

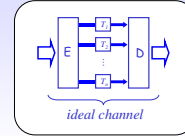
Transmitting Quantum Information through noisy channels

How much quantum information (measured in number of qubits) can one transmit reliably through a noisy quantum channel? Using asymptotic encoding and decoding schemes the optimal transmission rate defines the **Quantum Capacity**.

We derive explicit capacity formulas for important classes of quantum channels, including **Bosonic Channels** modeling **Optical Fibers with Photon Losses** and all **Extremal Qubit Channels** [1,2].

In these cases the derivation of an explicit expression is enabled by the possibility of representing the channels via a coupling to a **Small Environment** – they exhibit a property called **Degradability**.

For unknown channels we provide a method for determining and **certifying achievable rates from measurable moments** of the channel [2].



Quantum Capacity

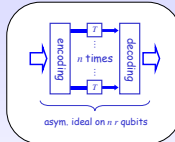
How much QI (measured in nr. of qubits) can be transmitted reliably per use of a given channel T ?

Quantum capacity: $Q(T) = \sup_{\text{codings}} r$

A Coding Theorem [4]:

$$Q(T) = \lim_{n \rightarrow \infty} \frac{1}{n} J(T^{\otimes n})$$

$$J(T) = \sup_{\rho} S(T(\rho)) - S(T \otimes \text{id}(\rho))$$



= 'coherent information'

Problems:

- J is not additive (regularization required)
- J is not concave (local extrema)
- Q is not known to be additive
- Q is not known to be convex

Extremal qubit channels

If a channel can be represented by a coupling to a qubit environment, the coding theorem can be considerably simplified due to the following [1]:

Theorem:

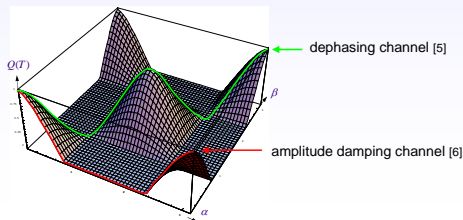
All $d_E=d=2$ channels with $Q>0$ are degradable. For higher dimensional systems still all channels in the $d_E=2$ neighborhood of any unitary are degradable.

(For larger environment this is generally false)

Normal form [7] for $d=2$: $T(\rho) = \sum_i K_i \rho K_i^\dagger$, $K_i = \begin{pmatrix} \cos \alpha & 0 \\ 0 & \cos \beta \end{pmatrix}$, $K_3 = \begin{pmatrix} 0 & \sin \beta \\ \sin \alpha & 0 \end{pmatrix}$

Capacity formula [1] $Q(T) = \max_{\rho \in (0,1)} h(p \cos^2 \alpha + (1-p) \sin^2 \beta) - h(p \sin^2 \alpha + (1-p) \sin^2 \beta)$

(maximum is attained for diagonal ρ)

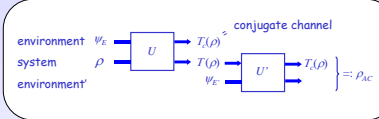


Literature

- [1] M.M. Wolf, D. Perez-Garcia, quant-ph/0607070
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- [3] M.M. Wolf, G. Giedke, J.I. Cirac, Phys. Rev. Lett. 96, 080502 (2006)
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Degradable Channels

If we represent a quantum channel by a coupling of a system to an environment, then it is called degradable if the output in the system is capable of simulating the output in the environment. Clearly, this can only be possible if the coupling to the environment is sufficiently small.



Definition: T is called **degradable** if there is a channel T' such that $T' T = T_c$

Main observation: If T is degradable, then J is a conditional entropy [5]

$$J(T) = \sup_{\rho} S(T(\rho)) - S(T_c(\rho))$$

$$S(\rho_{AC}) - S(\rho_C) = S(A|C)$$

Consequences:

- J becomes additive (no regularization required) [5]
- J becomes concave (no local extrema) [2]
- Q is provably additive
- Q is provably convex [1]

$$Q(T) = J(T)$$

Bosonic Gaussian Channels

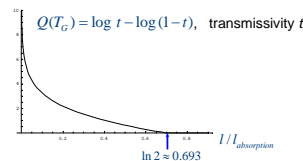


Gaussian channels describe the transmission of light in **optical fibers** as well as the storage of quantum information in **atomic ensemble quantum memories**.

Results [2]:

- Complete characterization of degradable Gaussian channels
- Capacity formula for all channels with single mode environment & the respective broadband channels
- Capacity is achieved by Gaussian input states
- Capacity upper bounds from convexity

Single mode channel with photon losses [2]:



The capacity vanishes at 70% of the absorption length.

Optimality of Gaussian input states follows from a central limit argument [3].

Certifying achievable rates for unknown channels

Effectively 'Gaussifying' an arbitrary channel by using a symmetrizing beam-splitter array in the encoding-decoding scheme allows to determine achievable rates for any channel in the following way:

- Send half of a two-mode squeezed state through the channel
- Measure the covariance matrix at the output
- Compute the capacity under the assumption the channel was Gaussian

