Pfaffian-like ground state for 3-body hard-core bosons in 1D lattices

T. Keilmann[1], _B. Paredes_[1,2], _and J. I. Cirac_[1]

[1]*Max–Planck Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching*
[2]*Institut für Physik, Johannes Gutenberg-Universität, Staudingerweg 7, D-55099 Mainz*

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Abstract

We propose a Pfaffian-like Ansatz for the ground state of bosons subject to 3-body infinite repulsive interactions in a 1D lattice. Our Ansatz consists of the symmetrization over all possible ways of distributing the particles in two identical Tonks-Girardeau gases. We support the quality of our Ansatz with numerical calculations and propose an experimental scheme based on mixtures of bosonic atoms and molecules in 1D optical lattices in which this Pfaffian-like state could be realized. Our findings may open the way for the creation of non-abelian anyons in 1D systems.

3-body hard-core bosons

We consider a system of bosonic atoms in a 1D lattice with repulsive 3-body on-site interactions. This system is described by the Hamiltonian:

$$H = -t\sum_{\ell}(a_{\ell}^{\dagger}a_{\ell+1} + h.c.) + U_3\sum_{\ell}(a_{\ell}^{\dagger})^3(a_{\ell})^3, \qquad (1)$$

where the operator $a_{\ell}^{\dagger}$ (a_{ℓ}) creates (annihilates) a boson on site ℓ, t is the tunneling probability amplitude, and U_3 is the on-site interaction energy. From now on we will consider the limit $U_3 \to \infty$. In this limit the Hilbert space is projected onto the subspace of states with occupation numbers $n_{\ell} = 0, 1, 2$ per site. We will refer to bosons subject to this condition as 3-hard-core bosons. The projected Hamiltonian has the form

$$H_3 = -t\sum_{\ell}(a_{3,\ell}^{\dagger}a_{3,\ell+1} + h.c.), \qquad (2)$$

where the 3-hard-core bosonic operators $a_{3,\ell}$ obey $(a_{3,\ell})^3 = 0$ and satisfy the CRs $[a_{3,\ell}, a_{3,\ell'}^{\dagger}] = \delta_{\ell,\ell'}\left(1 - \frac{3}{2}(a_{3,\ell}^{\dagger})^2(a_{3,\ell})^2\right)$.

Ansatz $|\Psi_3\rangle$ for the ground state of H_3

Our Ansatz for the ground state of Hamiltonian (2) is inspired by the form of the ground state for fractional quantum Hall bosons subject to three body interactions [1, 2]. We base our Ansatz state on two Tonks-Girardeau (T-G) gases [3]:

$$|\Psi_3\rangle = \mathscr{P}\left(|\Psi_2^{\uparrow}\rangle \otimes |\Psi_2^{\downarrow}\rangle\right), \qquad (3)$$

where $\mathscr{P}$ is a local operator of the form $\mathscr{P} = \mathscr{P}_{\ell}^{\otimes M}$, and $\mathscr{P}_{\ell}$ is an operator mapping the single-site 4-dimensional Hilbert space of two species of hard-core bosons to the three-dimensional one of 3-hard-core bosons:

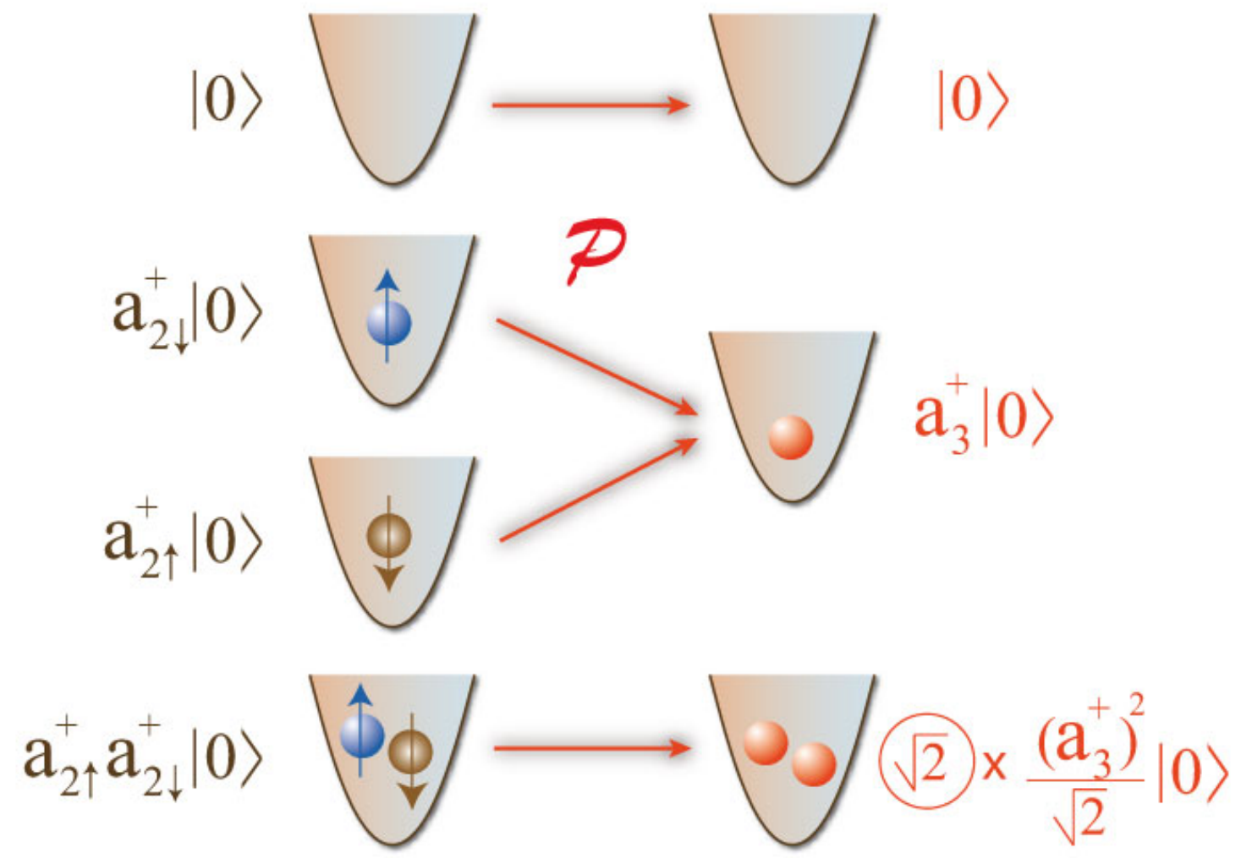

Fig. 1: Schematic representation of the operator $\mathscr{P}_{\ell}$

Characteristic properties of $|\Psi_3\rangle$

Taking into account the well known result for a T-G gas, i.e. the scaling of $\langle a_{\ell+\Delta}^{\dagger}a_{\ell}\rangle$ as $\Delta^{-1/2}$ for large Δ [3], we can derive the asymptotic behavior for the one-body and two-body correlation functions for the Ansatz (3): $\langle a_{\ell+\Delta}^{\dagger}a_{\ell}\rangle \to \Delta^{-1/4}$, $\langle a_{\ell+\Delta}^{\dagger}a_{\ell+\Delta}^{\dagger}a_{\ell}a_{\ell}\rangle \to \Delta^{-1}$. The two-body correlation is indeed the (one-particle) correlation function for on-site pairs. This means that whereas the system seen as a whole exhibits some kind of coherence (the spatial correlation decaying slowly as $\Delta^{-1/4}$), the underlying system of on-site pairs is in a much more disordered state (with a fast correlation decay as Δ^{-1}).

