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Resolved-sideband cooling and position measurement of a micromechanical oscillator close to the Heisenberg uncertainty limit

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The theory of quantum measurement of mechanical motion, describing the mutual coupling of a meter and a measured object, predicts a variety of phenomena such as quantum backaction, quantum correlations and non-classical states of motion. In spite of great experimental efforts, mostly based on nano-electromechanical systems, probing these in a laboratory setting has as yet eluded researchers. Cavity optomechanical systems, in which a high-quality optical resonator is parametrically coupled to a mechanical oscillator, hold great promise as a route towards the observation of such effects with macroscopic oscillators. Here, we present measurements on optomechanical systems exhibiting radiofrequency (62–122 MHz) mechanical modes, cooled to very low occupancy using a combination of cryogenic precooling and resolved-sideband laser cooling. The lowest achieved occupancy is $n \sim 63$. Optical measurements of these ultracold oscillators' motion are shown to perform in a near-ideal manner, exhibiting an imprecision–backaction product about one order of magnitude lower than the results obtained with nano-electromechanical transducers.

The observation of quantum phenomena in macroscopic mechanical oscillators[1,2] has been a subject of interest since the inception of quantum mechanics. It may provide insights into the quantum–classical boundary, and allow experimental investigation of the theory of quantum measurements[1,3,4], the origin of mechanical decoherence[5] and the generation of non-classical states of motion. Prerequisite to this regime are both preparation of the mechanical oscillator at low phonon occupancy and measurement sensitivity at the scale of the position spread Δx of the oscillator's ground-state wavefunction. Over the past decade, it has been widely perceived that the most promising approach to address these two challenges are electro-nanomechanical systems[2,6–11], which can be cooled with millikelvin-scale dilution refrigerators, and feature large displacements $\Delta x \sim 10^{-14}$ m that are resolvable with electronic transducers such as a superconducting single-electron transistor[7–9], a strip-line cavity[10] or a quantum interference device[12]. Fundamentally however, every linear continuous transducer induces a perturbation of the mechanical system by measurement backaction, leading to the Heisenberg uncertainty limit in the ideal case. Although low thermal occupation numbers have been achieved with nano-electromechanical systems, these experiments[9,11] have as yet remained far from attaining Heisenberg-uncertainty-limited measurements.

Here, we demonstrate cooling and measurement of the motion of a mechanical oscillator of mesoscopic dimensions—1,000 times more massive than the heaviest nanomechanical oscillators used so far—close to this limit. Imperative to these advances are two key principles of cavity optomechanics[13]: optical interferometric measurement of mechanical displacement at the attometre level[14,15] and the ability to use measurement-induced dynamical backaction[16–19] to achieve resolved-sideband laser cooling[11,20] of the mechanical degree of freedom close to its quantum ground state (63 ± 20 phonons). The cooling laser simultaneously acts as a highly ideal displacement transducer, exhibiting a measured imprecision–backaction product one order of magnitude lower than achieved in nano-electromechanical systems[9,21,22], thereby demonstrating the closest approach to the Heisenberg uncertainty product for continuous position measurements yet demonstrated.

The experimental setting of the present work is a cavity optomechanical system, which parametrically couples optical and mechanical degrees of freedom through radiation pressure. In the present case, toroidal microresonators[23] are used that exhibit (see Fig. 1) strong, inherent optomechanical coupling between high-quality-factor ($Q > 10^8$) optical whispering-gallery modes (WGMs) and the mechanical radial-breathing mode[24] (RBM), featuring high frequency (between 62 and 122 MHz for the resonators used in this work) and effective masses[25] of the order of 1–10 ng (see Fig. 1b). The quality factors of the RBM can reach values up to 80,000 if clamping losses are mitigated by modal engineering, for example in spoke resonators[26].

To achieve a regime of low mechanical-oscillator phonon occupancy, we apply laser cooling to a cryogenically precooled micromechanical oscillator with high frequency. Figure 1 shows a schematic diagram of the experiment. A chip with micro-resonators is inserted into a helium exchange gas cryostat. Piezoelectric actuators (Attocube GmbH) enable positioning of a tapered optical fibre used for evanescent coupling with a resolution sufficient to adjust the taper–toroid gap for critical coupling. The total optical loss through the cryogenic environment can reach values below 25%. Low-pressure (0.1–50 mbar) helium exchange gas is admitted into the sample chamber, thermalizing the sample with a heat exchanger through which ^{4}He is pumped from a reservoir of liquid ^{4}He. An exchange gas temperature of 1.65 K can be achieved. Owing to the low heat conductivity of glass, and possible light absorption, it is of prime importance to verify the thermalization of the mechanical oscillator to 1.65 K. To this end, we carry out noise thermometry (see the Methods section) using the RBM. A low-power (<2 μW) laser is tuned into resonance with a high-Q optical

