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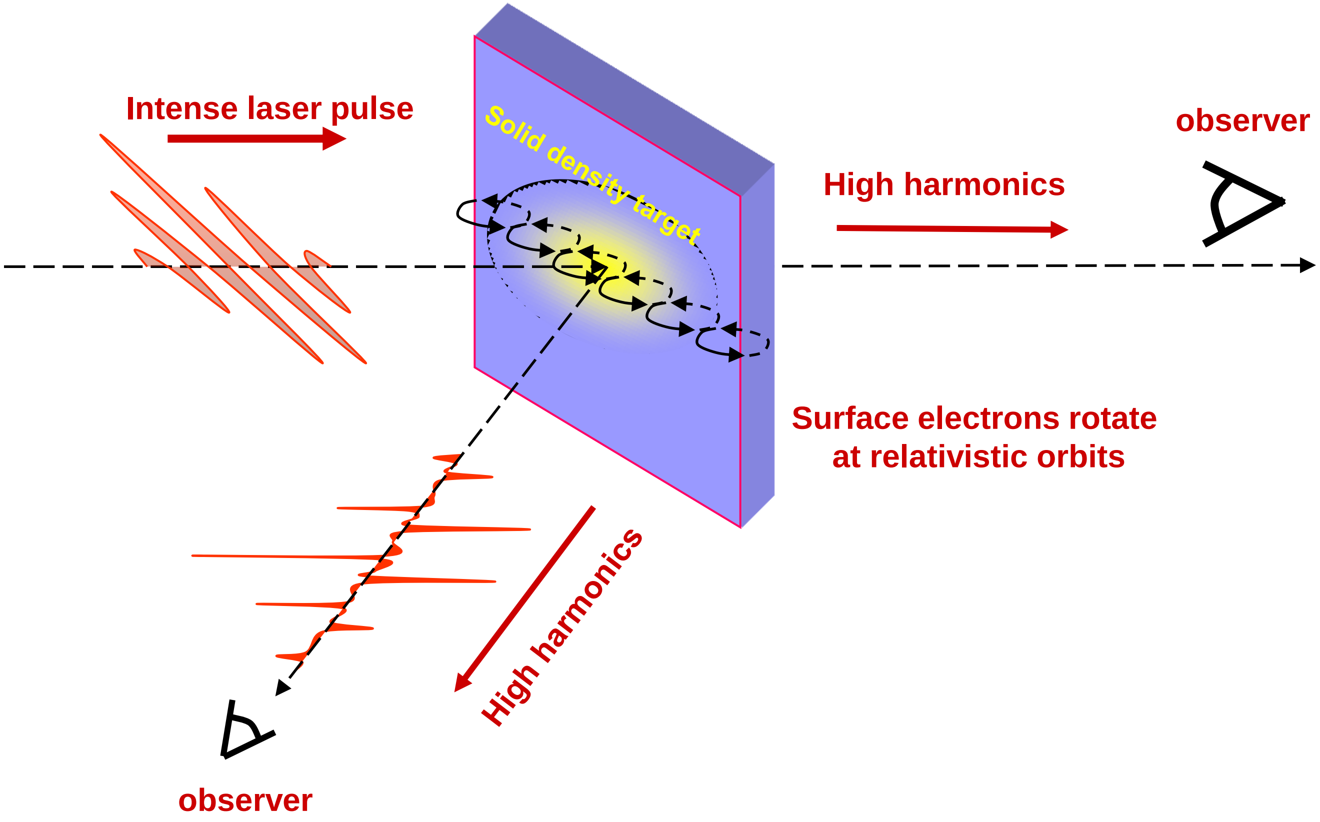
High Harmonic Generation from Overdense Plasmas

FILMITh, Garching 2012

Outline

- **Relativistic harmonics and attosecond pulses from overdense plasmas, spectrum $n^{-8/3}$**
- **Highly efficient regime of electron nanobunching, spectrum $n^{-6/5}$**
- **Spectral modulations and the femtosecond plasma surface dynamics**
- **2D dynamics and surface wave excitation**

Relativistic High Harmonics from solid state target surfaces



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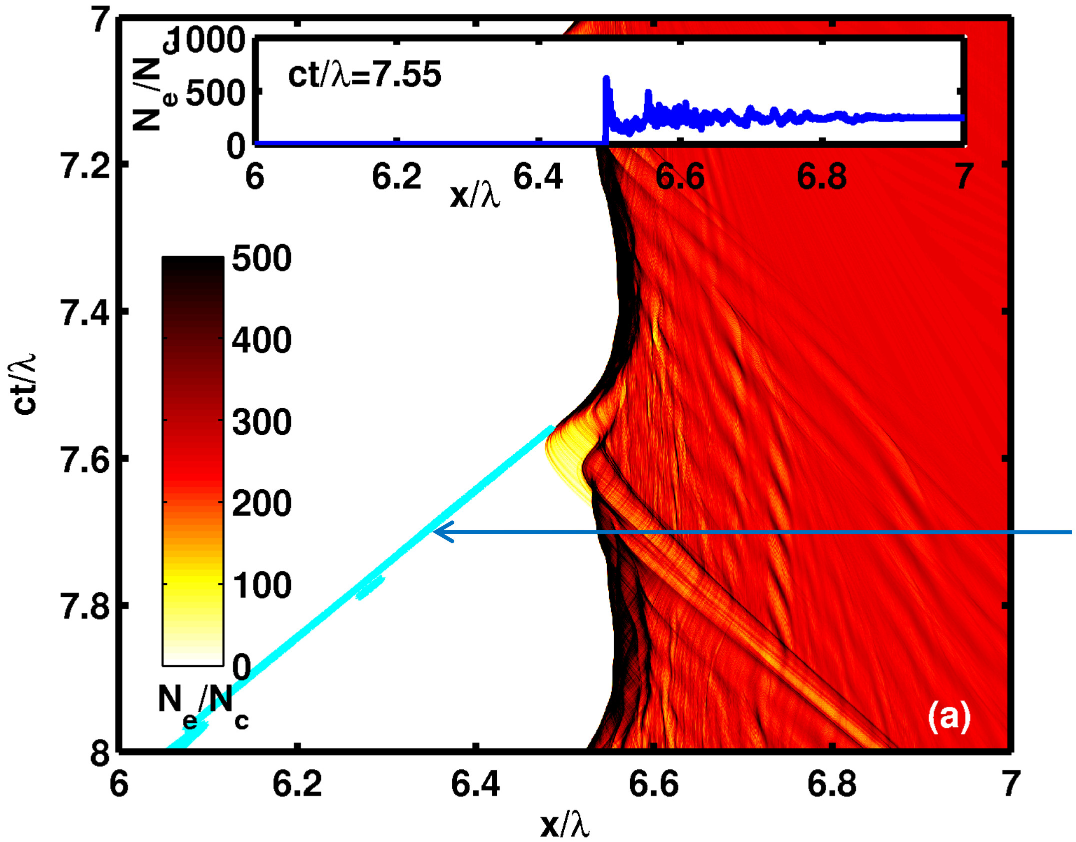
Relativistic Harmonics



