAGE UNITS
OPAIN-2 TABLE 0 NEEDS 10 STORAGE UNITS
OPAIN-2 TABLE 0 NEEDS 10 STORAGE UNITS
OPAIN-2 TABLE 0 NEEDS 10 STORAGE UNITS
OPAIN-3 9 TABLES LOADED

QUELIN-1 LASER PULSE CHARACTERISTICS
IDIR 1
MAXIMUM POWER 1.E+20
PULSE DURATION 3.E-10
WAWELENGTH 1.3E-4
FRACTION DUMPED AT CRITICAL 1.
CHARGE NUMBER 20.
ATOMIC MASS NUMBER 197.

RDGDT-1 GROUP DEFINITION TABLE

* GROUP DEFINITION TABLE

*.FREC A.	.FREC B.	G	..COS A	..COS B	..P1	..P2	..P3	..P4	..R1	..R2	..R3	..R4.
0.0001	200.	.5000000	0.	1.	1.	3.			2.	3.		
200.	400.	.5000000	0.	1.	1.	4.			2.	4.		
400.	600.	.5000000	0.	1.	1.	5.			2.	5.		
600.	800.	.5000000	0.	1.	1.	3.			2.	3.		
800.	1000.	.5000000	0.	1.	1.	5.			2.	5.		
1000.	1200.	.5000000	0.	1.	1.	5.			2.	5.		
1200.	1400.	.5000000	0.	1.	1.	6.			2.	6.		
1400.	1600.	.5000000	0.	1.	1.	6.			2.	6.		

SOUTIN-1 READ DATA FROM UNIT 18

* SELECTED SPECTRAL OUTPUT
U FOR GROUP 1 TO 999
S FOR GROUP 1 TO 999
FPLS FOR GROUP 1 TO 999
FMNS FOR GROUP 1 TO 999
FPLS FOR POINT 51
FMNS FOR POINT 51

INITIAL VALUES FOR SCHRITT

ISTEP =302 ITRY =302 TIME1 =3.00999999999989E-10

DENSITY

NUM	1	2	3	4	5
1	3.1888104	3.5832776	3.8950065	4.1390699	4.3183768
6	4.4501149	4.5461567	4.6244481	4.6915814	4.7507804
11	4.8012067	4.2382931	1.8239365	1.0849983	0.84257984
16	0.70127664	0.62751444	0.55298448	0.48603393	0.41770479
21	0.35707521	0.30569584	0.26635580	0.23569984	0.21105824
26	0.19182933	0.17673451	0.16407825	0.15317468	0.13454846
31	0.43547475	0.35616076	0.30166079	0.26176242	0.22852911
36	0.20265808	0.18273755	0.16677794	0.14913222	0.13223368
41	0.11731502	0.10348889	0.89956084E-01	0.76975852E-01	0.64634624E-01
46	0.52756755E-01	0.41149408E-01	0.29731780E-01	0.17103363E-01	0.28393520E-02

VELOCITY

NUM	1	2	3	4	5
1	-1470525.0	-1472780.3	-1455024.2	-1451384.7	-1450968.4
6	-1452075.7	-1452514.7	-1453412.4	-1455544.4	-1458185.8
11	-1461515.1	-1463860.6	-1407708.6	-1226454.1	-1107756.7
16	-1025034.0	-947518.51	-865739.04	-782099.80	-661261.45
21	-516038.19	-342515.91	-159804.08	31221.935	223482.82
26	407236.26	576147.81	729284.70	874061.59	1008883.8
31	1126922.8	1230167.1	1349059.1	1468626.5	1615423.8
36	1776529.7	1937170.6	2089651.7	2265163.3	2495862.4
41	2748032.3	2997573.9	3261488.7	3548073.8	3874509.8
46	4264417.4	4746363.1	5363849.2	6202747.6	7511562.4
51	16340748.				

ENERGY

NUM	1	2	3	4	5
1	0.12181426E+11	0.37449611E+11	0.48878406E+11	0.54511757E+11	0.59101919E+11
6	0.62685281E+11	0.65369739E+11	0.67915022E+11	0.70515583E+11	0.73194097E+11
11	0.75725917E+11	0.18183122E+12	0.14292415E+13	0.24277803E+13	0.32354249E+13
16	0.40783381E+13	0.52799085E+13	0.68084825E+13	0.87457933E+13	0.10866565E+14
21	0.13088663E+14	0.15308505E+14	0.17473138E+14	0.19587215E+14	0.21588904E+14
26	0.23431059E+14	0.25183420E+14	0.26906361E+14	0.28582221E+14	0.32463027E+14
31	0.11805731E+14	0.13892594E+14	0.15734629E+14	0.17301848E+14	0.18732443E+14
36	0.20002714E+14	0.21123096E+14	0.22352537E+14	0.23796102E+14	0.24974766E+14
41	0.26070712E+14	0.27121971E+14	0.28160741E+14	0.29194669E+14	0.30254598E+14
46	0.31354745E+14	0.32520455E+14	0.33877768E+14	0.38829478E+14	0.10075684E+15