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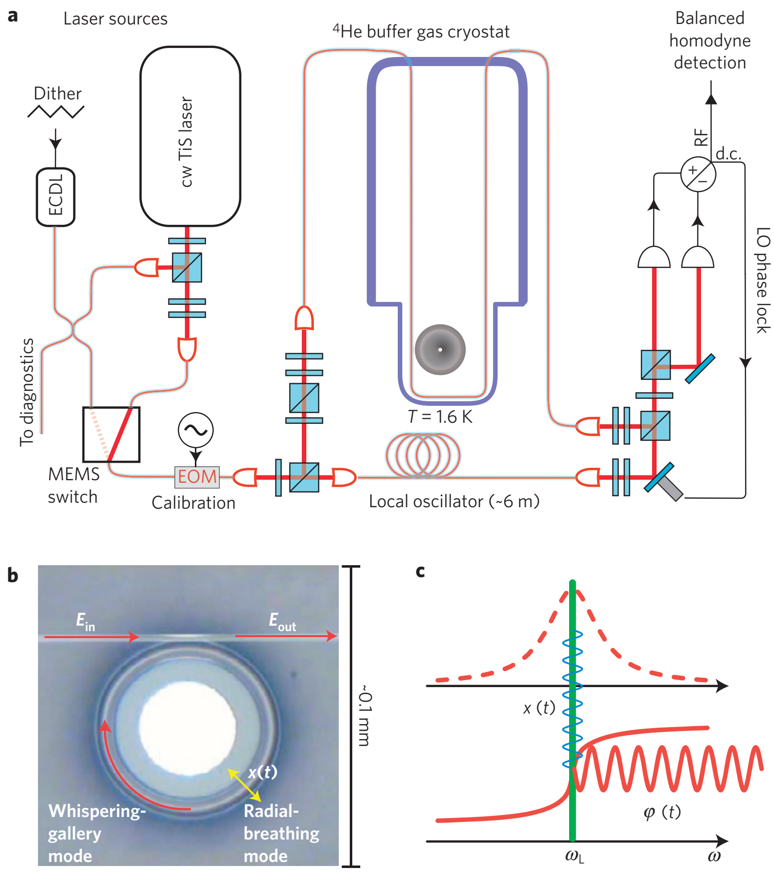

Figure 1 | Cryogenic cooling and displacement measurements of a micromechanical oscillator. a, A silica microtoroid is held in a 1.65 K-cold ^{4}He atmosphere. The toroid supports both high-Q optical WGMs and a mechanical RBM, parametrically coupled to an optical resonance frequency. High-Q optical resonances are identified using a frequency-agile external-cavity diode laser (ECDL). To probe the mechanical oscillator, the laser input is switched to a low-noise Ti:sapphire laser. **b**, A small fraction of the laser light is sent into the cryostat and evanescently coupled to the WGM by means of a tapered fibre placed in the near-field of the WGM. Balanced homodyne measurement of the laser phase as transmitted through the taper is implemented using a Mach–Zehnder fibre interferometer (phase plates and polarizing beam splitters are only schematically indicated). **c**, A modulation $x(t)$ of the radius of the cavity induces a modulation of the phase $\varphi(t)$ of the laser light emerging from the cavity. This phase shift is detected by comparison with a phase reference, derived from the same laser in a beam splitter followed by a balanced detector.