Parameters:
$N_e = 250\ N_c$
step profile
a_0=60

Harmonic emission

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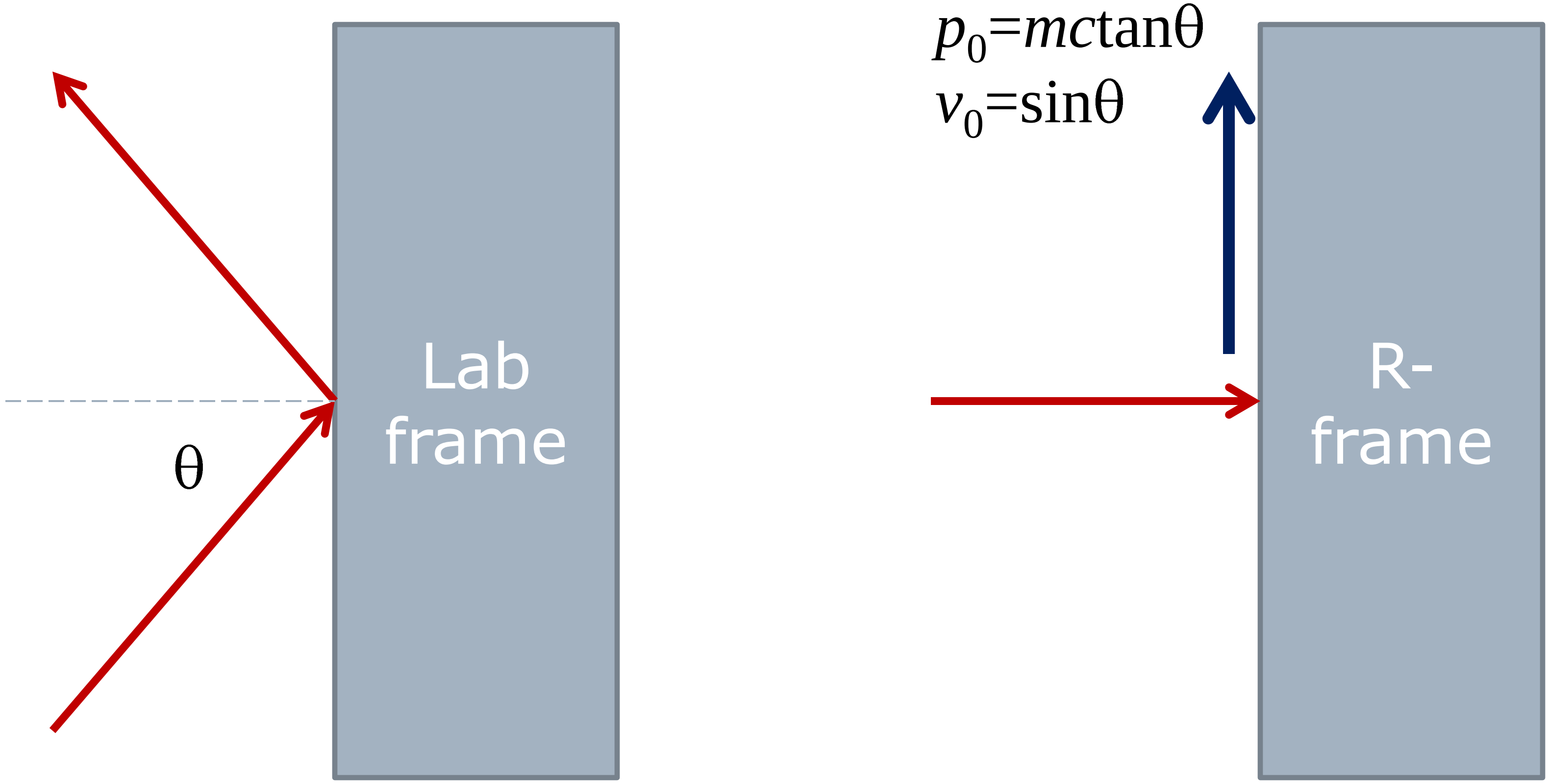
Oblique incidence in 1D geometry



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Analytic description of HHG

Electric field

$$\mathbf{E}_\perp(t,x)=\frac{2\pi}{c}\int_x^{+\infty}\left[\mathbf{j}_\perp\left(t+\underset{\substack{\uparrow\\ \text{Reflected}\\ \text{wave}}}{\frac{x-x'}{c}},x'\right)-\mathbf{j}_\perp\left(t-\underset{\substack{\uparrow\\ \text{Incoming}\\ \text{wave}}}{\frac{x-x'}{c}},x'\right)\right]dx'$$

So that at the left of plasma $\mathbf{E}_\perp(t,x)=\mathbf{E}_i(t-\frac{x}{c})+\mathbf{E}_r(t+\frac{x}{c})$

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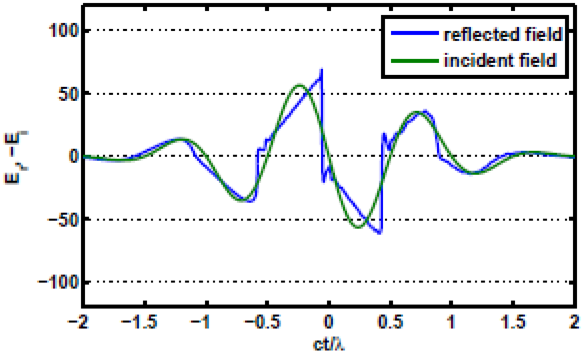
The ROM boundary condition

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)

- Skin layer evolution time long compared to skin length (e.g. step-like profile), then follows ARP boundary condition

$$E_i\left(x_{ARP}(t)-ct\right)+E_r\left(x_{ARP}(t)+ct\right)=0$$

- Reflected field phase modulation of incident field
    - same maxima and minima
    - same sequence of monotonic intervals



Normal incidence, $N_e = 250\,N_c$, sharp edged profile, $a_0 = 60$

Analytical derivation of the BGP spectrum

Fourier transform of the reflected wave

$$E_r(\omega) = -\int E_i\left(t - \frac{x_{ARP}}{c}\right) e^{i\omega(t + x/c)} \left(1 + \frac{\dot{x}_{ARP}}{c}\right) dt$$

We assume for the incident wave $E_i(t) = a(t)\left[e^{i\omega_0 t} + e^{-i\omega_0 t}\right]/2$

The reflected wave is thus $E_r(\omega) = E_+ + E_-$

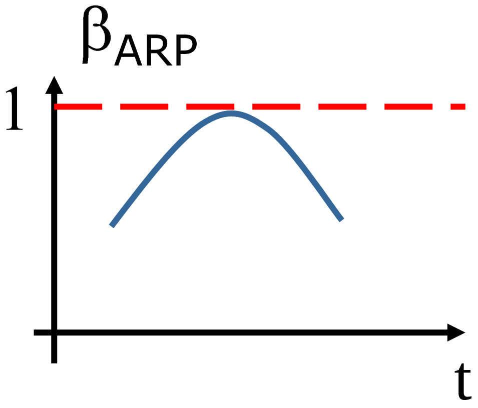
$$E_\pm(\omega) = -\int a\left(t - \frac{x_{ARP}}{c}\right) e^{i\left[\omega\left(t + \frac{x_{ARP}}{c}\right) \pm \omega_0\left(t - \frac{x_{ARP}}{c}\right)\right]} \left(1 + \frac{\dot{x}_{ARP}}{c}\right) dt$$

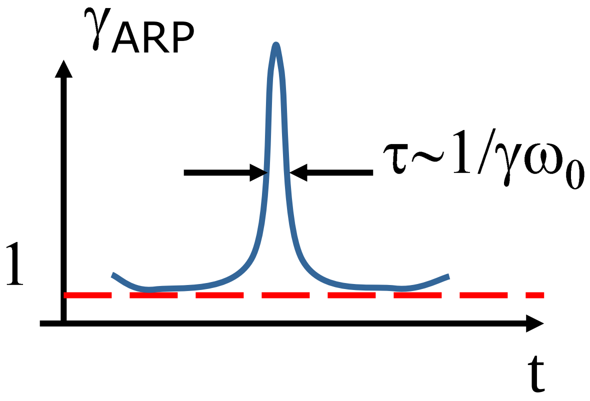
Stationary points and the γ-spikes

The stationary phase points correspond to the instants when the apparent reflection point (ARP) moves towards the observer with maximum velocity.