INTERPOLATED EOS VALUES

PRESSURE

NUM	1	2	3	4	5
1	0.22831943E+12	0.53648125E+12	0.74465759E+12	0.90966765E+12	0.10433685E+13
6	0.11483623E+13	0.12278169E+13	0.12968882E+13	0.13584532E+13	0.14149244E+13
11	0.14676187E+13	0.15893065E+13	0.15945608E+13	0.14904131E+13	0.14782550E+13
16	0.14630052E+13	0.14997139E+13	0.15103301E+13	0.15291509E+13	0.15264656E+13
21	0.15132184E+13	0.14818663E+13	0.14518666E+13	0.14225343E+13	0.13890416E+13
26	0.13572543E+13	0.13298644E+13	0.13056294E+13	0.12838877E+13	0.12607801E+13
31	0.12152745E+13	0.11518473E+13	0.10902462E+13	0.10306916E+13	0.96648003E+12
36	0.90807141E+12	0.85831484E+12	0.82220370E+12	0.77426723E+12	0.71953868E+12
41	0.66558695E+12	0.61018314E+12	0.55018024E+12	0.48764577E+12	0.42397115E+12
46	0.35834914E+12	0.28966452E+12	0.21783787E+12	0.14324702E+12	0.64729805E+11

TEMPERATURE

NUM	1	2	3	4	5
1	0.10393349	0.22728805	0.24842190	0.21263835	0.17987049
6	0.15259567	0.13057139	0.11358951	0.10269770	0.96345844E-01
11	0.90852133E-01	1.0042349	6.6892864	10.418635	12.573919
16	14.601058	16.831716	19.275582	21.975214	24.645195
21	27.215466	29.641496	31.716570	33.451706	35.075066
26	36.552286	37.642591	38.684697	39.686161	42.089560
31	45.654841	49.440571	52.795589	55.445982	57.750956
36	59.822572	61.680208	63.143224	64.671751	66.286238
41	67.787424	69.227397	70.650265	72.066500	73.518348
46	75.025288	76.622033	78.481228	85.263889	162.15837

VALUES AFTER HYDRO

DENSITY

NUM	1	2	3	4	5
1	3.1878661	3.5731007	3.8918374	4.1380716	4.3185288
6	4.4496642	4.5472476	4.6277140	4.6964341	4.7560941
11	4.7987035	4.1279011	1.7529562	1.0667886	0.83347332
16	0.69541632	0.62121525	0.54762410	0.47927295	0.41111052
21	0.35069219	0.30032399	0.26163394	0.23152552	0.20753270
26	0.18886233	0.17417754	0.16177324	0.15107795	0.13298576
31	0.43150279	0.35310206	0.29943826	0.25969990	0.22682526
36	0.20132384	0.18171123	0.16576445	0.14807818	0.13133315
41	0.11661068	0.10290752	0.89479610E-01	0.76578798E-01	0.64300174E-01
46	0.52481290E-01	0.40934744E-01	0.29579559E-01	0.17024692E-01	0.28247340E-02

VELOCITY

NUM	1	2	3	4	5
1	-1486672.0	-1484054.8	-1463841.2	-1459064.6	-1457866.6
6	-1458017.3	-1457638.4	-1458428.9	-1460485.4	-1463155.8
11	-1465720.1	-1464654.9	-1407354.9	-1223343.3	-1108499.3
16	-1023973.9	-953502.32	-868282.20	-784233.96	-659751.77
21	-511216.67	-333406.91	-149642.71	41929.046	236506.18
26	420643.88	589070.96	742631.74	887075.77	1022632.7
31	1140012.5	1242955.2	1361340.7	1480946.2	1628497.8
36	1788315.4	1947178.8	2097186.9	2275128.0	2507061.2
41	2758986.7	3008913.7	3273795.4	3560934.1	3887625.8
46	4277936.7	4760507.8	5378636.1	6218092.2	7527756.0
51	16367406.				

