Quantum Simulations with Trapped Ions

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Abstract

Trapped ions interacting with lasers are suitable to build quantum simulators with present technology. Using this toolbox we proposed two schemes to realize some models in condensed matter physics, e.g. quantum spin models and Bose-Hubbard model[1,2].

- ***Quantum spin models with trapped ions[1,3]***

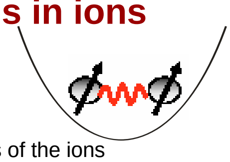
A standing wave coupling the ions' internal states to the vibrational modes induces effective spin-spin interactions, which allow us to study quantum phase transitions in **Ising** and **Heisenberg** models.

- ***The Bose-Hubbard model with phonons[2,4]***

Radial phonons in a Paul trap can be used to simulate ***Bose Hubbard model*** under certain conditions.

spins and phonons in ions

- Ions are trapped by harmonic potentials



- Effective spins: internal electronic states of the ions

$$H = B^x \sum_j \sigma_j^x + B^z \sum_j \sigma_j^z$$

- Phonons: vibrational modes of the chain

$$H = \sum_{n,\alpha=x,y,z} \hbar\Omega_n a^+_{\alpha,n} a_{\alpha,n}$$

$$\beta = \frac{e^2}{d_0^3} / m\omega^2$$

- Soft limit $\beta >> 1$
- hard limit $\beta << 1$

Quantum spin models with trapped ions: (1)

- An off-resonant standing wave couples the internal states to the vibrational modes

- The upper level experiences an ac-Stark shift

$$H_{ac-Stark} = F\sigma^z x$$

- Some canonical transformation

$$H' = e^{-S} H e^{S}$$

- effective spin-spin interactions, Ising-like Haniltonian

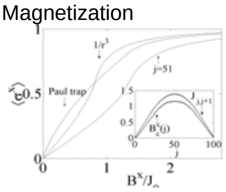
$$H = \frac{1}{2} \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z + B_x \sum_i \sigma_i^x$$

- hard limit $\beta \ll 1$

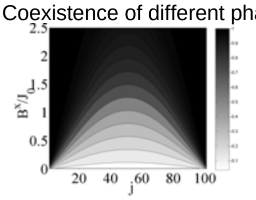
$$J_{ij} = \frac{J}{|i-j|^3}$$

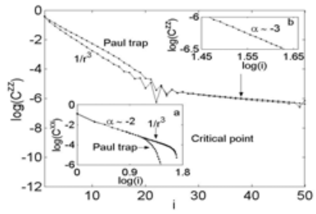
Quantum spin models with trapped ions: (2)

- This model can be addressed numerically with the DMRG, N=100 ions.
- Same universality as nearest-neighbor Ising model.
- Coexistence of antiferromagnetic and paramagnetic phases in linear Paul traps.
- Correlations can be directly measured.
- Magnetization



- Coexistence of different phases

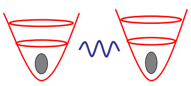




- Correlation functions :
  - at Critical point : long-range algebraic decay
  - at noncritical point : exponential decay
  - at long distance : power-law decay as interactions

The Bose-Hubbard Model with phonons: (1)

- Trapping potential much larger than Coulomb interaction, phonons are conserved.



$$H_0 = \sum_j \hbar\omega_0 b_j^+ b_j + H_{\text{Coul}}$$

- Tunneling: $H_{\text{Coul}} \sim \sum_{i,j} t_{ij}(b_i^+ b_j + b_i b_j^+)$ $\qquad \omega_0 >> t_{i,j} = \frac{e^2}{d_{i,j}^3}$

- Phonon-phonon interactions are position dependent ac-Stark shift.

$$H_{\text{p-p}} = \sum_n \sigma_n^z \frac{\Omega^2}{\Delta} \cos(k(x_n - x_{eq})) \to \sum_n U a_n^{+2} a_n^2$$

- Radial Phonons in Paul trap fulfill the above conditions, thus we can get a Bose-Hubbard Hamiltonian:

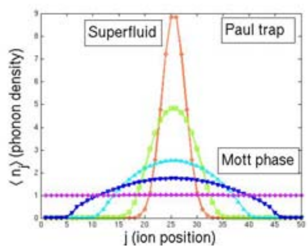
$$H = \sum_{i,j} t_{ij}(b_i^+ b_j + b_i b_j^+) + \sum_i (\omega_x + \omega_{x,i}) b_i^+ b_i + U \sum_i b_i^{+2} b_i^2$$

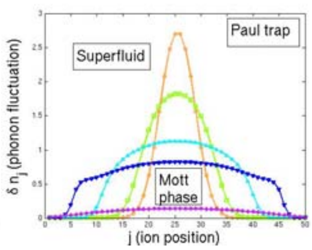
•The sign and strength of the interaction can be chosen at will in the experiment.

The Bose-Hubbard Model with phonons: (2)

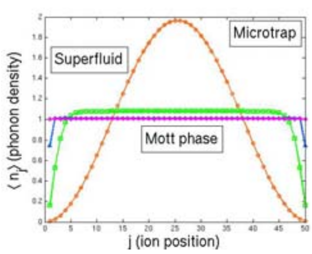
- Numerical calculations with the DMRG, Coulomb chain with N=50 sites and N=50 phonons.

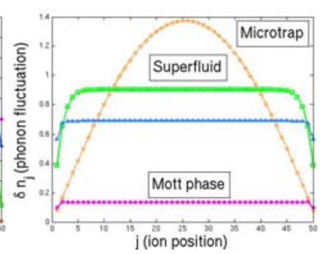
1. Superfluid-Mott insulator phase transition in Paul traps:





2. Superfluid-Mott insulator phase transition in Microtraps:





[1] D. Porras and J.I. Cirac, Phys. Rev. Lett. **92**, 207901 (2004). [3] X.-L. Deng, D. Porras and J.I. Cirac, Phys. Rev. A **72**, 063407 (2005).
[2] D. Porras and J.I. Cirac, Phys. Rev. Lett. **93**, 263602 (2004). [4] X.-L. Deng, D. Porras and J.I. Cirac, in preparation.