The corresponding ARP gamma factor

$$\gamma_{ARP} = \left(1-\dot{x}_{ARP}^{2}\right)^{-1/2}$$





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General form of the spectrum

The spectrum can be calculated for an arbitrary order *n* of the surface velocity maximum

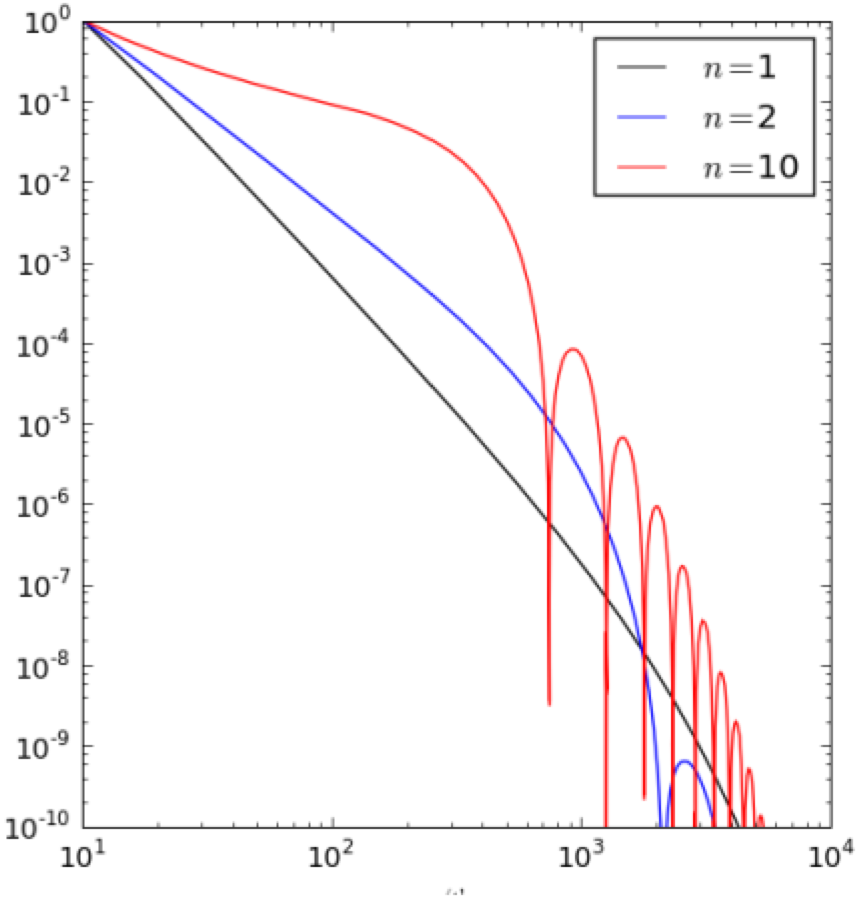
$$I_{\mathrm{n}}(\omega) \propto \omega^{-\frac{4n+4}{2n+1}} \left[\mathrm{Gai}_n\left(\frac{\omega\gamma^{-2} - 4\omega_0}{2\left(\alpha\omega\right)^{1/(2n+1)}} \right) - \mathrm{Gai}_n\left(\frac{\omega\gamma^{-2} + 4\omega_0}{2\left(\alpha\omega\right)^{1/(2n+1)}} \right) \right]^2$$

Where $\mathrm{Gai}_{\mathrm{n}}$ is the generalized Airy function:

$$\mathrm{Gai}_{\mathrm{n}}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[i\left(xt + t^{2n+1}/(2n+1) \right) \right] dt$$

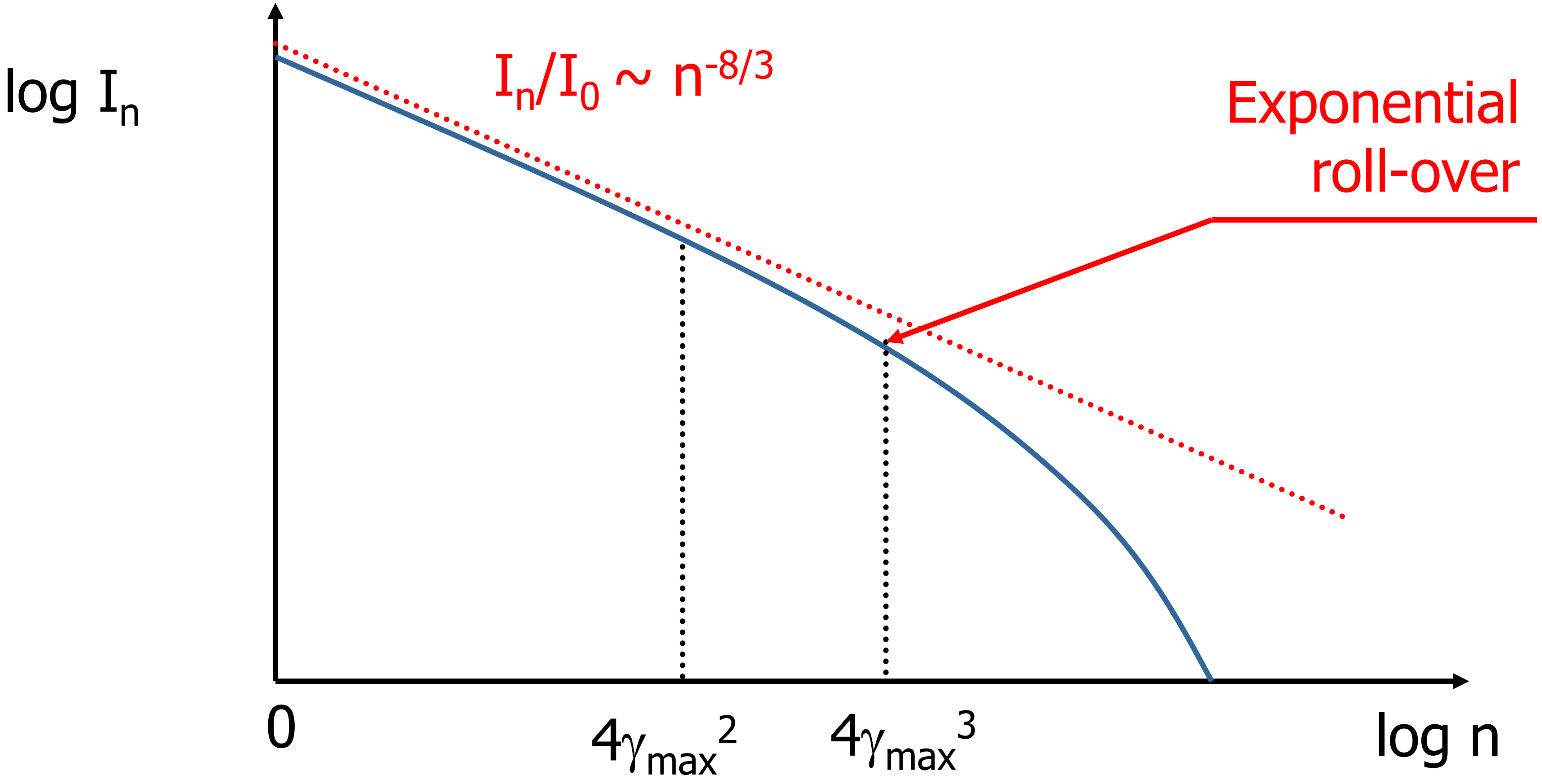
General form of the spectrum

The spectrum shape
for orders *n*=1,2,10
of the surface
velocity maximum.
α=1, γ=8 have been used



The BGP Spectrum

Baeva, Gordienko, Pukhov, *Phys. Rev. E***74**, 046404 (2006)