ENERGY

NUM	1	2	3	4	5
1	0.12160167E+11	0.37027061E+11	0.48723057E+11	0.54458757E+11	0.59110427E+11
6	0.62659135E+11	0.65434579E+11	0.68113341E+11	0.70815598E+11	0.73528084E+11
11	0.75566153E+11	0.17228305E+12	0.13952514E+13	0.24046409E+13	0.32164082E+13
16	0.40608306E+13	0.52558007E+13	0.67818533E+13	0.87016677E+13	0.10808305E+14
21	0.13012068E+14	0.15222358E+14	0.17375414E+14	0.19479105E+14	0.21477786E+14
26	0.23320524E+14	0.25073526E+14	0.26793552E+14	0.28466435E+14	0.32353352E+14
31	0.11780115E+14	0.13864649E+14	0.15707864E+14	0.17270645E+14	0.18700740E+14
36	0.19973073E+14	0.21096609E+14	0.22322451E+14	0.23759201E+14	0.24937507E+14
41	0.26036487E+14	0.27088701E+14	0.28128210E+14	0.29161858E+14	0.30220516E+14
46	0.31319132E+14	0.32483581E+14	0.33840104E+14	0.38790813E+14	0.10063901E+15

TEMPERATURE

NUM	1	2	3	4	5
1	0.10388425	0.22603636	0.24810562	0.21256567	0.17987691
6	0.15256830	0.13059366	0.11369276	0.10289796	0.96369580E-01

11	0.90819261E-01	0.98449245	6.5518237	10.336391	12.513139
16	14.566179	16.778371	19.221343	21.883404	24.523054
21	27.062452	29.465637	31.549431	33.269333	34.898171
26	36.371887	37.493961	38.538502	39.533688	41.954990
31	45.568985	49.336717	52.704066	55.329152	57.640278
36	59.724703	61.596510	63.073104	64.621206	66.235203
41	67.740543	69.181825	70.605704	72.021556	73.471665
46	74.976506	76.571524	78.429637	85.210926	162.02168

EXTERNAL ENERGY DEPOSITION

EXTERN

NUM	1	2	3	4	5
1	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
6	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
11	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
16	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
21	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
26	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
31	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
36	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
41	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
46	0.00000000	0.00000000	0.19713199E+22	0.27877664E+23	0.20694261E+26

VALUES AFTER RADIATION

TEMPERATURE

NUM	1	2	3	4	5
1	0.10388425	0.22603636	0.24810562	0.21256567	0.17987691
6	0.15256830	0.13059366	0.11369276	0.10289796	0.96369585E-01
11	0.91014988E-01	1.1767005	6.9477798	10.564971	12.696484
16	14.737773	17.009206	19.487616	22.207711	24.902455
21	27.476745	29.913505	31.939914	33.674698	35.308307
26	36.778842	37.817587	38.859753	39.848554	42.191794
31	45.705247	49.549195	52.903568	55.532880	57.835585
36	59.903339	61.761023	63.200365	64.748093	66.345477
41	67.840740	69.275740	70.694996	72.107497	73.556062
46	75.059094	76.652143	78.487254	84.421466	162.14734

VALUES AFTER THERMAL HEATING (WAW=3.2033857547175E-2)

TEMPERATURE INCREMENT

NUM	1	2	3	4	5
1	0.34610535E-08	-0.89211881E-09	-0.52570959E-08	0.11722489E-08	0.12486830E-08
6	0.95340936E-09	0.67115728E-09	0.52694255E-09	0.29923860E-09	0.88532701E-10
11	0.13473217E-04	0.32174673E-02	0.20958358E-02	-0.51018730E-03	0.10174789E-02
16	0.19113343E-02	0.31101671E-02	0.40638432E-02	0.37919626E-02	0.32769491E-02
21	0.28361567E-02	-0.10097688E-02	-0.74866993E-03	0.13450819E-02	0.11075681E-03
26	-0.66151066E-02	0.21038476E-02	0.13544617E-02	0.32966589E-02	0.45760536E-01
31	0.56090202E-01	-0.14384612E-01	-0.23508260E-01	-0.12323587E-01	-0.90596251E-02
36	-0.76850745E-02	-0.12201129E-01	0.52473452E-03	-0.62199410E-03	-0.29328339E-02
41	-0.22659625E-02	-0.17270905E-02	-0.17351783E-02	-0.13945733E-02	-0.14181400E-02
46	-0.14344282E-02	-0.84119874E-03	0.19663645E-01	0.85504266	0.19908431E-01

