# Quantum and classical effects of radiation in relativistic laser plasma

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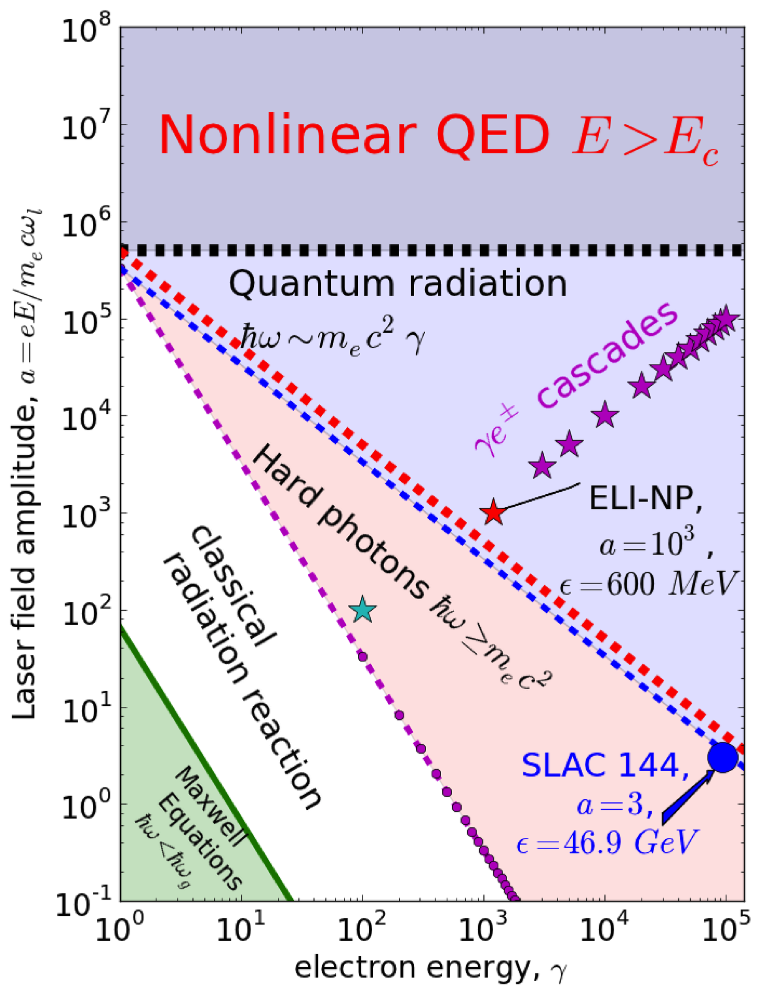
## Problem statement




- Quantum efficiency parameter

$$\chi = \frac{e\hbar}{m^3c^4}\sqrt{\left(\frac{\varepsilon\vec{E}}{c} + \vec{p}\times\vec{H}\right)^2 - (\vec{p}\cdot\vec{E})^2}$$

- Vacuum breakdown $\chi >> 1$
- Cascades $\chi \sim 1$
- Quantum effects of radiation reaction

$$\hbar\omega \sim \gamma mc^2, \quad a \sim \sqrt{\frac{2mc^2}{3\hbar\omega_0}} \sim 600$$

- Classical effects of radiation reaction

$$\Delta\varepsilon_{rad} = \frac{2\pi I_{rad}}{\omega_0} \geq \gamma mc^2 \; a \sim 300$$

