Quantum Theory of Nuclear Spin Cooling in Quantum Dots

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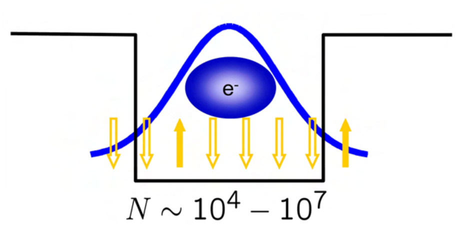
Abstract

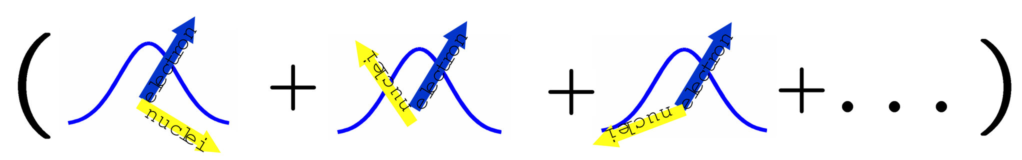
We study theoretically the cooling of an ensemble of nuclear spins coupled to the spin of a localized electron in a quantum dot. We obtain a master equation for the state of the nuclear spins interacting with a sequence of polarized electrons that allows us to study quantitatively the cooling process including the effect of nuclear spin coherences, which can lead to "dark states" of the nuclear system in which further cooling is inhibited. We show that the inhomogeneous Knight field mitigates this effect strongly and that the remaining dark state limitations can be overcome by very few shifts of the electron wave function, allowing for cooling far beyond the dark state limit. Numerical integration of the master equation indicates, that polarizations larger than 90% can be achieved within a millisecond timescale.

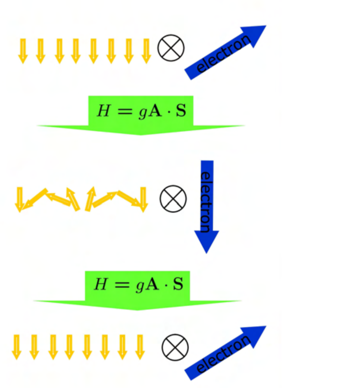
Introduction

The spin of an electron in a quantum dot is a promising candidate for a qubit. The hyperfine interaction with the surrounding nuclear spins is in many situations of interest the strongest coupling to the environment.



→ Decoherence of the electron spin due to averaging over nuclear fields





The highly polarized states resulting from our cooling procedure are interesting not only for the purpose of reducing decoherence, but also in their own right: The long nuclear coherence times suggest the possibility to let the nuclear ensemble play an active role in quantum information processing, e.g. as a quantum memory.

Basic Idea

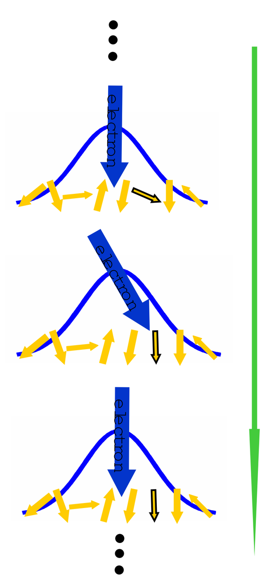
▷ Use strong coupling of electron and nuclei for nuclear spin state preparation

▷ $N \gg 1$ → electron spin needs to be repolarized during cooling procedure

→ Consider iteration of the simple steps

- Initialization of the electron in $|\downarrow\rangle$
- Interaction with nuclei for time Δt

▷ Nuclear spins "see" many polarized electrons, which effectively act as a $T = 0$–bath for the nuclei.