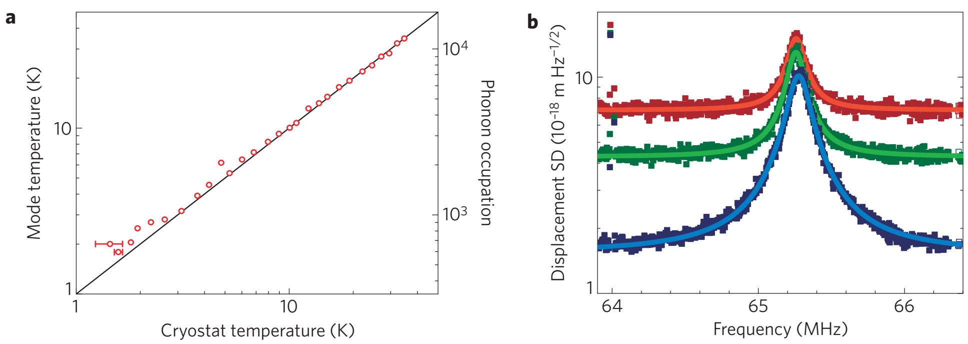

Figure 2 | Thermalization and probing of a micromechanical oscillator. a, Using noise thermometry, the effective temperature of the mechanical mode is determined as a function of the temperature of the cryostat (exchange gas). The temperature of the 62 MHz mode follows the cryostat temperature in a linear manner, down to an occupation of less than 1,000 phonons. **b**, In spite of the low phonon occupation (770 in the case of this 65.3 MHz oscillator), displacement monitoring with high signal-to-noise ratio is possible using optical techniques (sensitivity at the attometre level in the present case). The optical power used to interrogate the mechanical oscillator was about 3 μW (red trace), 10 μW (green trace) and 100 μW (blue trace). The measurement background originates from the laser quantum phase noise.

mode. Fluctuations of the cavity radius—as induced by thermal noise of the RBM—induce resonance frequency fluctuations, which are imprinted as phase fluctuations on the laser light transmitted by the tapered fibre (see Fig. 1c). A phase-sensitive detection scheme enables measurement of the Lorentzian displacement noise spectrum $S_{xx}^{\mathrm{th}}[\Omega]$ of the thermal (Brownian) motion of the RBM, where $\Omega/2\pi$ is the analysis Fourier frequency. The spectrum is characterized by its resonance frequency $\Omega_{\mathrm{m}}/2\pi$, mechanical damping rate $\Gamma_{\mathrm{m}}/2\pi$ and peak displacement amplitude $S_{xx}^{\mathrm{th}}[\Omega_{\mathrm{m}}]$ (see the Methods section). Figure 2a shows the resulting (effective)

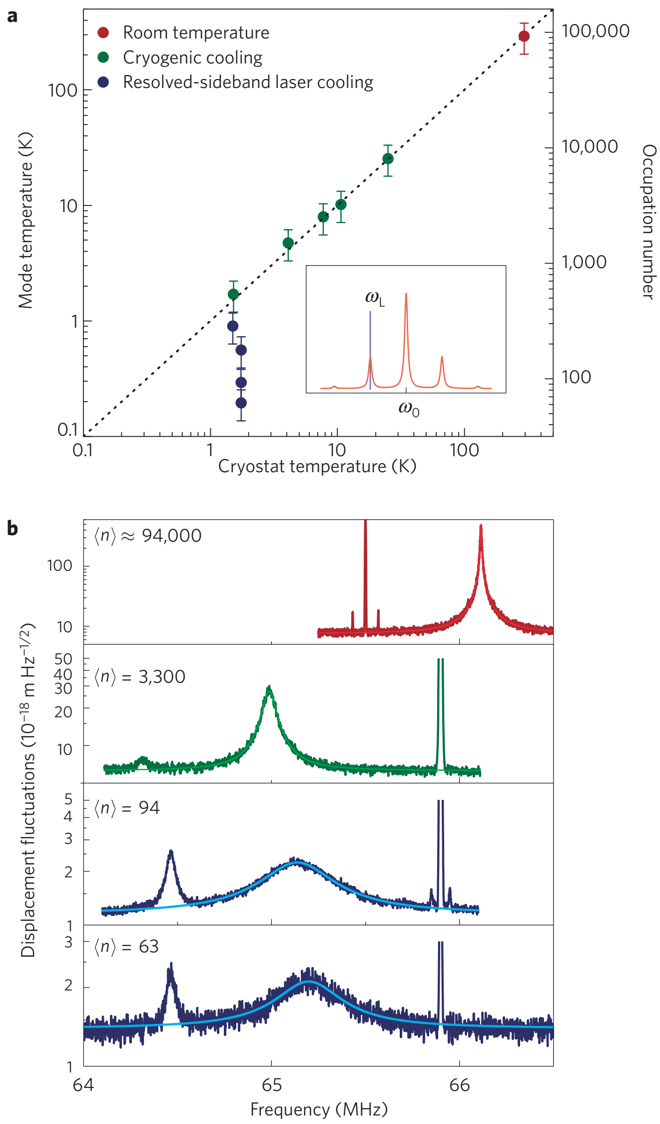

Figure 3 | Cryogenic precooling and resolved-sideband laser cooling. **a**, Mode temperature, derived from noise thermometry, compared with cryostat temperature, measured with a calibrated silicon diode close to the sample. Combined with precooling in the cryostat, resolved-sideband cooling reduces the occupation of the oscillator further to $\langle n \rangle = 63 \pm 20$ phonons. The inset illustrates the laser (blue line) detuned to the lower mechanical sideband of the cavity spectrum (red line). **b**, Displacement noise spectra of the RBM with four different average occupation numbers, together with a Lorentzian fit. The resonance frequency and intrinsic damping of the mode are modified under cryogenic cooling from room temperature (red curve) to 10 K (green curve) and 1.7 K (blue curves) owing to phonon coupling to structural defect states[27]. Extra resolved-sideband cooling (blue curves) induces extra damping and a small frequency shift. For the two traces shown, the launched laser powers were approximately 190 and 200 μW. The sharp calibration peak and a second mechanical mode (with a higher effective mass) at slightly lower frequency are also visible. It was verified that the contribution of the latter to the noise at the resonance frequency of the RBM is negligible.