### Nonlinear Compton scattering laser-plasma interaction

- at $I >> 10^{18}\, W/cm^2$ field is locally constant
- Crossed field approximation for ultra-relativistic particles

$$(E^2 - H^2)/E_S^2 \ll \chi_e^2, \quad \vec{E}\cdot\vec{H}/E_S^2 \ll \chi_e^2$$

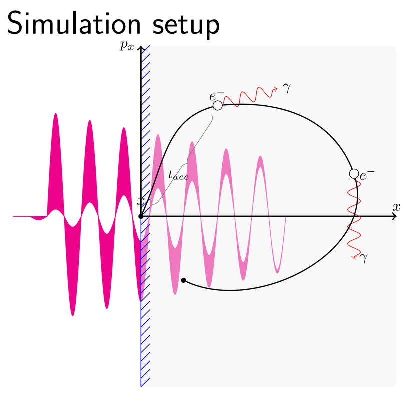
- Motion is quasi-classical in external classical field
- Dominating process is multi-photon Compton scattering

$$e^{\pm} + n\hbar\omega_l \rightarrow e^{\pm} + \gamma$$

- Quantum probability of emission calculated using Volkov states.

$$\frac{dw_\gamma}{d\varepsilon_\gamma} = -\frac{\alpha m^2 c^4}{\hbar\varepsilon_e^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)\, d\xi + \left(\frac{2}{x} + \chi_\gamma\sqrt{x}\right)\mathrm{Ai}'(x)\right\},$$

Simulation setup



where $x = [\chi_\gamma/\chi_e(\chi_e - \chi_\gamma)]^{2/3}$ with $0 \leq \chi_\gamma < \chi_e$, $\mathrm{Ai}(x)$ is the Airy function.

### Transport equations for QED plasma [1], [2]

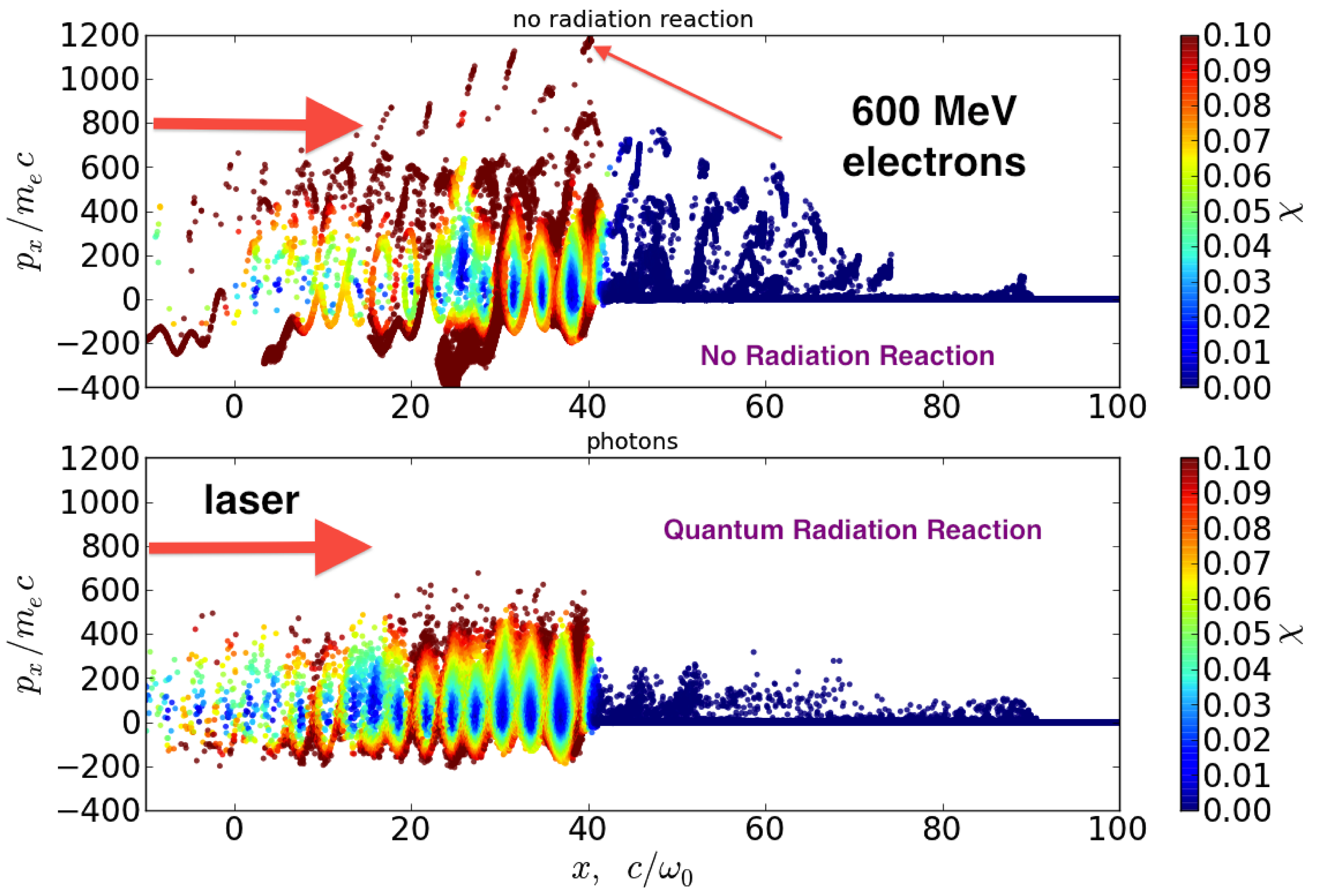
- Kinetic equation for relativistic plasma