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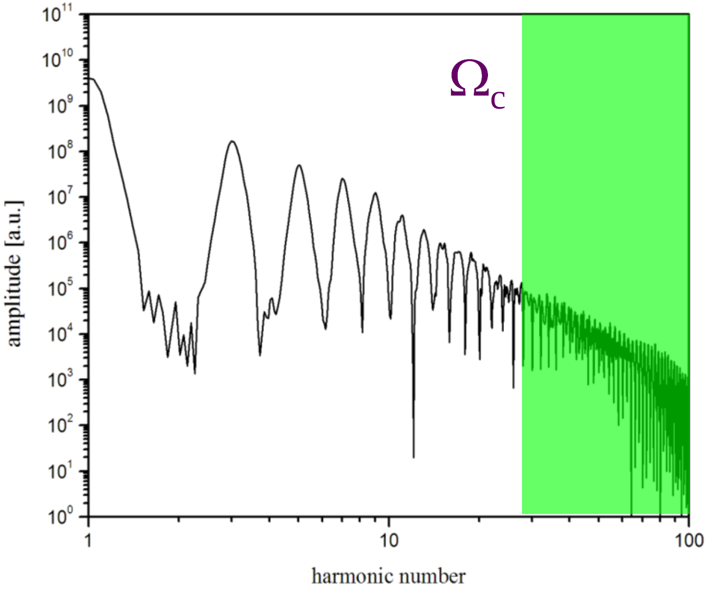
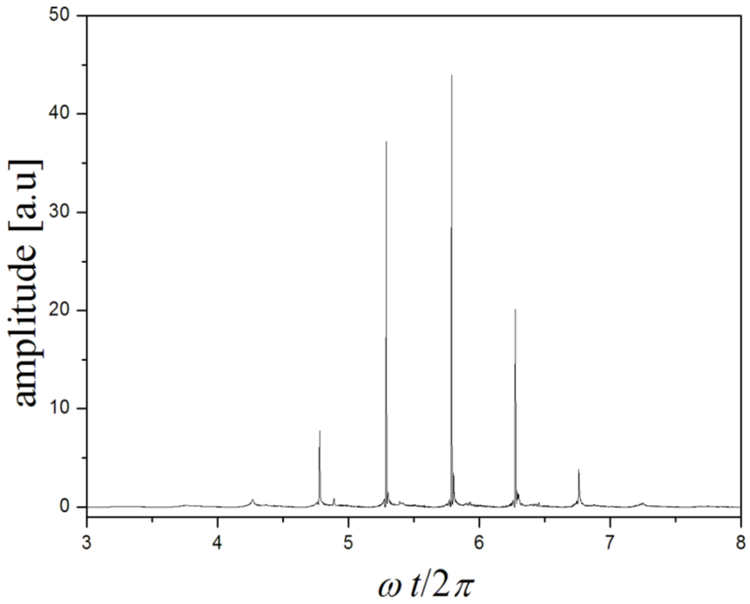
(Sub-)Attosecond Pulses

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)

- After proper filtering of RHHG one obtains *a train of (sub-)attosecond pulses*





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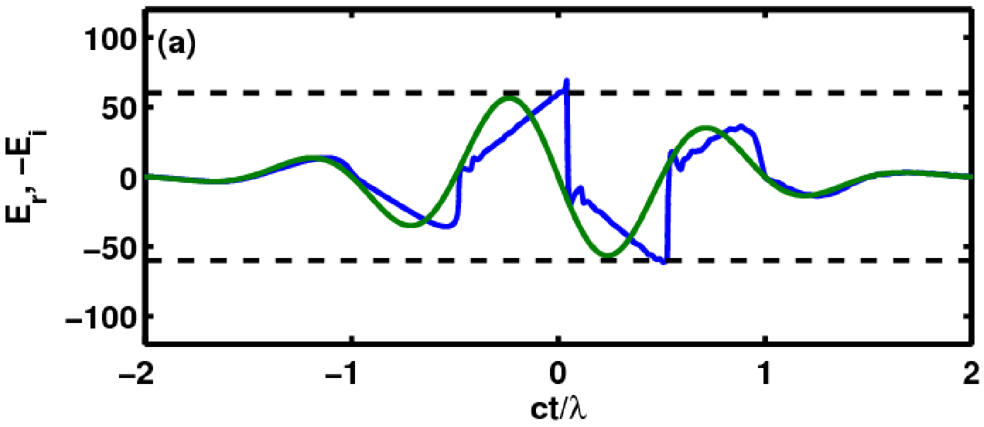
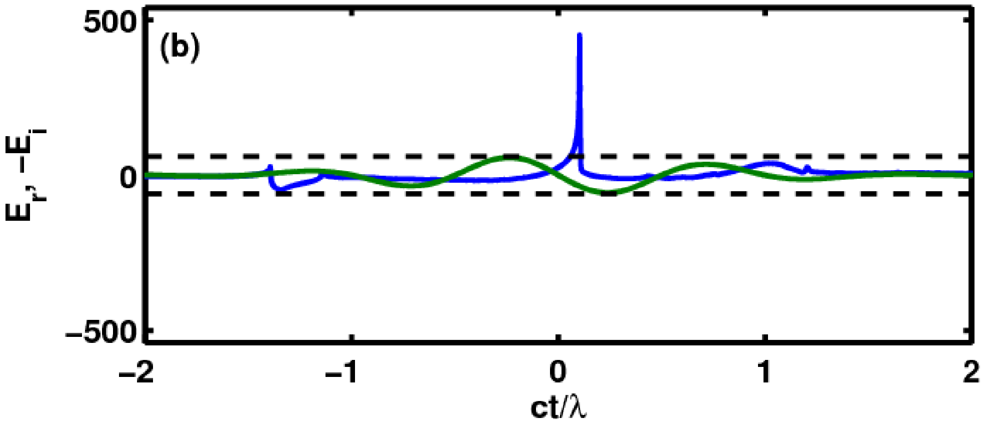
How universal is the -8/3 spectrum?



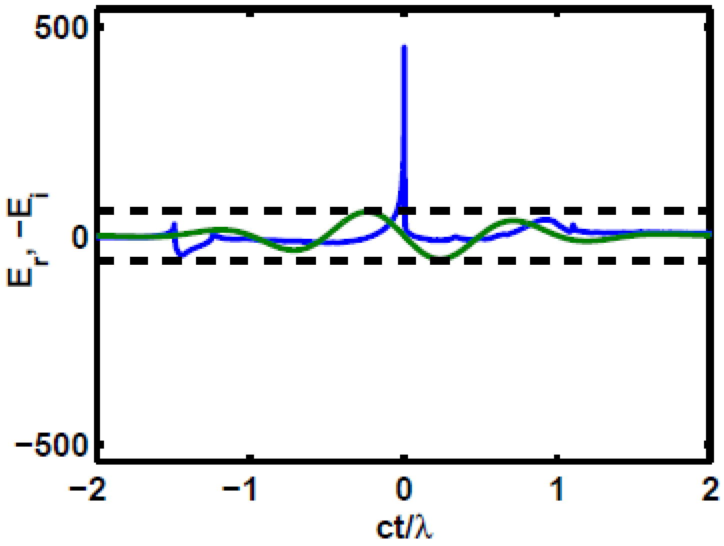
BGP case:
pure phase modulation
of the laser field

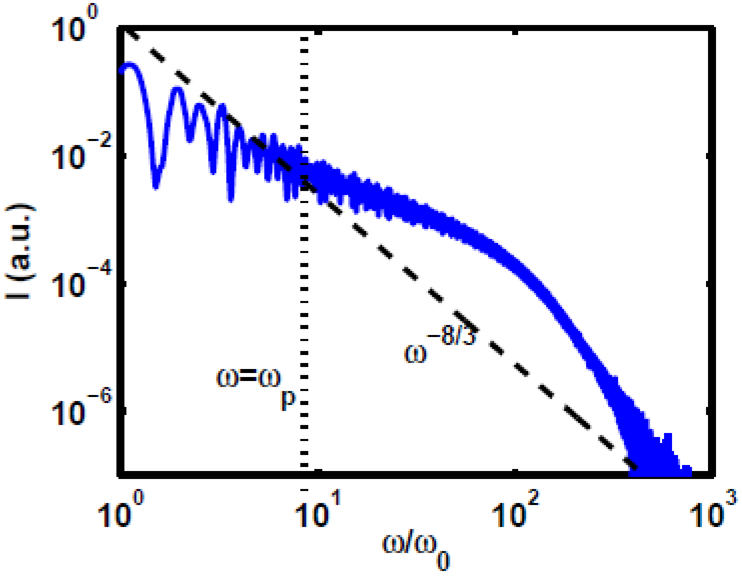


Clear violation of
the boundary condition
oblique incidence
finite density gradient

Violation of the boundary condition. Much flatter spectrum, isolated pulse

Simulation parameters: plasma density ramp $\propto \exp(x/(0.33\,\lambda))$ up to a maximum density of $N_e = 95\,N_c$ (lab frame), oblique incidence at 63° angle (p-polarised). Laser field amplitude is $a_0 = 60$.