mode temperature as derived by means of the equipartition theorem from the independently calibrated noise spectra, where phase-sensitive detection was accomplished using the Pound–Drever–Hall technique. Importantly, the temperature of this 62 MHz mode follows the one of the exchange gas, demonstrating that excellent thermalization is achieved, a key prerequisite for the experiments described from here on. For a 62 MHz oscillator, thermalization to 1.65 K entails an initial average occupancy of $\langle n \rangle = k_B T_{RBM}/\hbar\Omega_m \approx 560$, where T_{RBM} is the temperature of the RBM (see the Methods section). For the 122 MHz oscillator used later in this work, this temperature corresponds to $\langle n \rangle \approx 280$. Note that despite the modest precooling to 1.65 K, this occupancy would be reached for a 1 MHz nanomechanical oscillator thermalized to a dilution refrigerator temperature below 30 mK. This emphasizes the significant advantage of working with high-frequency oscillators.

Measuring the mechanical displacement associated with such a massive oscillator at low phonon number requires high sensitivity, in particular, because the mechanical quality factor $Q = \Omega_m/\Gamma_m$ of silica is typically reduced to ∼2,000 at 1.65 K owing to losses originating from phonon coupling to structural defect states[27] (note that damping by the exchange gas is negligible, and that the mechanical Q factor improves again at lower temperatures[28]). The required attometre-level displacement sensitivity can (so far) be achieved only with optical transducers. We use balanced homodyne spectroscopy[15] based on a quantum-noise-limited titanium:sapphire laser (in both amplitude and phase), which is resonantly coupled to a WGM resonance in the vicinity of a wavelength of 780 nm. The laser's phase shift introduced by the mechanical fluctuations is detected interferometrically, by comparison with a high-power (2–5 mW) optical phase reference (local oscillator). Frequency analysis of this signal yields the thermal-noise displacement spectrum $S_{xx}^{th}[\Omega]$, on top of a measurement background. Figure 2b shows data obtained from the RBM of a different, 55-μm-diameter micro-resonator cooled to 2.4 K, or $\langle n \rangle \approx 770$. The background of this measurement is at a level of $\sim 1.5 \times 10^{-18}$ m $Hz^{-1/2}$, which is a factor of only 5.5 ± 1.5 times higher than the standard quantum limit[1,29], given by $\sqrt{S_{xx}^{SQL}[\Omega_m]} = \sqrt{\hbar/m_{eff}\Gamma_m\Omega_m}$ for a measurement at the mechanical resonance frequency. This proves the counter-intuitive notion, that measurements close to the standard quantum limit are possible, in spite of the here used strategy of using high-frequency and comparatively massive oscillators.

To further decrease the number of thermal quanta of the mechanical oscillator, we use cooling through radiation-pressure dynamical backaction as predicted[16,30] and experimentally demonstrated[17–19] in 2006. Similar to the atomic physics[31] case, ground-state cooling requires accessing the resolved-sideband regime[20,32,33], which necessitates the mechanical oscillator frequency to exceed the cavity decay rate $\kappa/2\pi$ (that is, $\Omega_m \gg \kappa$). This regime is moreover prerequisite for schemes such as two-transducer quantum non-demolition measurements[1,34] or the preparation of a mechanical oscillator in a squeezed state of motion[21]. Operation in the resolved-sideband regime is accomplished using a cavity with a narrow resonance (5.5 MHz intrinsic decay rate and 9 MHz mode splitting), which is broadened to a $\kappa/2\pi \approx 19$ MHz-wide resonance owing to fibre coupling (corresponding to a loaded finesse of ∼70,000). The laser is subsequently tuned to the lower mechanical sideband, that is, red-detuned by 65.2 MHz, the resonance frequency of this sample's RBM. For this detuning, the circulating power is reduced by a factor of $4\Omega_m^2/\kappa^2 + 1 \approx 50$. At the same time, the sensitivity to mechanical displacements is slightly reduced. In the ideal case of a highly overcoupled cavity, with unity detection efficiency and no excess noise except for the laser's intrinsic quantum noise, the imprecision noise spectral density, that is, the background of the measurement caused by shot noise in the detection process, is given by:

$$S_{xx}[\Omega] = \frac{\hbar\omega}{16 g_0^2 \cdot P_{in}} \cdot \left(\frac{\Delta^2 + (\kappa/2)^2}{\kappa/2}\right)^2 \times \left(1 + \Omega^2 \frac{\Omega^2 + (\kappa/2)^2 - 2\Delta^2}{(\Delta^2 + (\kappa/2)^2)^2 + \Omega^2(\kappa/2)^2}\right) \quad (1)$$