TEMPERATURE

NUM	1	2	3	4	5
1	0.10388425	0.22603636	0.24810562	0.21256567	0.17987691
6	0.15256831	0.13059366	0.11369276	0.10289796	0.96369585E-01
11	0.91028461E-01	1.1799180	6.9498757	10.564461	12.697501
16	14.739685	17.012316	19.491680	22.211503	24.905732

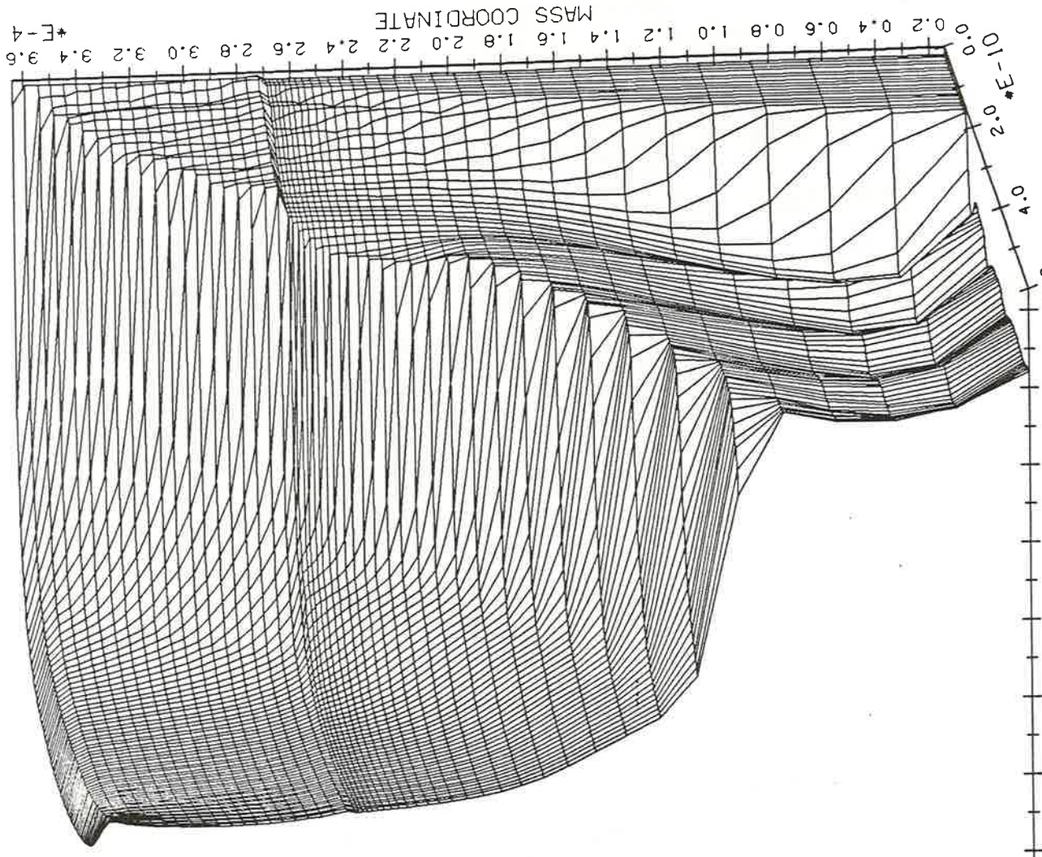
21	27.479581	29.912496	31.939166	33.676043	35.308418
26	36.772227	37.819691	38.861108	39.851851	42.237554
31	45.761337	49.534810	52.880060	55.520557	57.826525
36	59.895654	61.748822	63.200890	64.747471	66.342544
41	67.838474	69.274013	70.693261	72.106103	73.554643
46	75.057660	76.651301	78.506918	85.276509	162.16725