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{\gamma m}\cdot\frac{\partial}{\partial\vec{r}} + \vec{F}\cdot\frac{\partial}{\partial\vec{p}}\right) f(\vec{r},\vec{p},t) = \int d^3k\, W_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p}+\vec{k})\, f(\vec{r},\vec{p}+\vec{k},t) - f(\vec{r},\vec{p},t)\int d^3k\, W_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p})$$

- Kinetic equation for hard photons ($\hbar\omega \geq m_e c^2$)

$$\left(\frac{\partial}{\partial t} + \frac{\partial\omega}{\partial\vec{k}}\cdot\frac{\partial}{\partial\vec{r}}\right) f_\gamma(\vec{x},\vec{k},t) = \int d^3p\, \mathcal{W}_\gamma^{\vec{E},\vec{B}}(\vec{k},\vec{p})\, [f(\vec{r},\vec{p},t)]$$

### Hard photon emission in laser plasma dynamics [4]



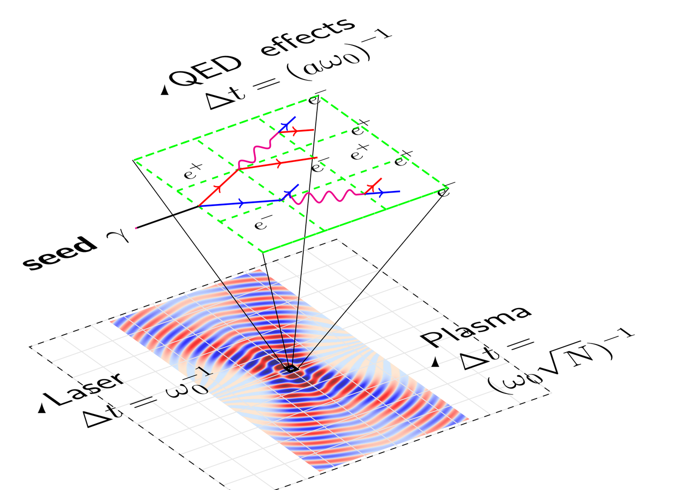

*Radiation reaction reduces a gain of energy by electrons.*
*Simulation parameters $a = 100$, $n_p = 10n_{cr}$*

## Adaptive Simulation of Radiation Effects

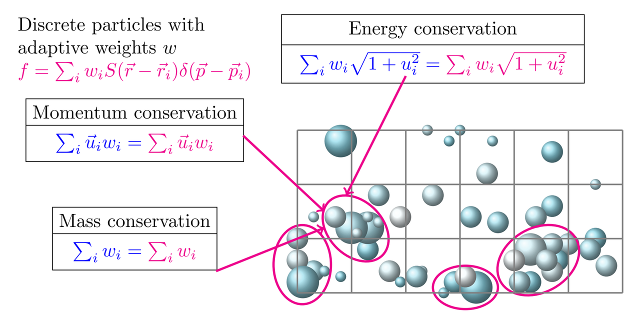
### Numerical issues

**Pros:**
- Allows to resolve dynamically emerging plasma effects
- Extends the range of wave modes supported by grid.
- Improves accuracy and safes computational power.