Master Equation for the Cooling Process

Fermi contact interaction between the electron and the nuclear spins

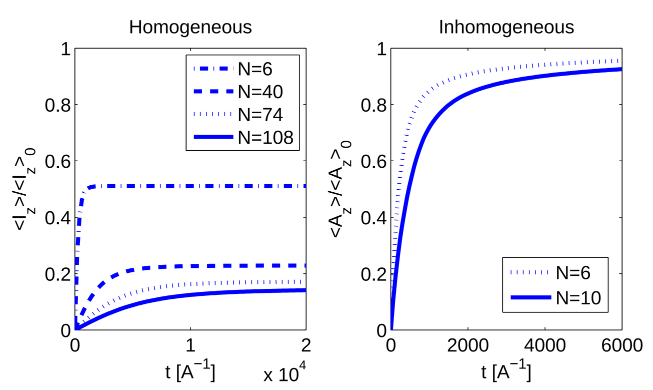
$$\hat{H} = \frac{g}{2}\left(\hat{A}^+\hat{S}^- + \hat{S}^+\hat{A}^-\right) + g\hat{A}^z\hat{S}^z + B_{ext}\hat{S}_z.$$

The overall coupling strength between the spins is set by $A \approx 90\mu$eV (GaAs), the individual couplings are $\alpha_i \propto |\psi_e(x_i)|^2 = \mathcal{O}(1/N)$. $\mathbf{S}$ is the spin operator for the electron spin and $\hat{\mathbf{A}}_\beta = \sum_i g_i \hat{\mathbf{I}}_i^\beta$ are the three components of the collective nuclear operators ($\beta = \pm, z$), where $g_i = \alpha_i/\sqrt{\sum_i \alpha_i^2}$, such that $\sum_i g_i^2 = 1$, and $g = A\sqrt{\sum_i \alpha_i^2}$.
For $g\Delta t \ll 1$ we get for the state of the nuclei

$$\rho_{t+\Delta t} - \rho_t = i\frac{g\Delta t}{2}[\hat{A}_z - \langle\hat{A}_z\rangle, \rho_t] - \frac{g^2(\Delta t)^2}{8}[\hat{A}_z - \langle\hat{A}_z\rangle, [\hat{A}_z - \langle\hat{A}_z\rangle, \rho_t]] - \frac{g^2(\Delta t)^2}{8}\left(\hat{A}_+\hat{A}_-\rho_t + \rho_t\hat{A}_+\hat{A}_- - 2\hat{A}_-\rho_t\hat{A}_+\right).$$

- terms in the first row can lead to dephasing of nuclear states
- terms in second row are polarizing terms
- similar equations have been studied in the literature on superradiance
- mean value of Overhauser field is compensated by external field
- finite electron spin polarization and depolarization are neglected

Exact Treatment



▷ For homogeneous couplings, α_i = const, collective nature of coupling prevents complete polarization, in contrast to expectations from naive spin temperature description.

▷ For homogeneous couplings the attainable polarization decreases with increasing particle number.

▷ The reason are singlet like dark states which "trap" spin excitation

$$(\sigma_1^- + \sigma_2^-)(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = 0.$$

Dark States

Spin-trapping states for homogeneous couplings in the Dicke basis are

$$|D\rangle = |J, -J, \beta\rangle, \qquad 0 \leq J \leq N/2,$$

with the first entry being the total angular momentum and the second its z-projection. The achievable polarization in the homogeneous case is

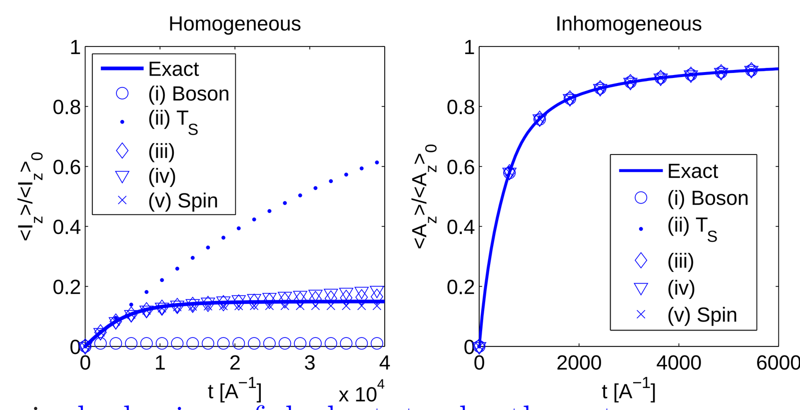
$$\frac{\langle I_z\rangle_{ss}}{\langle I_z\rangle_0} = \frac{2}{2^N N}\sum_J J(2J+1)D_J = \sqrt{\frac{8}{\pi N}} + \mathcal{O}(1/N).$$

Taylor *et al.* (PRL, 2003): Homogeneous dark states can be mapped 1:1 to inhomogeneous ones.