- Violation of ARP boundary condition $\Longrightarrow$ no ROM
- No spectral cut-off at plasma frequency $\Longrightarrow$ no CWE

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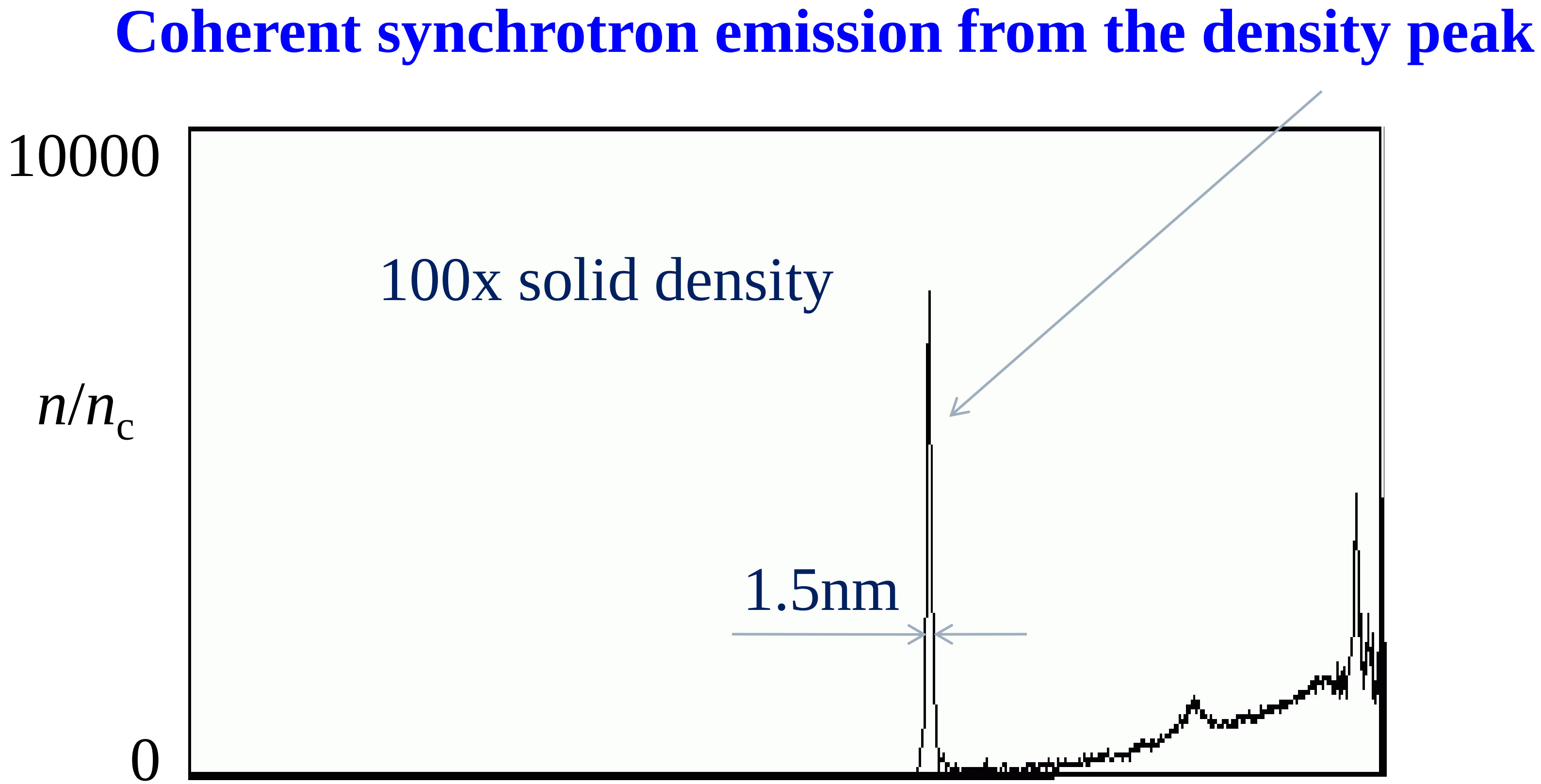
DadB, Pukhov, Phys. Plasmas 17, 033110 (2010)

Nanobunching in electron density



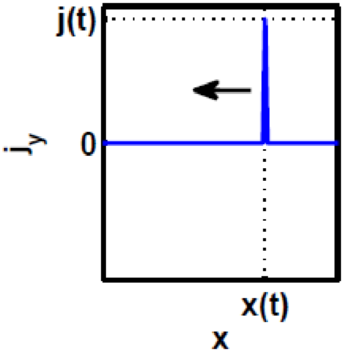
DadB, Pukhov, Phys. Plasmas 17, 033110 (2010)

1D Coherent Synchrotron

- 1D current distribution generates E-field:

$$E_{sy}(t,x) = 2\pi \int_{-\infty}^{+\infty} j\left(t + \frac{x - x'}{c}, x'\right) dx'$$

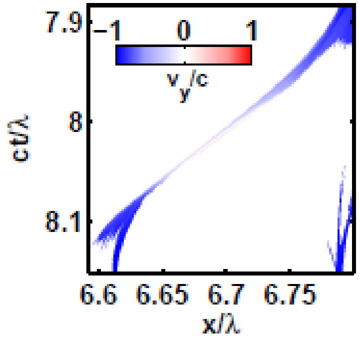
- Assume infinitely thin current layer $j(t,x) = j(t)\delta(x - x_{el}(t))$ and Fourier-transform to obtain spectrum:



$$\tilde{E}_{sy}(\omega) = 2\pi \int_{-\infty}^{+\infty} j(t)\, e^{-i\omega(t + x_{el}(t)/c)} dt$$

- Asymptotic behaviour ($\omega \gg 1$) of spectrum depends on stationary phase point $t = 0$:

$$\dot{x}_{el}(0) \approx -c$$



Simulation parameters: $a_0 = 60$, plasma density ramp $\propto \exp(x/(0.33\,\lambda))$, maximum density $N_e = 95\,N_c$, p-polarised incidence at 63°

- 2nd order zero in v_y
- Bunch width: $\delta_{FWHM} \approx 0.0015\,\lambda$, roughly Gaussian profile

Excellent Agreement between Theory and Simulation

- Theoretical Spectrum:

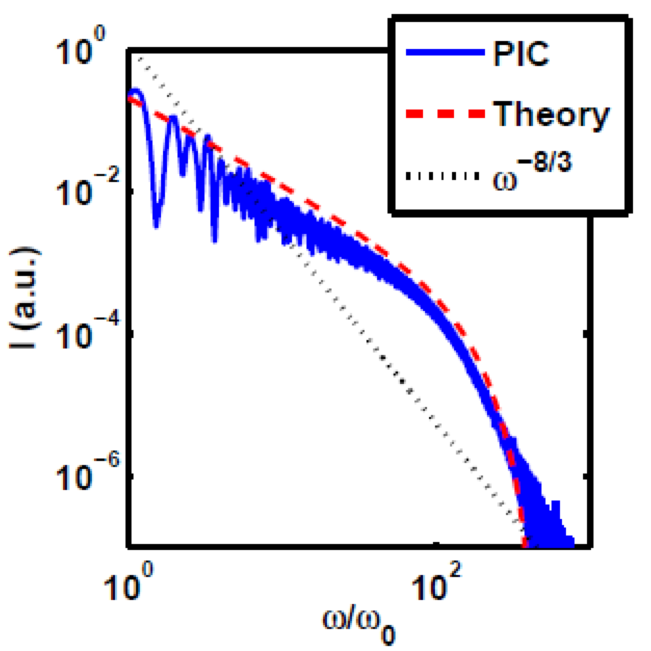
$$I(\omega) \propto \omega^{-6/5} \times \left[S''\left(\left(\frac{\omega}{\omega_{rs}}\right)^{4/5}\right)\right]^2 \times \exp\left(-\left(\frac{\omega}{\omega_{rf}}\right)^2\right)$$

using

$\omega_{rs} = 800$

$\omega_{rf} = 225$

compatible with PIC data



Two relativistic regimes of HHG

1. BGP case:
   plasma boundary stays “conjunct”
   the skin layer emits as a whole
   → the universal **$n^{-8/3}$** spectrum

2. Nanobunching of plasma electrons,
   Coherent Synchrotron Emission (**CSE**)
   → much flatter spectra, **$n^{-4/3}$ or $n^{-6/5}$**