MULTI-3 NORMAL EXIT WITH ISTEP=601 AND ITRY=601

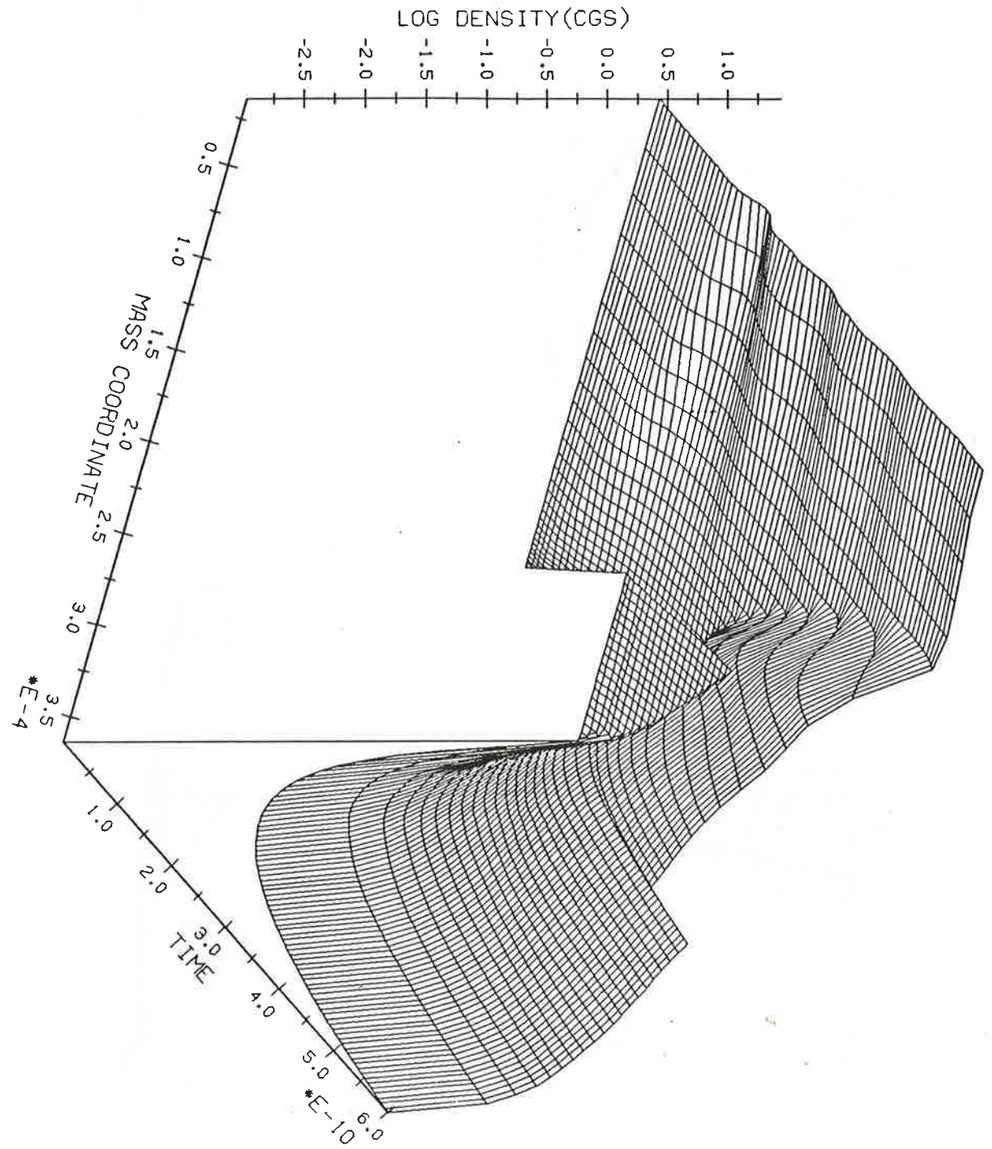
```
ENTRY P3D

ENTRY ZEIGEN

SECTION 1 NUMBER OF VALUES 324

NAME       NUMBER      INDEX          VALUE
.PROGRAM        1          0          MULTI
.VERSION        1          0          1.4
.JOB            1          0          RES008
CMC            50          1       0.14097614E-04
                           50       0.36408750E-03
CMI            51          1       0.00000000
                           51       0.36650000E-03
SOCFRE         16       1001        100.00005
                        2008        1500.0000
SOFCMI        204       1001       0.00000000
                        4051       0.36650000E-03

SECTION 2 NUMBER OF VALUES 371

NAME       NUMBER      INDEX          VALUE
TIME            1          0       0.10000000E-11
R              50          1        2.7000000
                          50        19.112519
T              50          1       0.25848886E-01
                          50       0.33499435E-01
P              50          1        1052918.1
                          50       0.77646354E+10
SOC            16       1001        1298.7823
                        2008       0.00000000
SOF           204       1001       0.61541086E-04
                        4051      -0.25916719E-11

NUMBER OF RECORDS: 158
NUMBER OF WORDS  : 57737

EXIT ZEIGEN

ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA: CMC          1.       50.      1.    MASS COORDINATE
DATA: TIME         1.     1000.      1.         TIME
DATA: T                                    LOG TEMP. (EV)
DATA:   -5.       15.      0.01
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 50
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 3800


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA: CMC          1.       50.      1.    MASS COORDINATE
DATA: TIME         1.     1000.      1.         TIME
DATA: R                                    LOG DENSITY(CGS)
DATA:    30.      30.    0.0001
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 50
```