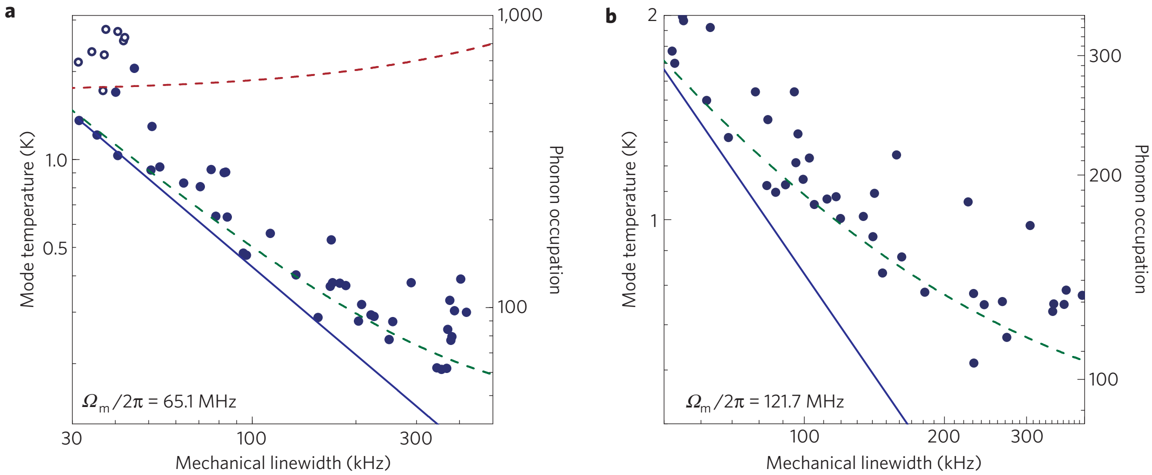

Figure 4 | Resolved-sideband laser cooling and heating by absorption. a,**b**, Data for a RBM at 65.1 MHz (**a**) and at 121.7 MHz (**b**) during two separate runs for varying injected power, laser detuning and coupling efficiency. The mode temperature and the corresponding phonon occupation number (ordinates) are reduced as the detuned laser induces extra damping, and therefore increases the linewidth of the oscillator's thermal noise spectrum (abscissa). The filled points correspond to mechanical spectra taken with a detuned cooling laser, and the open points in **a** correspond to measurements with the laser tuned close to the optical resonance. Note that the scatter reflects the varying operation conditions; the uncertainty in the phonon occupation for each data point is <30%. The deviation from a linear cooling behaviour (blue line) indicates a backaction heating effect, which is compatible with heating of the structure originating from residual photon absorption. The green dashed lines indicate the expected mode temperature taking heating of the reservoir into account; the corresponding reservoir temperature is indicated with the red dashed line (**a**). For the 65 MHz oscillator, the known[27] temperature dependence of the intrinsic quality factor of the oscillator is also taken into account. For this device, extrapolation to zero laser power yields a quality factor of 2,600 at the cryostat base temperature.

where the optomechanical coupling is given by $g_0 = \mathrm{d}\omega/\mathrm{d}x$ and $\mathrm{d}\omega/\mathrm{d}x = -\omega/R$ in the present embodiment. Moreover, R is the cavity radius, P_{in} is the launched input laser power, $\omega/2\pi$ is the optical resonance frequency and $\Delta/2\pi$ is the detuning from the cavity resonance. In the resolved-sideband case $|\Delta| = \Omega_{\mathrm{m}} \gg \kappa$, this expression simplifies to

$$S_{xx}[\Omega_{\mathrm{m}}] \approx \frac{\hbar\omega\Omega_{\mathrm{m}}^2}{4g_0^2 P_{\mathrm{in}}}$$

at the mechanical resonance frequency, which is only a factor of 4 higher than in the resonant readout case ($\Delta = 0, \Omega_{\mathrm{m}} \gg \kappa$). We note the interesting result that this expression does not depend on optical linewidth in the deeply resolved-sideband regime.