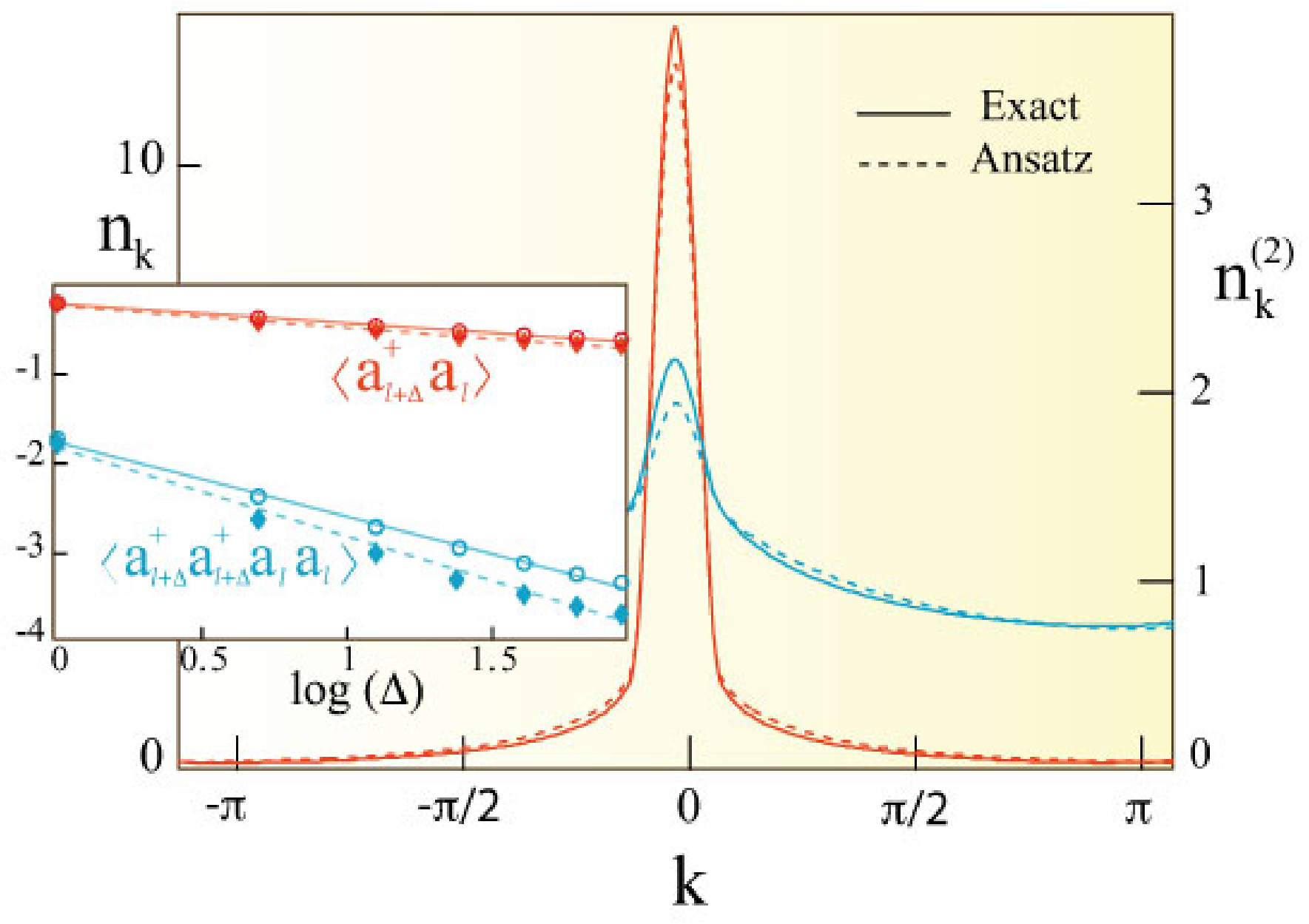

Fig. 2: Quasi-momentum distribution of particles $n_k \propto \sum_{\ell,\Delta} e^{-ik\Delta}\langle a_{\ell+\Delta}^{\dagger}a_{\ell}\rangle$ (orange, left axis) and of on-site pairs $n_k^{(2)} \propto \sum_{\ell,\Delta} e^{-ik\Delta}\langle a_{\ell+\Delta}^{\dagger}a_{\ell+\Delta}^{\dagger}a_{\ell}a_{\ell}\rangle$ (blue, right axis). Results are shown both for the exact ground state (solid) and the Ansatz (dashed). The inset shows the long-distance scaling of the correlation functions $\langle a_{\ell+\Delta}^{\dagger}a_{\ell}\rangle \sim \Delta^{-\alpha_1}$ (orange), and $\langle a_{\ell+\Delta}^{\dagger}a_{\ell+\Delta}^{\dagger}a_{\ell}a_{\ell}\rangle \sim \Delta^{-\alpha_2}$ (blue), for the exact ground state (circles, $\alpha_1 = 0.22$, $\alpha_2 = 0.83$) and the Ansatz (diamonds, $\alpha_1 = 0.24$, $\alpha_2 = 0.99$). Parameters are $M = 20$ lattice sites and filling factor $\nu = N/M = 1$.