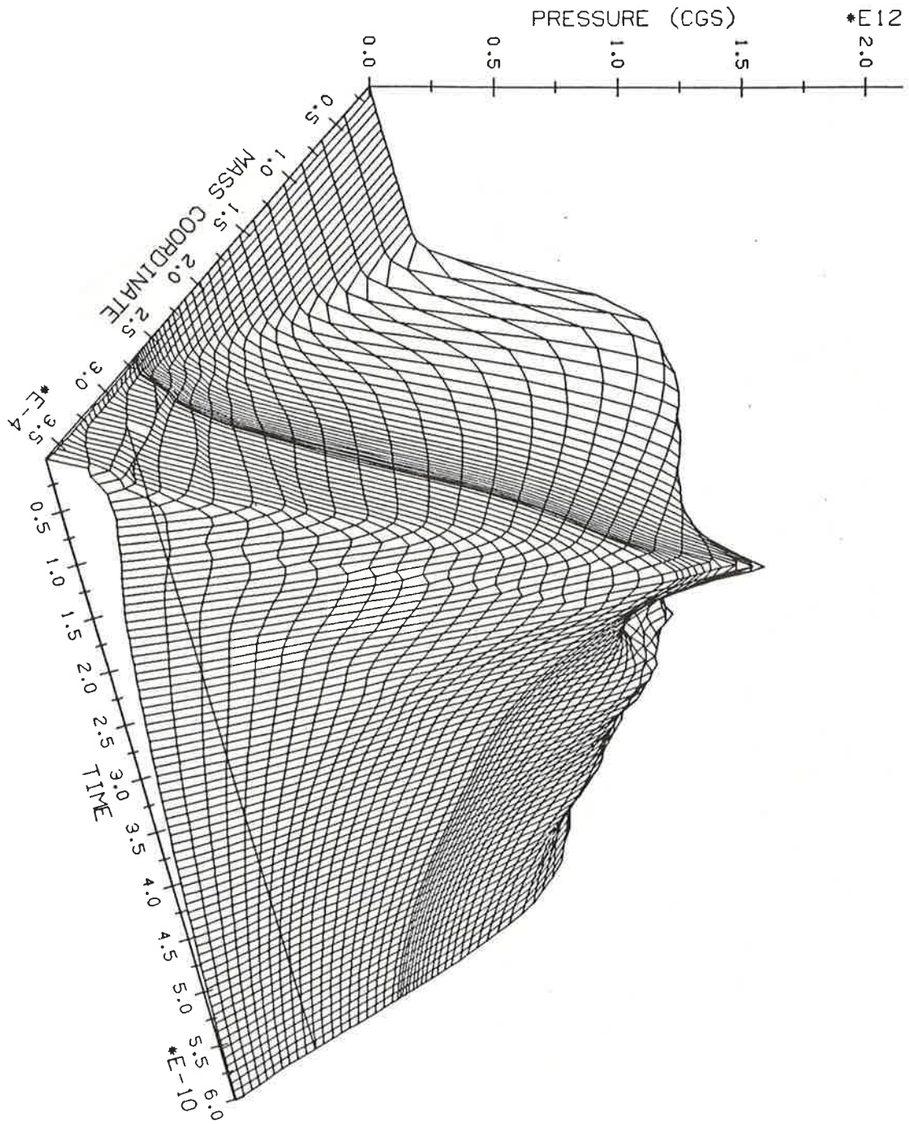


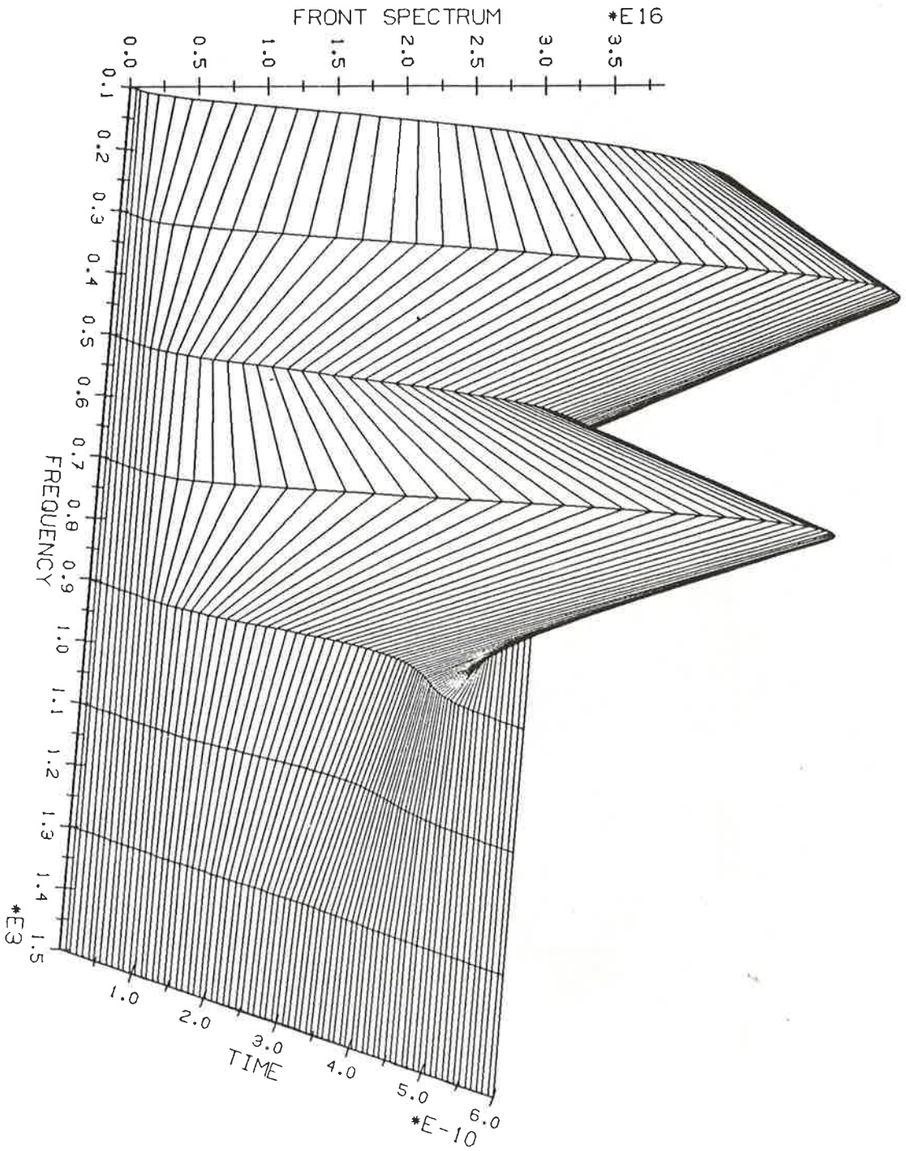
```
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 3800


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA: CMC          1.       50.      1.    MASS COORDINATE
DATA: TIME         1.     1000.      1.         TIME
DATA: P                                     PRESSURE (CGS)
DATA:    60.      30.    .0
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 50
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 3800


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA: SOCFRE       1.        8.      1.       FREQUENCY
DATA: TIME         1.     1000.      1.         TIME
DATA: SOC                                   FRONT SPECTRUM
DATA:     10      30.    .0
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 8
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 608


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA:CSOCFRE       9.       16.      1.       FREQUENCY
DATA:CTIME         1.     1000.      1.         TIME
DATA:CSOC                                   MANHATTAN
DATA:C     10      30.    .0
DATA:C........00000000........00000000........00000000
DATA: SOFCMI       1.       51.      1.    MASS COORDINATE
DATA: TIME         1.     1000.      1.         TIME
DATA: SOF                                   LOG RAD. ENERGY
DATA:    -10       30.    .0001
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 51
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 3876


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA: SOFCMI      52.      102.      1.    MASS COORDINATE
DATA: TIME         1.     1000.      1.         TIME
DATA: SOF                                   RADIATION FLUX
DATA:    -10       30.    .0
EXIT LESEN

LADEN-8 NUMBER OF X-VALUES 51
        NUMBER OF Y-VALUES 76
        NUMBER OF Z-VALUES 3876


ENTRY LESEN
DATA:C........00000000........00000000........00000000
DATA:CSOFCMI    103.      153.      1.    MASS COORDINATE
```