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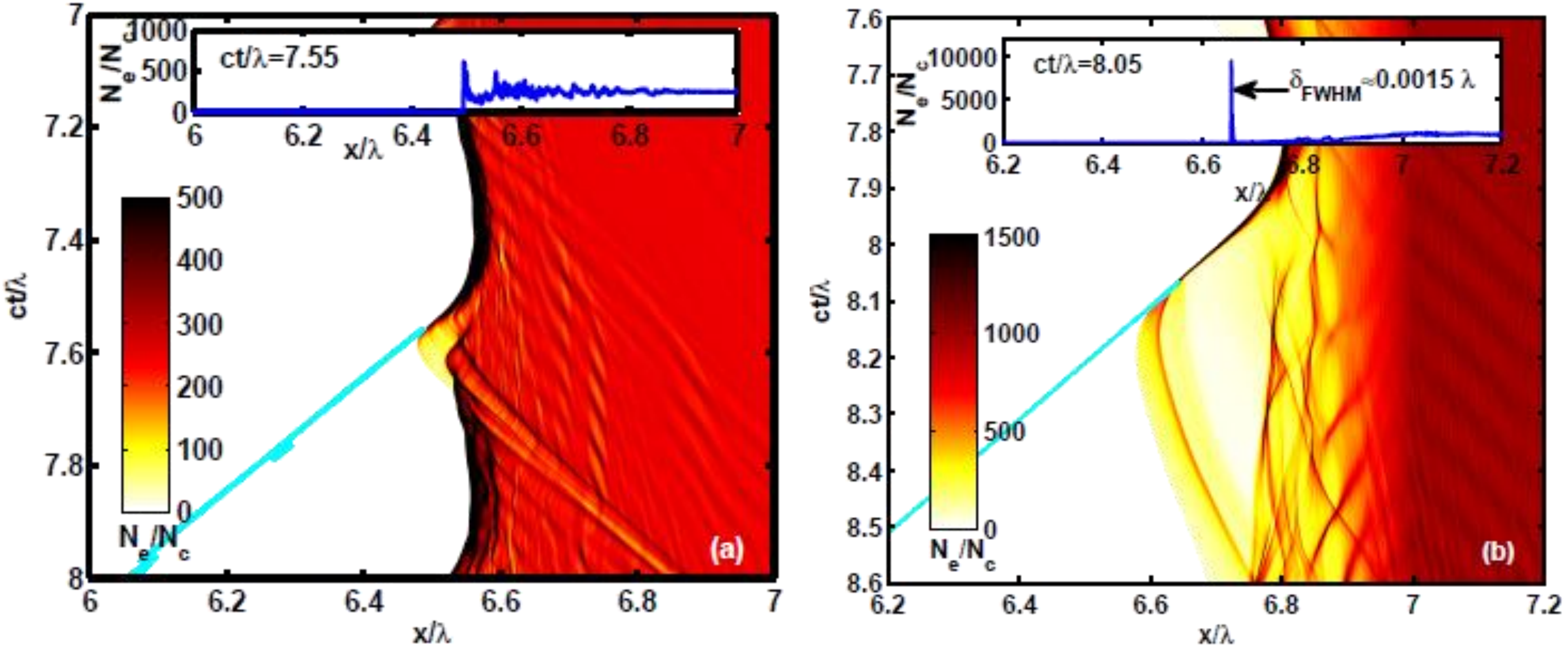
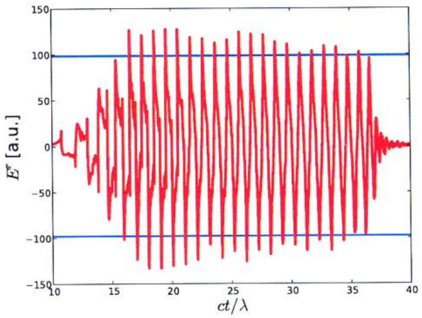
BGP vs CSE case



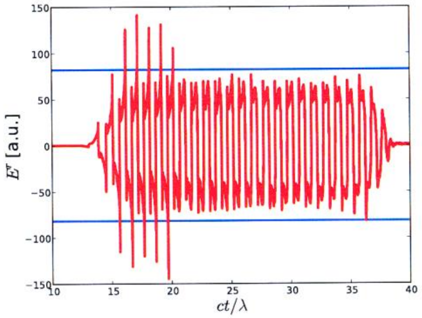
Pictures: electron density and contour lines (cyan) of the emitted harmonics radiation ($\omega > 4.5\omega_0$).
Simulation parameters: $a_0 = 60$ in both cases, (a) normal incidence, $N_e = 250\,N_c$, sharp edged profile;
(b) plasma density ramp $\propto \exp(x/(0.33\,\lambda))$, maximum density $N_e = 95\,N_c$, p-polarised incidence at 63°

Boundary condition violation

Normal incidence

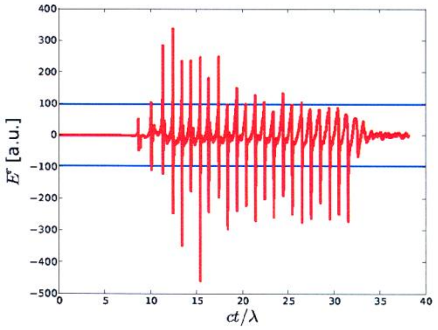


(a) $L = 0.464\lambda$ und $a_0 = 98$

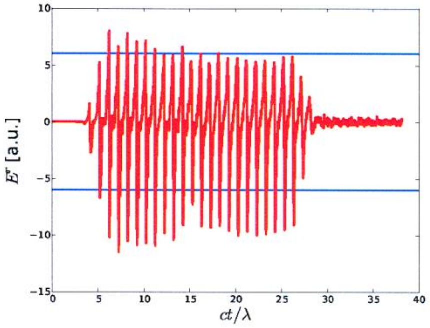


(b) $L = 0.107\lambda$ und $a_0 = 82$

Oblique incidence



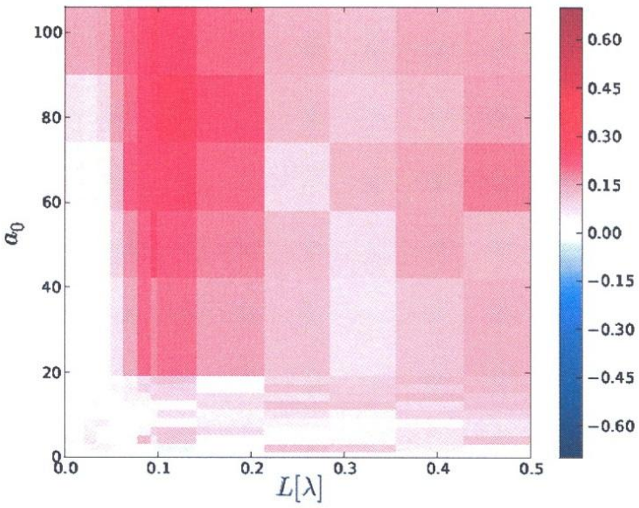
(a) $L = 0.25\lambda$ und $a_0 = 98$



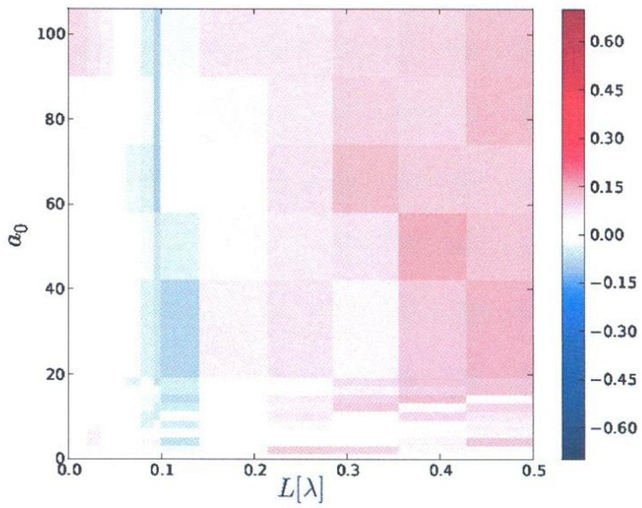
(b) $L \approx 0.08\lambda$ und $a_0 = 6$

