1. *Quantum mechanics is local; Teleportation*

   a) Consider the teleportation set-up. $|\Psi\rangle = |\phi\rangle |\Phi_+\rangle$. Show that the reduced state of the third system (the system of the "receiver") is $\propto \mathbb{1}$, after the Bell-measurement of the first two systems. Since this is the completely mixed state, it implies that the receiver has *no information* about either the measurement result of the input state $|\phi\rangle$ prior to receiving data from the sender.

   b) Show in general that in any two-party state $\rho_{AB}$, the reduced state at $B$ is unaffected by any operation performed by $A$.

2. *Quantum gates*

   Construct a $C^{(n)} - U$ operation ($U$ conditioned on $n$ control bit being "1") from single-qubit unitaries (SQUs) and CNOTs.
   *Hint: for the multiply-controlled operation it is helpful to proceed in two steps: (i) consider $n = 2$ and make use of controlled-$V$ and controlled-$V^\dagger$ gates with $V^2 = U$. (ii) To extend to larger $n$, introduce some auxiliary qubits ("ancillas") prepared in $|0\rangle$. How can one make one of these ancilla bits flip if and only if all the control-bits are "1"? Make use of the the $C^{(2)} - X$ (doubly controlled-NOT or "Toffoli"-gate).*

3. *Schmidt decomposition*

   a) Show that any pure bipartite state $|\psi\rangle$ on $\mathbb{C}^d \otimes \mathbb{C}^d$ can be transformed into "Schmidt form"

   $$\lambda_0 |0,0\rangle + \lambda_1 |1,1\rangle + ... + \lambda_{d-1} |d-1, d-1\rangle\,, \quad \lambda_0 \geq \lambda_1 \geq \dots \lambda_{d-1} \geq 0$$

   by local unitaries, i.e., show that for any $|\psi\rangle = \sum_{kl} \psi_{kl} |k,l\rangle$ there exist unitary matrices $U, V$ such that $U \otimes V |\psi\rangle = \sum_{n=0}^{d-1} \lambda_n |nn\rangle$.
   *Hint: Consider the singular value decomposition of the matrix $\psi_{kl}$.*

   b) The positive numbers $\lambda_k$ are called the "Schmidt coefficients" of the state $|\psi\rangle$. They characterize all non-local properties of the state. Show that they cannot be changed by local unitaries. What Schmidt-coefficients characterize separable states? Which Schmidt-coefficients correspond to the maximally entangled state of a $\mathbb{C}^d \otimes \mathbb{C}^d$-system?

4. *Entanglement transformations*

   Show that the pure bipartite state $|\lambda\rangle = \lambda_0 |00\rangle + \lambda_1 |11\rangle$ can be converted into the maximally entangled state $|\Phi_+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ by a *local* generalized measurement $\mathcal{M} = \{M_k\}$.
   Show that the converse conversion $|\Phi_+\rangle \to |\lambda\rangle$ is possible with certainty (if, depending on the measurement result suitable local unitaries are applied to the resulting state).
   *Hint: a two-element POVM $\{M_1, M_2\}$ at one of the two sites suffices.*