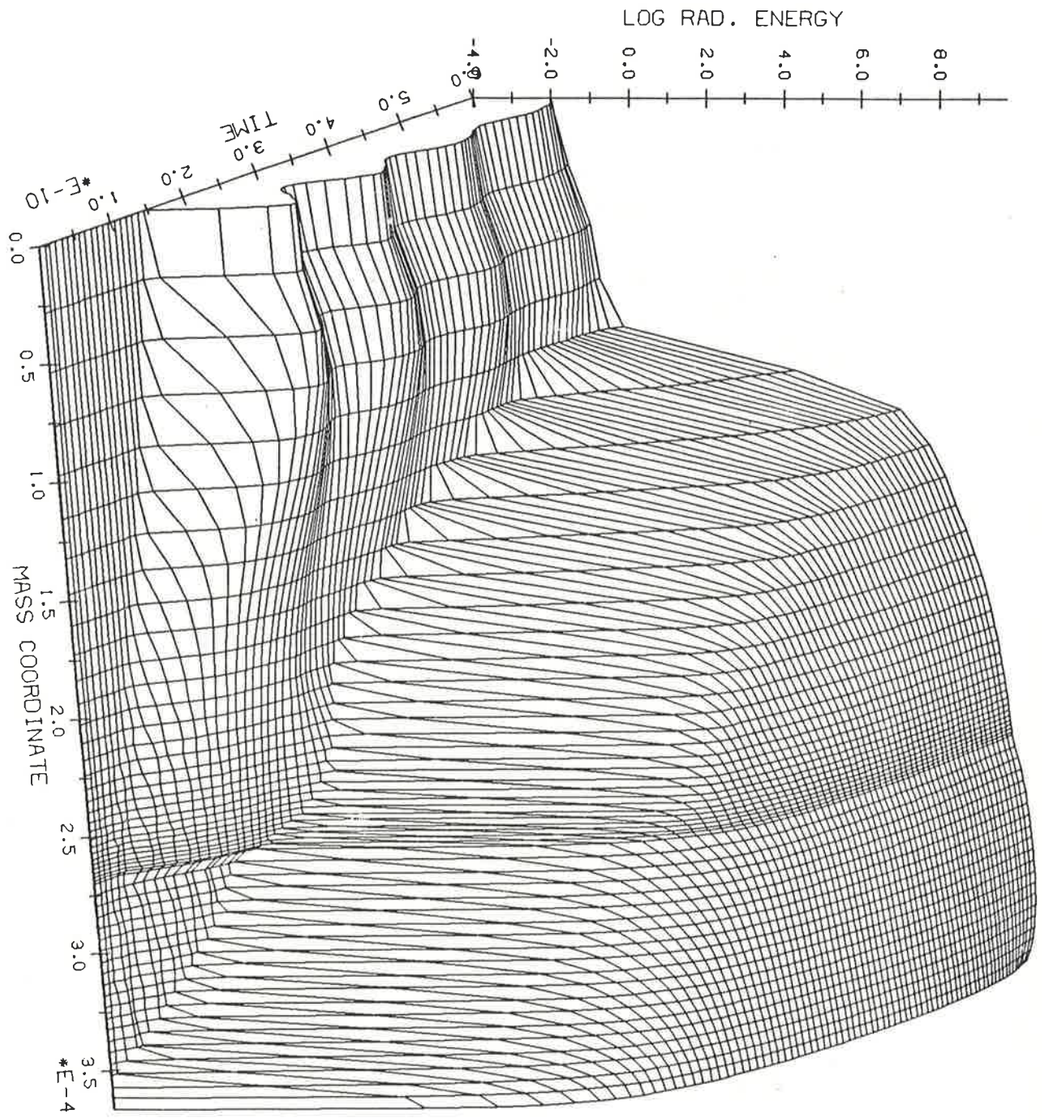



```
DATA:CTIME       1.    1000.      1.        TIME
DATA:CSOF                                  F PLUS
DATA:C   -10     30.     .0
DATA:C.....OOOOOOOO.....OOOOOOOO.....OOOOOOOO
DATA:CSOFCMI   154.     204.      1.   MASS COORDINATE
DATA:CTIME       1.     1000.     1.        TIME
DATA:CSOF                                  F MINUS
DATA:C   -10     30.     .0
END OF FILE
EXIT LESEN

EXIT P3D
```


LOG DENSITY(CGS)
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
MASS COORDINATE
0.5
1.0
1.5
2.0
2.5
3.0
3.5
*E-4
TIME
1.0
2.0
3.0
4.0
5.0
6.0
*E-10


PRESSURE (CGS)
*E12
0.0
0.5
1.0
1.5
2.0
MASS COORDINATE
0.5
1.0
1.5
2.0
2.5
3.0
3.5
*E-4
TIME
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
*E-10


FRONT SPECTRUM
*E16
FREQUENCY
TIME
*E-10
*E3


LOG RAD. ENERGY
TIME
*E-10
MASS COORDINATE
*E-4

IPP-CRAY 25/02/86 11:35:08 RES007 D2-03 006