**Cons:**
- Spurious reflection of waves on non-uniformity (solved)
- Violation of conservation at grid interfaces (solved)
- Algorithmic complexity.




### Adaptation in momentum space




**Todo list:**
- Optimization of splitting/clustering methods for particles.
- Reduction of noise in charge and current deposition methods.
- Criteria for grid adaptation for the Maxwell equations with nonlinear source terms.
- Matching of adaptive algorithms in momentum and real space.

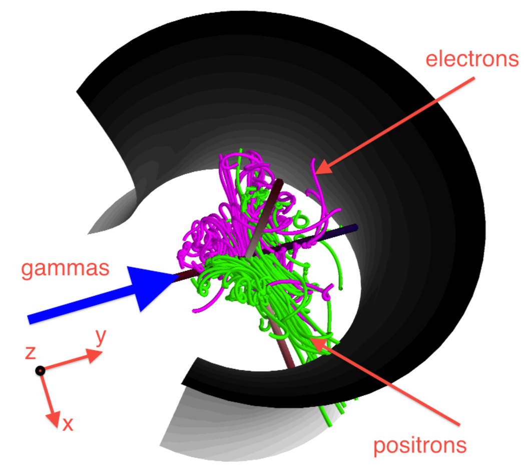
### Simulation of QED cascades using adaptive particles

- Nonlinear Compton scattering

$$e^- + n\hbar\omega_l \rightarrow e^{-'} + \gamma$$

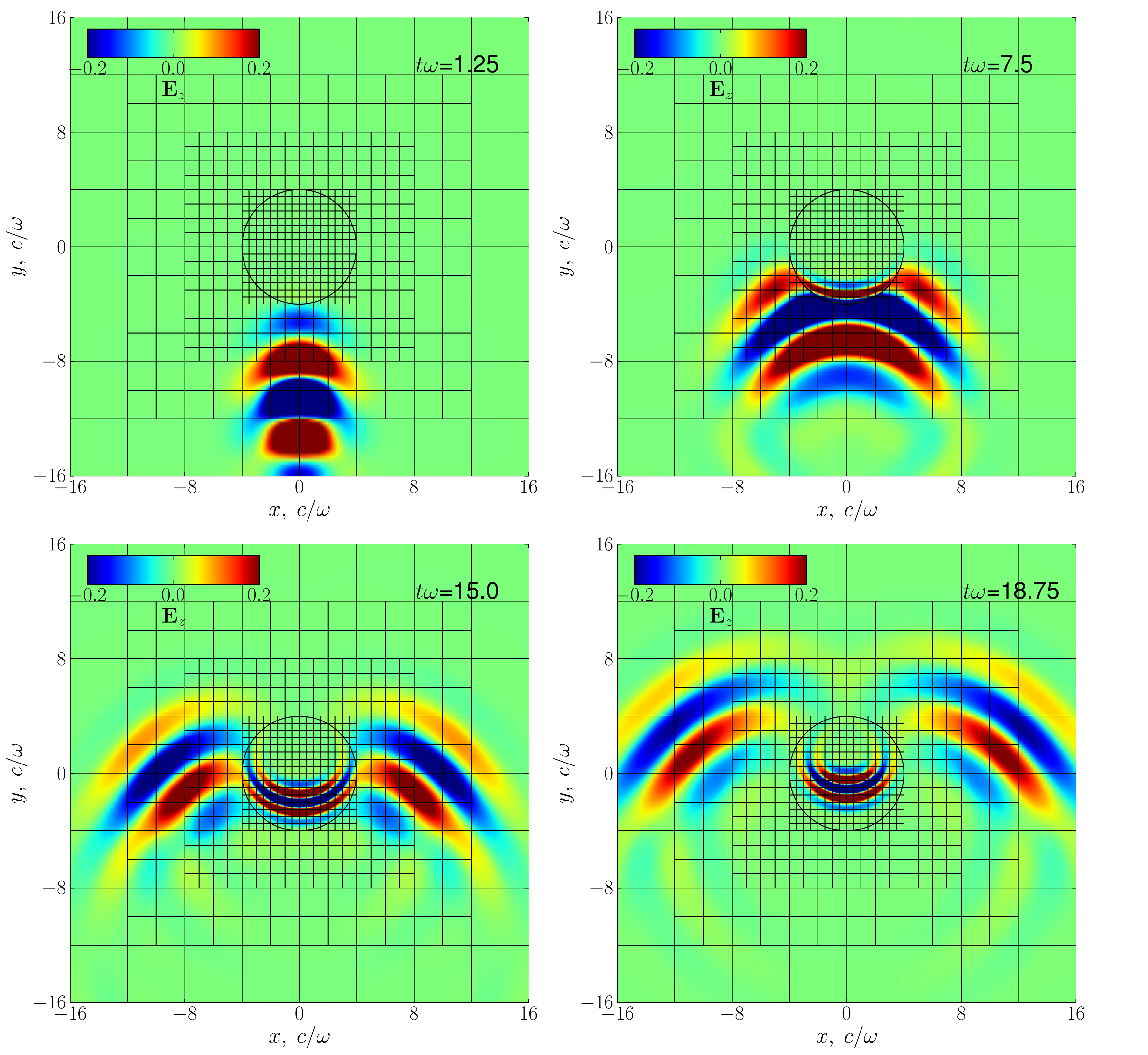
- Stimulated pair production:

$$\gamma + n\hbar\omega_l \rightarrow e^+ + e^-$$




$$\frac{dW_{cr}(\varepsilon_e)}{d\varepsilon_e} = \frac{\alpha m^2 c^4}{\hbar\varepsilon_\gamma^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)\, d\xi + \left(\frac{2}{x} - \chi_\gamma\sqrt{x}\right)\mathrm{Ai}'(x)\right\},$$

### Reflection free AMR-Maxwell: laser pulse and refractive disc




## Conclusions

We report on our accomplishments in development of *a novel numerical methodology for modeling of the high-frequency radiation reaction effects in ultra-relativistic plasma using the adaptive mesh refinement technique*. The research of the radiation reaction lies at the border between laser plasma physics and quantum electrodynamics. The radiation reaction arises from the interaction of electrons with their own electromagnetic field and *unresolved* high-frequency photons emitted by other particles and becomes important at high electromagnetic intensity. There is still no single simulation method which is applicable for both classical relativistic plasma processes and quantum electrodynamical effects at such conditions. We suggest to plan to bring these scientific disciplines together by employing the adaptive mesh refinement technique in kinetic plasma simulation.

[1] Elkina N. V., Fedotov A. M., Kostyukov I. Yu., Legov M. V., Narozhny N. B., Nerush E. N. , and Ruhl H., QED cascades induced by circularly polarized laser fields, Phys. Rev. ST. Accel. 14, 054401 (2011).
[2] Nerush E. N., I. Kostyukov I. Yu., Fedotov A. M., Narozhny N. B., Elkina N., and Ruhl H., Laser Field Absorption in Self-Generated Electron-Positron Pair Plasma, Phys. Rev. Lett. 106, 035001 (2011).
[3] Hadad Y., Labun L., Rafelski J., Elkina N., Klier C., and Ruhl H., Effects of radiation reaction relativistic laser acceleration, Phy. Rev. D 82, 096012 (2010).
[4] N. Elkina, H. Ruhl, " The effects of radiation reaction on streaming instabilities in laser plasma", in preparation.