Boundary condition violation parameter study, normal incidence

Head of the pulse



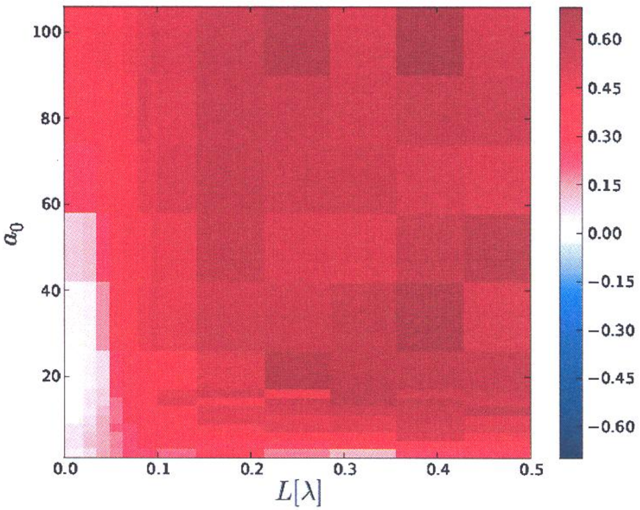
Bulk pulse



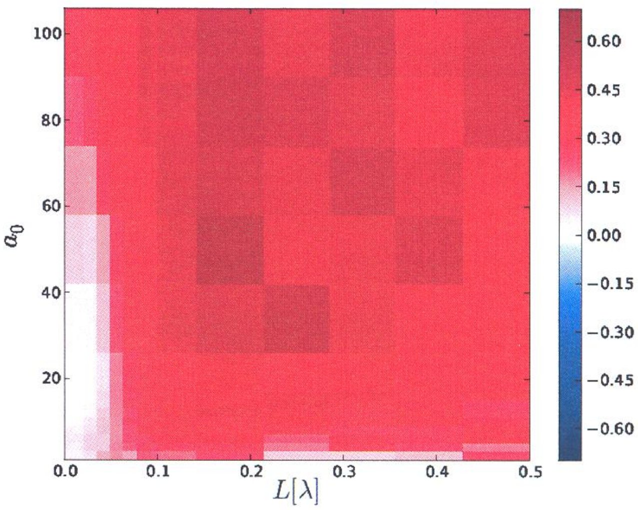
$$\frac{\max|E_r| - \max|E_i|}{\max|E_r| + \max|E_i|}$$

Boundary condition violation parameter study, oblique incidence

Head of the pulse



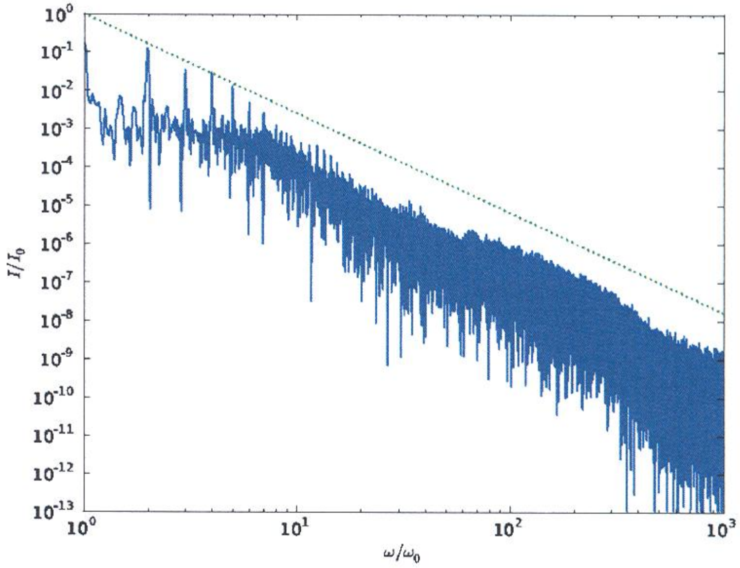
Bulk pulse



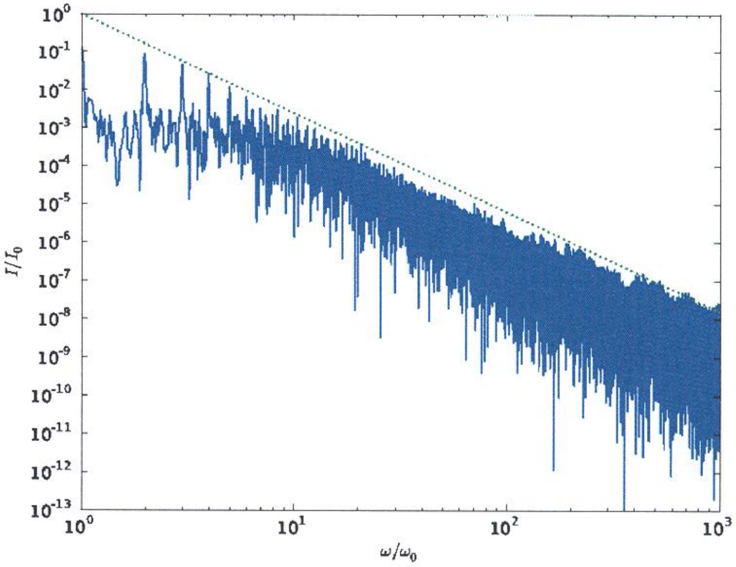
$$\frac{\max|E_r|-\max|E_i|}{\max|E_r|+\max|E_i|}$$

Boundary condition violation influence on spectra oblique incidence

a_0=6



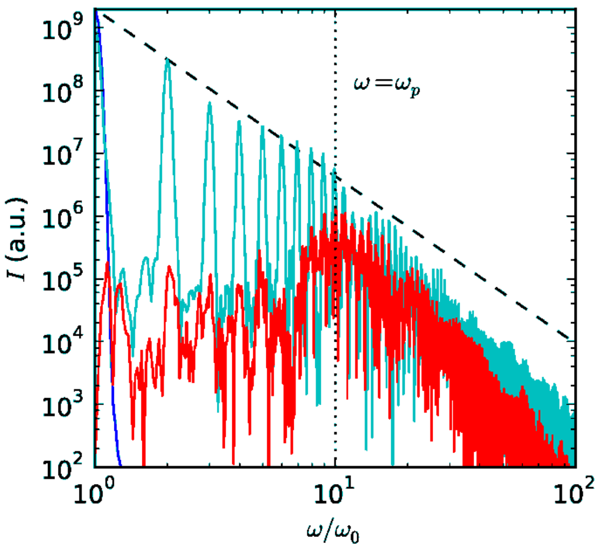
a_0=66

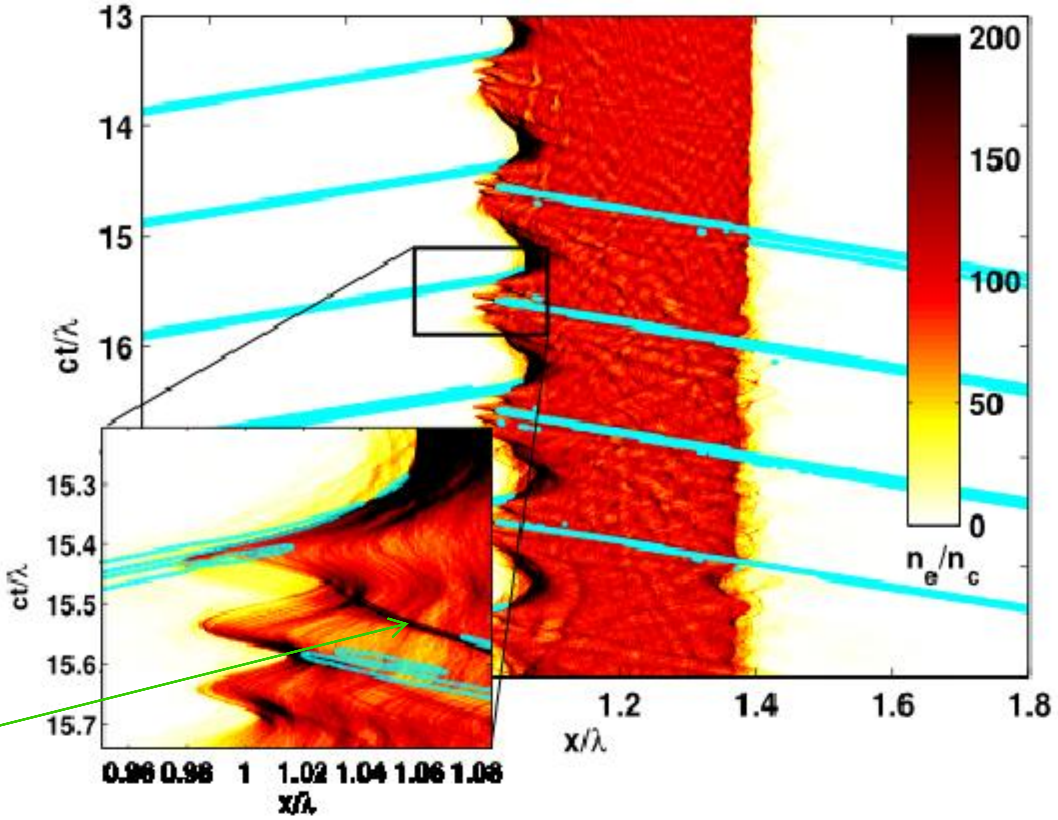


Hardly any influence on the spectra.