As shown in previous work[11,20], the laser detuned to the lower sideband leads to a significant reduction of the thermal occupation, as demonstrated by the reduced area underneath the peaks associated with the oscillator's thermal noise (see Fig. 3b). The underlying physical mechanism giving rise to cooling is enhanced anti-Stokes scattering into the cavity mode, whereby each scattering process annihilates a thermal phonon. In the resolved-sideband regime[20,32], ground-state cooling is possible in principle, and the minimum occupation that can be reached is given by $\langle \bar{n} \rangle = \kappa^2/16\Omega_{\mathrm{m}}^2 \ll 1$ (refs 32, 33). As the laser cools the resonator out of equilibrium with the thermal reservoir seen by the oscillator (at temperature T_{res}), however, heating through the reservoir competes with laser cooling and leads to a final occupation of $\langle n_{\mathrm{f}} \rangle \approx (\Gamma_{\mathrm{m}}/(\Gamma_{\mathrm{m}} + \Gamma_{\mathrm{cool}}))k_{\mathrm{B}}T_{\mathrm{res}}/\hbar\Omega_{\mathrm{m}}$, where Γ_{m} is the intrinsic damping rate and Γ_{cool} is the laser-induced cooling rate[19,20] with

$$\Gamma_{\mathrm{cool}} \approx \frac{2g_0^2 P_{\mathrm{in}}}{m_{\mathrm{eff}}\omega\Omega_{\mathrm{m}}^3}$$

in the resolved-sideband case. In the case of the data shown in Fig. 3, a strong increase in the damping with a concomitant reduction of the thermal occupation can be observed. Small frequency shifts due to the cooling laser are also observed as expected[19]; their magnitude however is small enough to leave the performance of the cooling unaffected, so that re-adjustment of the detuning is not necessary. The highest attained total damping rate is $\Gamma_{\mathrm{eff}}/2\pi = (\Gamma_{\mathrm{m}} + \Gamma_{\mathrm{cool}})/2\pi = 370$ kHz, reached with a launched power of $\sim$ 0.2 mW. Evaluation of the calibrated thermal noise spectrum reveals an effective mode temperature of 200 ± 60 mK, which corresponds to an average occupation as low as $\langle n_{\mathrm{f}} \rangle = 63 \pm 20$ quanta (see the Methods section). This is the lowest reported occupancy for a cavity optomechanical cooling experiment reported so far; lower occupancy, $\langle n_{\mathrm{f}} \rangle = 25$, has been attained only in the context of conventional dilution refrigeration of nanomechanical oscillators[9], albeit with a signal-to-background ratio well below unity. Backaction cooling techniques applied to nanomechanical oscillators at millikelvin temperatures achieve occupation numbers about one order of magnitude higher[9,11].

We next consider the reported measurements from the perspective of the theory of quantum measurement[1]. For linear continuous measurements such as those used here, the total displacement uncertainty arises from two intrinsic sources of noise: measurement imprecision and measurement backaction. Imprecision is due to fluctuations at the output of the measurement device, which are not related to mechanical oscillator motion. In the case of an optical interferometric measurement as reported here, the imprecision can be reduced to the laser quantum noise in the detection process (see equation (1)). Measurement backaction, on the other hand, describes the perturbation of the mechanical oscillator by the measurement. For a mechanical oscillator, this occurs in the form of a fluctuating force, characterized by a spectral density $S_{FF}[\Omega]$.

The backaction-force noise of the measurement on the mechanical mode can have various contributions, such as classical fluctuations[20] of laser frequency or amplitude, inducing fluctuations of the radiation-pressure force; or absorption of the measurement laser heating the reservoir, and entailing extra thermal Langevin force fluctuations. All of these effects are usually referred to as 'excess backaction', as they can all be eliminated in principle. Fundamentally, the lowest possible force noise for a coherent input is caused by unavoidable quantum noise[1,35] and is termed quantum backaction:

$$S_{FF}^{\mathrm{qba}}[\Omega] = \frac{\hbar}{2\omega} g_0^2 P_{\mathrm{in}} \left(\frac{\kappa^2}{(\kappa/2)^2 + \Delta^2} \right) \left(\frac{1}{(\kappa/2)^2 + (\Delta - \Omega)^2} + \frac{1}{(\kappa/2)^2 + (\Delta + \Omega)^2} \right)$$

and

$$S_{FF}^{\mathrm{qba}}[\Omega_{\mathrm{m}}] \approx \frac{2 g_0^2 P_{\mathrm{in}} \hbar}{\omega \Omega_{\mathrm{m}}^2}$$