▷ Dark States may seriously hinder polarization of nuclei in quantum dots.

Inhomogeneous Couplings

▷ The inhomogeneous couplings, $\alpha_i \neq$ const, increase the performance of the cooling scheme dramatically



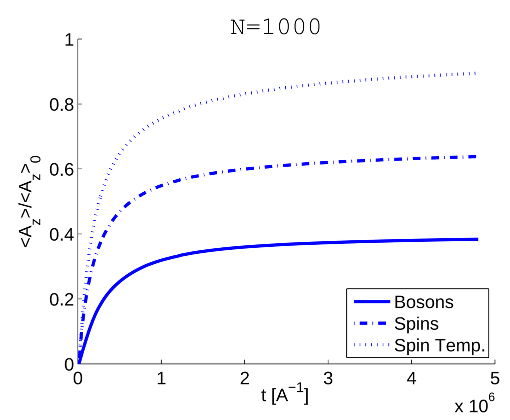
The reason is dephasing of dark states by the z-term

$$e^{iA_z t}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = (e^{-i\delta_g t}|\uparrow\downarrow\rangle - e^{i\delta_g t}|\downarrow\uparrow\rangle) \rightarrow (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

▷ Note that spins with equal couplings α_i are not dephasing

▷ The polarization on "shells" of spins will still be limited by dark states

Cooling Time and Final Polarization



▷ 2D Gaussian wave function

▷ Steady state is reached in time

$$\tau_D = \mathcal{O}\left(\frac{4N^{3/2}}{A(g\Delta t)}\right)$$

⟹ Cooling process can be fast in comparison to dipolar diffusion

▷ Taking $A = 100\mu$eV$\sim$ 40ps approximately $N = 0.5 \times 10^4$ spins cooled to more than 90% within a millisecond.

▷ Larger Δt gives larger cooling rate

Even in the presence of the dephasing z-part of the hyperfine interaction, the obtained polarization is too low for the most interesting applications.

▷ Idea: Induce further dephasing of dark states by changing the spatial profile of the electron wave function.

Approximation Schemes for the Equations of Motion

The master equation implies for the correlations $\gamma_{ij} = \langle\sigma_i^+\sigma_j^-\rangle$

$$\frac{\Delta\gamma_{ij}}{\Delta t} = \xi_{ij}\gamma_{ij} - \kappa\sum_k g_k\left(g_j\langle\sigma_i^+[\sigma_j^-, \sigma_j^+]\sigma_k^-\rangle - g_i\langle\sigma_k^+[\sigma_i^+, \sigma_i^-]\sigma_j^-\rangle\right),$$