We can also obtain analytical expressions for the relative occupation of single and doubly occupied sites. The average number of doubly occupied sites is $n_2 = \langle a_{\ell}^{\dagger}a_{\ell}^{\dagger}a_{\ell}a_{\ell}\rangle/2 = \nu^2/2$, and the one of single occupied sites is given by $n_1 = \langle a_{\ell}^{\dagger}a_{\ell}(2 - n_{\ell})\rangle = \nu(2-\nu)$. This distribution is clearly different from the Poissonian, for which we have $n_2^{\text{Po}}/n_1^{\text{Po}} = \nu/2$:

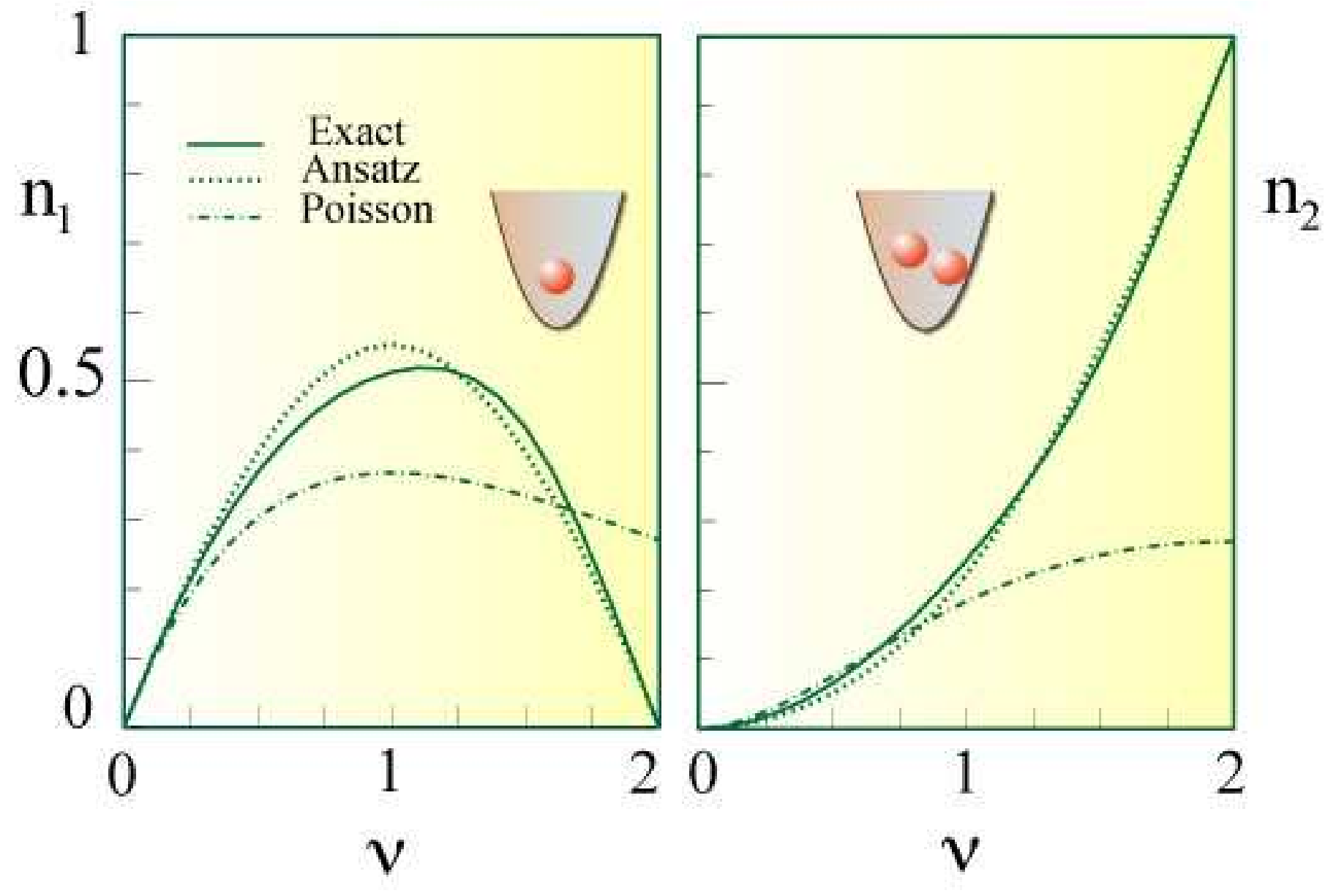

Fig. 3: Average occupation of sites with one (left fig.) and two particles (right fig.) for the exact, Ansatz and Poissonian distributions, vs. the filling factor ν. The system size is $M = 20$.

To further determine the quality of the Ansatz, we have calculated numerically overlaps with the exact ground state:

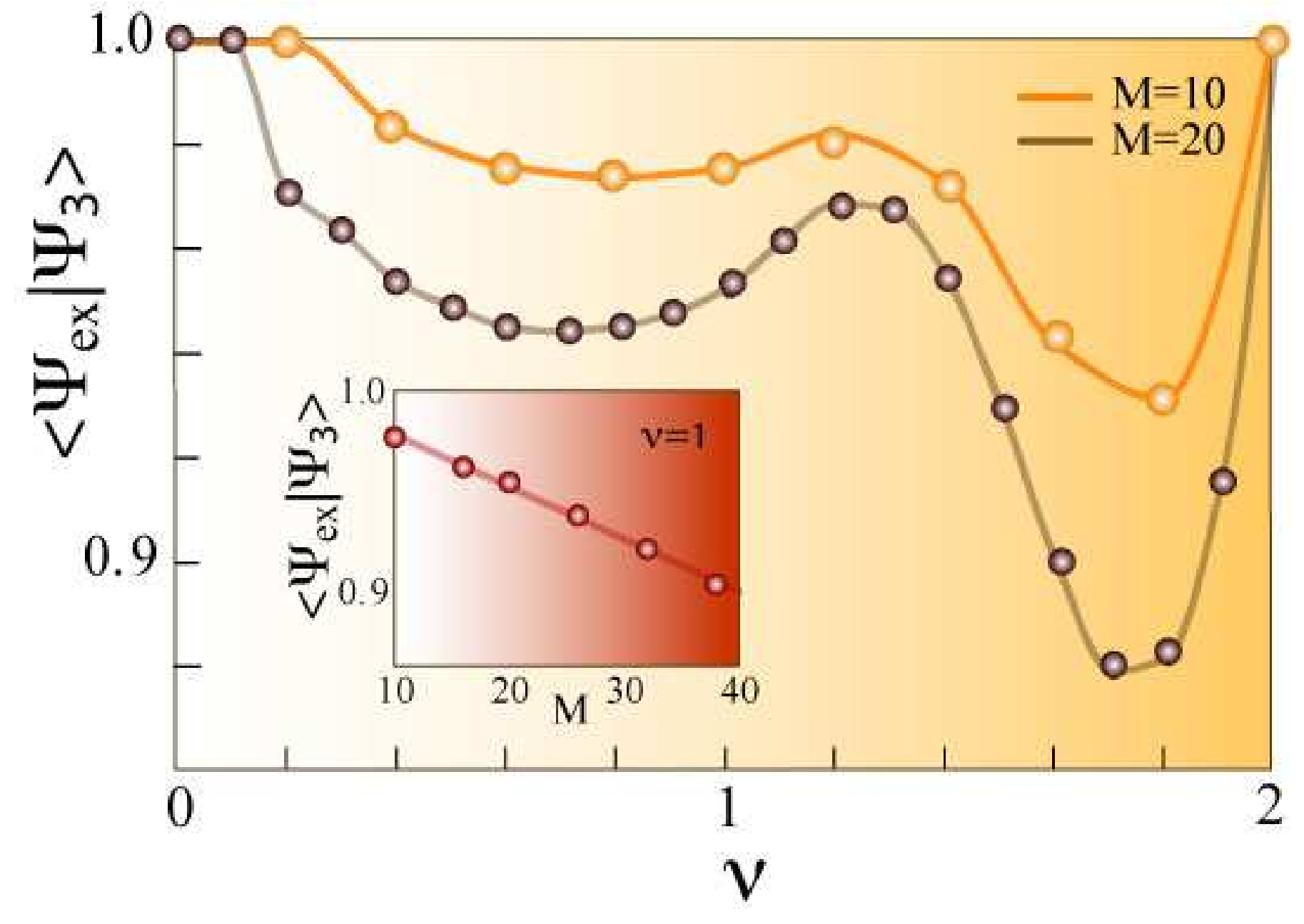

Fig. 4: Overlap $\langle\Psi_{\mathbf{ex}}|\Psi_3\rangle$. The main plot shows the overlap's dependence on ν for fixed $M = 10, 20$, the inset the decrease of the overlap with M, well staying above 0.9, for fixed $\nu = 1$.

Experimental proposal

Inspired by Cooper's ideas [4] for 2D rotating Bose gases we present an experimental scheme for the realization of Hamiltonian (2). Let us consider a system of bosonic atoms and diatomic Feshbach molecules trapped in a 1D optical lattice.

The Hamiltonian of the system is $H = H_K + H_F + H_I$ [5], where

$$\begin{aligned} H_K &= -t_a\sum_i(a_i^{\dagger}a_{i+1} + \text{h.c.}) - t_m\sum_i(m_i^{\dagger}m_{i+1} + \text{h.c.}), \\ H_F &= \sum_i \Delta m_i^{\dagger}m_i + \frac{U_{aa}}{2}a_i^{\dagger}a_i^{\dagger}a_ia_i + \frac{g}{\sqrt{2}}(m_i^{\dagger}a_ia_i + \text{h.c.}), \\ H_I &= U_{am}\sum_{i=1}^{M} m_i^{\dagger}a_i^{\dagger}a_im_i + \frac{U_{mm}}{2}\sum_{i=1}^{M} m_i^{\dagger}m_i^{\dagger}m_im_i\,. \end{aligned} \qquad (4)$$

We consider the limit in which $\gamma^2 = g^2/2\Delta^2 \ll 1$. Within this limit the formation of molecules is highly suppressed due to the high energy offset, Δ. However, virtual processes in which two atoms on the same lattice site go to the bound state, form a molecule and separate again, give rise to an effective 3-body interacting atomic Hamiltonian of the form:

$$\begin{aligned} H_{\text{eff}} = &- t_a\sum_i(a_i^{\dagger}a_{i+1} + \text{h.c.}) + U_{am}\gamma^2\sum_i(a_i^{\dagger})^3(a_i)^3 \\ &- t_m\gamma^2\sum_i(a_i^{\dagger 2}a_{i+1}^2) + (U_{aa} - \frac{g^2}{\Delta})\sum_i(a_i^{\dagger})^2(a_i)^2, \end{aligned} \qquad (5)$$

where we have neglected higher order terms in γ^2. Assuming $U_{aa} = g^2/\Delta$, and $t_m\gamma^2 \ll t_a$, H_{eff} reduces to Hamiltonian (1) with $t = t_a$ and $U_3 = U_{am}\gamma^2$. Finally, assuming $U_{am}\gamma^2 \gg t_a$ we end up with the desired Hamiltonian (2) for 3-hard-core bosons. The assumptions we have imposed are consistent with present experimental conditions for 1D optical lattices loaded with Rb-87 atoms and Feshbach molecules.

In conclusion, we have shown that the ground state of 3-hard-core bosons in a 1D lattice can be well described by a Pfaffian-like state which is a cluster of two T-G gases. We have shown that such a state may be accessible with current technology with atoms and molecules in optical lattices. We believe that our findings may open a new path for the creation of non-abelian anyons.

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