in the deeply resolved-sideband regime ($|\Delta| = \Omega_{\mathrm{m}} \gg \kappa$). Note that the spectra of imprecision and backaction noise, as well as their possible correlation $S_{xF}[\Omega]$, reflect properties of the measurement device (the cavity pumped with a laser field) independent of the mechanical oscillator. Quantum mechanics requires the product of imprecision (S_{xx}) and backaction (S_{FF}) to obey $\sqrt{S_{xx}[\Omega] S_{FF}[\Omega]} \geq \hbar/2$; an equality can be reached in the ideal case and in the absence of correlations of imprecision and backaction noise. This relation can also be considered as a manifestation of the Heisenberg uncertainty principle applied in the context of continuous position measurements[1].

We next estimate how closely our measurements approach this fundamental imprecision–backaction product. For the parameters of the measurement corresponding to $\langle n_{\mathrm{f}} \rangle = 63 \pm 20$, we calculate the smallest possible force noise $\sqrt{S_{FF}^{\mathrm{qba}}[\Omega_{\mathrm{m}}]} \approx 1\,\mathrm{fN\,Hz^{-1/2}}$ The challenge of reaching this limit lies in the careful elimination of the aforementioned sources of excess backaction. To suppress the classical force noise added by laser frequency and amplitude fluctuations, we use a titanium:sapphire continuous-wave laser source. Note that the mechanical oscillator is sensitive only to fluctuations at Fourier frequencies around its resonance frequency, where this laser source is quantum limited in both amplitude and phase. A second contribution arises from photon absorption of the cavity structure[27]. Importantly, this contribution can be reduced by operating in the resolved-sideband regime.

To illustrate the instrumental role of the resolved-sideband regime for suppressing the effect of absorption-induced heating, we compare the cooling run just described with a further, independent run with a smaller sample ($\Omega_{\mathrm{m}}/2\pi = 122$ MHz, $Q \approx 2{,}200$) exhibiting significantly broader linewidth (fibre coupling broadened to $\kappa/2\pi = 155$ MHz), as shown in Fig. 4. In this case, a significant deviation from the linear cooling behaviour is observed, with higher mode temperatures measured than expected from the cooling rate. It is possible to model this deviation by taking an intracavity-power-dependent heating of the mechanical mode into account. From both data sets, we extract a heating effect below $10\,\mathrm{K\,W^{-1}}$ of circulating optical power. A refined estimate can be obtained by taking the increase of intrinsic mechanical damping with increasing temperature into account, which has been independently measured[27]. With the measured $\mathrm{d}\Gamma_{\mathrm{m}}/\mathrm{d}T \approx 2\pi \cdot 16\,\mathrm{kHz\,K^{-1}}$ for a resonance frequency of 65 MHz below 2 K, we infer heating of the reservoir by about $5\,\mathrm{K\,W^{-1}}$ of circulating power. Although quantitative modelling would have to take characteristics of the optical mode and heat transfer in the gas-cooled sample into account, we note that similar values of laser-induced heating were extracted from studies of the optical multi-stability at low temperature at a wavelength of 1.5 μm (ref. 27).

This analysis can also be applied to the measurement with the lowest obtained occupancies of $\langle n_{\mathrm{f}} \rangle = 63 \pm 20$. From the experimental data (see Fig. 3b), we infer a total fluctuating thermal Langevin force experienced by the optically cooled resonator given by $S_{FF}^{\mathrm{the}}[\Omega] \approx 2 m_{\mathrm{eff}} \Gamma_{\mathrm{eff}} \hbar \Omega_{\mathrm{m}} \langle n_f \rangle = 2 m_{\mathrm{eff}} \Gamma_{\mathrm{m}} k_{\mathrm{B}} T_{\mathrm{res}}$ that heats the oscillator towards equilibrium with the reservoir at temperature T_{res}. We find $\sqrt{S_{FF}^{\mathrm{the}}[\Omega_{\mathrm{m}}]} \approx 8\,\mathrm{fN\,Hz^{-1/2}}$ for this measurement. Note that the reservoir temperature T_{res} is increased from the cryostat temperature by 400 mK owing to photon-absorption-induced heating, which entails extra Langevin force fluctuations. The part of this force that is related to the temperature increase caused by the measurement (and cooling) laser translates to total excess backaction-force fluctuations of only $\sqrt{S_{FF}^{\mathrm{ba}}[\Omega_{\mathrm{m}}]} \approx 4\,\mathrm{fN\,Hz^{-1/2}}$. To quantify to what extent a quantum-mechanically ideal measurement has been carried out, we evaluate the imprecision–backaction product using the measured imprecision of $\sqrt{S_{xx}[\Omega]} \approx 1.4 \times 10^{-18}\,\mathrm{m\,Hz^{-1/2}}$. Certainly, $\sqrt{S_{xx}[\Omega_{\mathrm{m}}] S_{FF}^{\mathrm{the}}[\Omega_{\mathrm{m}}]} \approx 220 \cdot \hbar/2$ represents an upper bound in this regard, effectively considering all thermal noise as measurement backaction. This conservative upper bound is already a factor of 4 better than the closest approach made with nano-electromechanical systems[9,21,22]. Considering only the force noise added by the laser due to heating, a product of $\sqrt{S_{xx}[\Omega_{\mathrm{m}}] S_{FF}^{\mathrm{ba}}[\Omega_{\mathrm{m}}]} \approx 100 \cdot \hbar/2$ is found. This represents the closest approach to the Heisenberg uncertainly limit for continuous position measurements so far and confirms the highly ideal operation of optical transducers even at low occupation numbers.