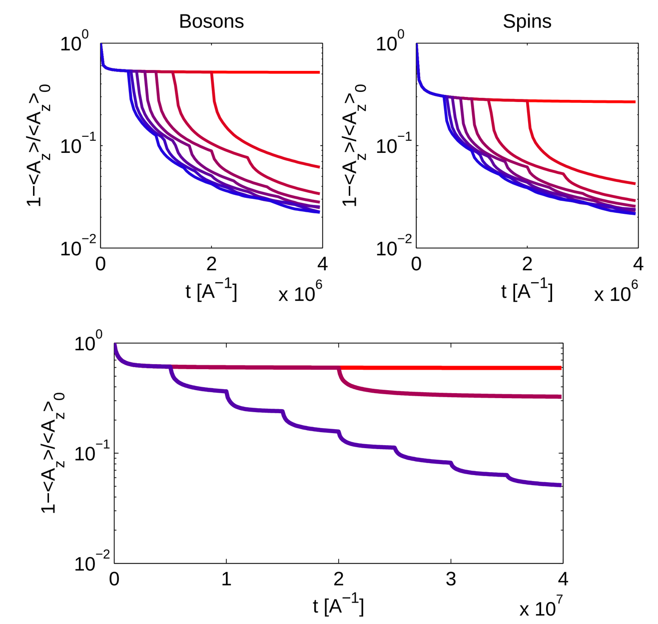
with $\xi_{ij} = ig(g_j - g_i)/2 - g^2\Delta t(g_j - g_i)^2/8$ and $\kappa = g^2\Delta t/8$.
One obtains a hierarchy of equations, for which we devise several approximate descriptions.
We present exemplarily two approaches we have taken:

▷ At high polarizations the spin system can be described by an effective *bosonic model*. In this case the equations of motion close $\dot{\gamma}_{ij} = \xi\gamma_{ij} - \kappa\sum_k g_k(g_i\gamma_{kj} + g_j\gamma_{ik})$. At low polarizations this model is expected to give a lower bound on the performance of the cooling scheme.

▷ Following the literature on superradiance (Andreev *et al.*, 1993), the higher order expectation values can be *factorized* in analogy to the Wick theorem $\langle\sigma_k^+\sigma_i^z\sigma_j^-\rangle = -\frac{1}{2}\gamma_{kj} - \gamma_{ki}\gamma_{ij} + \gamma_{kj}\gamma_{ii}$.

Changing Modes

- Dark states (can) become bright when electronic wave function is changed



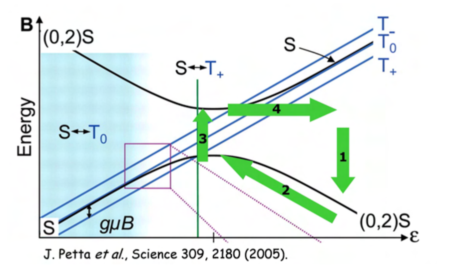
▷ 2D Gaussian wave function
▷ Upper plots
  ▷ $N = 200$
  ▷ Random Gaussians
  ▷ Shifts $\lesssim$ 1.5 lattice sites
▷ Lower plot
  ▷ $N = 1000$
  ▷ Two modes iterated (centers $\{(1/3, 1/3), (-3.15, -1.5)\}$)
  ▷ Bosonic scheme

▷ Different schemes converge for enhanced cooling protocols

▷ 3-5 (random) shifts of wave function enable cooling above 90%

Possible Experimental Realization

Double Dots



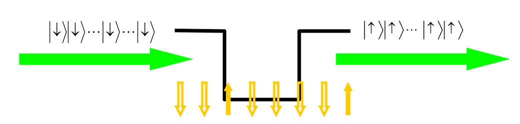
▷ Initialize S(0,2)
▷ Adiabatically (1 ns) move to resonance between S and T_+
▷ Evolve with HF interaction for time Δt
▷ Ramp back to (0,2) region again

Self-Assembled Dots

- Polarized light → polarized electrons (Imamoglu *et al.*, PRL 2003).
- Polarized electrons in turn lead to nuclear polarization

Spin Polarized Currents

▷ Let polarized electrons tunnel into dot
▷ Evolve with HF interaction for time Δt
▷ Push electron out by appropriate gating



Conclusions

We have studied
▷ **nuclear spin polarization in quantum dots**
▷ **limitations due to destructive quantum interference**

We have shown
▷ **consistent quantum theory for nuclear spin cooling in QDs**
▷ **mechanism that overcome limitations due to dark states**
▷ **possibility to achieve above 90%-polarizations**

High polarization will set the stage for
▷ **cavity-QED in quantum dots**
▷ **probing mesoscopic quantum dynamics**
▷ **nuclear spin state engineering and quantum info processing**