We expect that further significant improvements on our results are readily attainable by eliminating technical imperfections such as optical loss, operating more deeply in the resolved-sideband regime[20], in a colder cryogenic environment (such as ^{3}He) and with reduced mechanical dissipation by using, for example, crystalline resonators[36]. The properties of the system demonstrated here—a massive oscillator, prepared with very low average occupation, which is monitored with an optical motion transducer close to the fundamental Heisenberg uncertainty principle for the backaction–imprecision product—are pivotal for a variety of experiments and protocols involving photons and phonons, such as radiation-pressure squeezing[37,38], entanglement[39] or quantum non-demolition measurements[40,41].

Methods

This section describes the experimental protocol for measurement and calibration of the noise spectra. Calibration proceeds by first applying a known phase modulation (modulation depth $\delta\varphi$) to the probing laser using a $LiNbO_3$ modulator. In the phase-sensitive detection scheme, this induces a measurement signal identical to a mechanical displacement of magnitude $\delta x_{\mathrm{eq}} = \delta\varphi \cdot R \Omega_{\mathrm{mod}}/\omega$, where ω is the laser frequency, Ω_{mod} is the modulation frequency (chosen close to the mechanical frequency) and R is the cavity radius. The displacement noise spectra are acquired with a high-end radiofrequency spectrum analyser by averaging typically 100 traces containing 8,000 points. The resolution bandwidth (RBW) is chosen to be sufficiently small compared with the mechanical modes' spectral width, typically 10 kHz. In the measured spectra, the peak at Ω_{mod} thus corresponds to an equivalent displacement noise spectral density $S_{xx}[\Omega_{\mathrm{mod}}]$ of $\delta x_{\mathrm{eq}}^2/\mathrm{RBW}$ enabling us to calibrate the measured spectra in absolute terms. Finite-element modelling of the mechanical modes of silica microtoroids[15,26] enables us to identify the noise peak caused by the RBM. If the temperature of the sample is known, it is possible to derive from this measurement the effective mass of the mechanical mode as well, by using the equipartition theorem, which states $S_{xx}^{\mathrm{th}}[\Omega_{\mathrm{m}}] = 2 k_{\mathrm{B}} T_{\mathrm{RBM}} / m_{\mathrm{eff}} \Omega_{\mathrm{m}}^2 \Gamma_{\mathrm{m}}$, where $S_{xx}^{\mathrm{th}}[\Omega_{\mathrm{m}}]$, Ω_{m} and Γ_{m} can be directly extracted from the measured spectra. Systematic uncertainties of this mass calibration arise from dynamical backaction- or absorptive heating-induced temperature changes, and imperfections of the modulation scheme and are estimated to be at the level of 1.1 dB (30%). For most samples, reference measurements were carried out at room temperature—where laser-induced heating effects cause only negligible relative temperature changes—to derive the effective mass of the oscillator, which usually agrees with finite-element modelling within the experimental uncertainty. Once the effective mass is determined, the equipartition theorem can subsequently be used to determine the temperature T_{TBM} of the mode in a cryogenic environment and/or in the presence of laser cooling, with an uncertainty given by the uncertainty of the effective mass. The resulting phonon occupancy is subsequently calculated as $\langle n \rangle \approx k_{\mathrm{B}} T_{\mathrm{RBM}} / \hbar \Omega_{\mathrm{m}}$. In addition to calibration purposes, the laser modulation is also used for detuning control of the laser in the resolved-sideband regime: demodulating the homodyne signal at the modulation frequency (or, alternatively, at the mechanical resonance frequency sensing the thermal noise), a locally maximum signal is detected when the laser is resonant with the mechanical sideband.

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