BGP spectra energetically dominate over nanobunching

Forward harmonic emission





Nanobunches?

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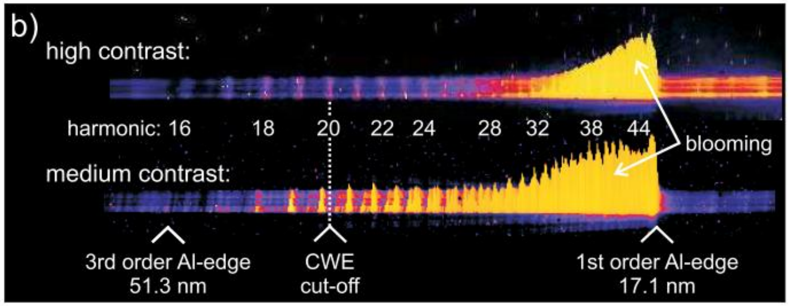
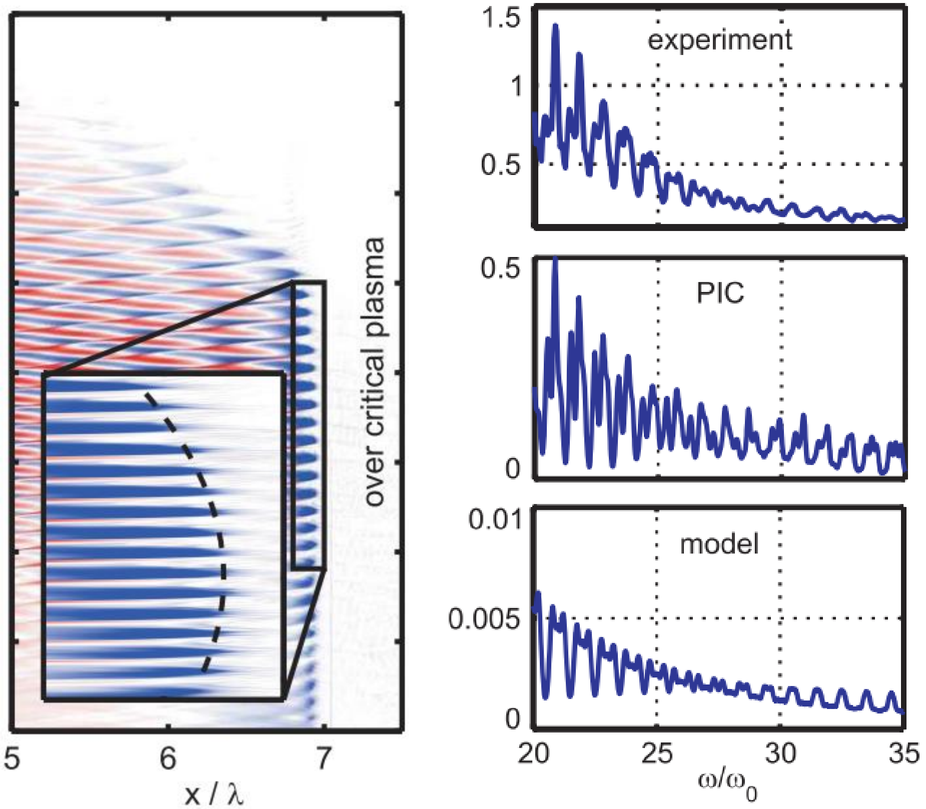
HHG experiment on D'Arcturus

M. Behmke et al., *Phys. Rev. Lett.*, **106**, 185002 (2011)

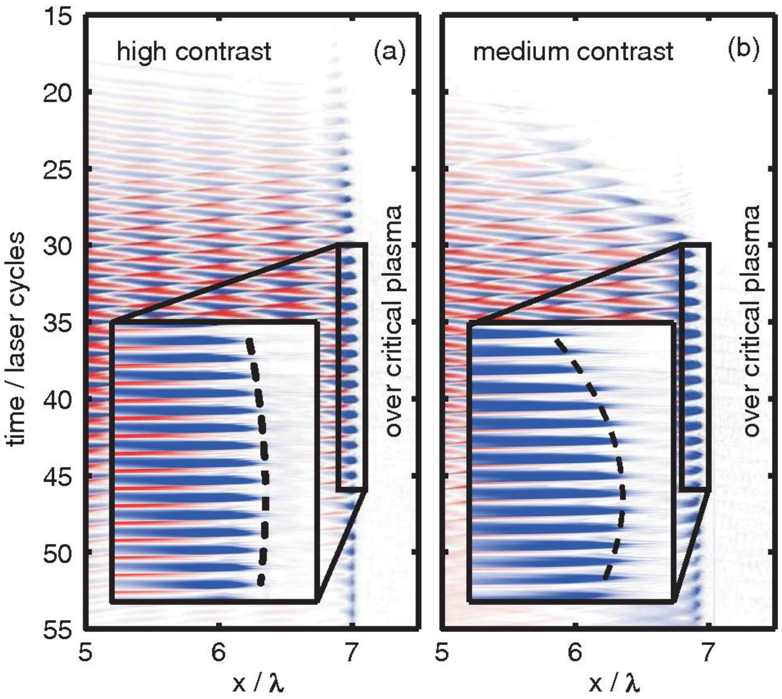


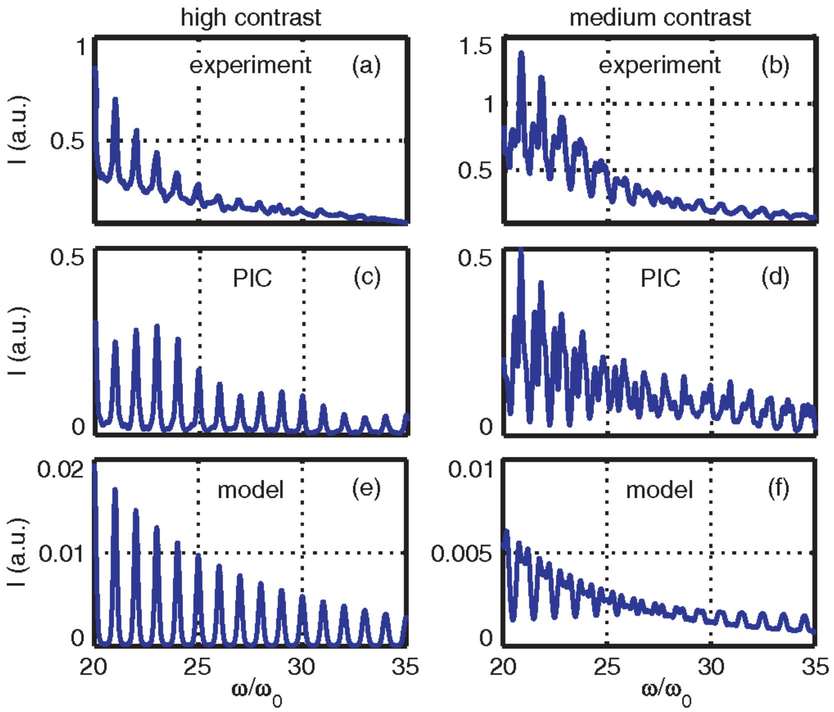


The harmonic spectrum contains information on the femtosecond dynamics of relativistic plasma

Spectral modulations and surface dynamics

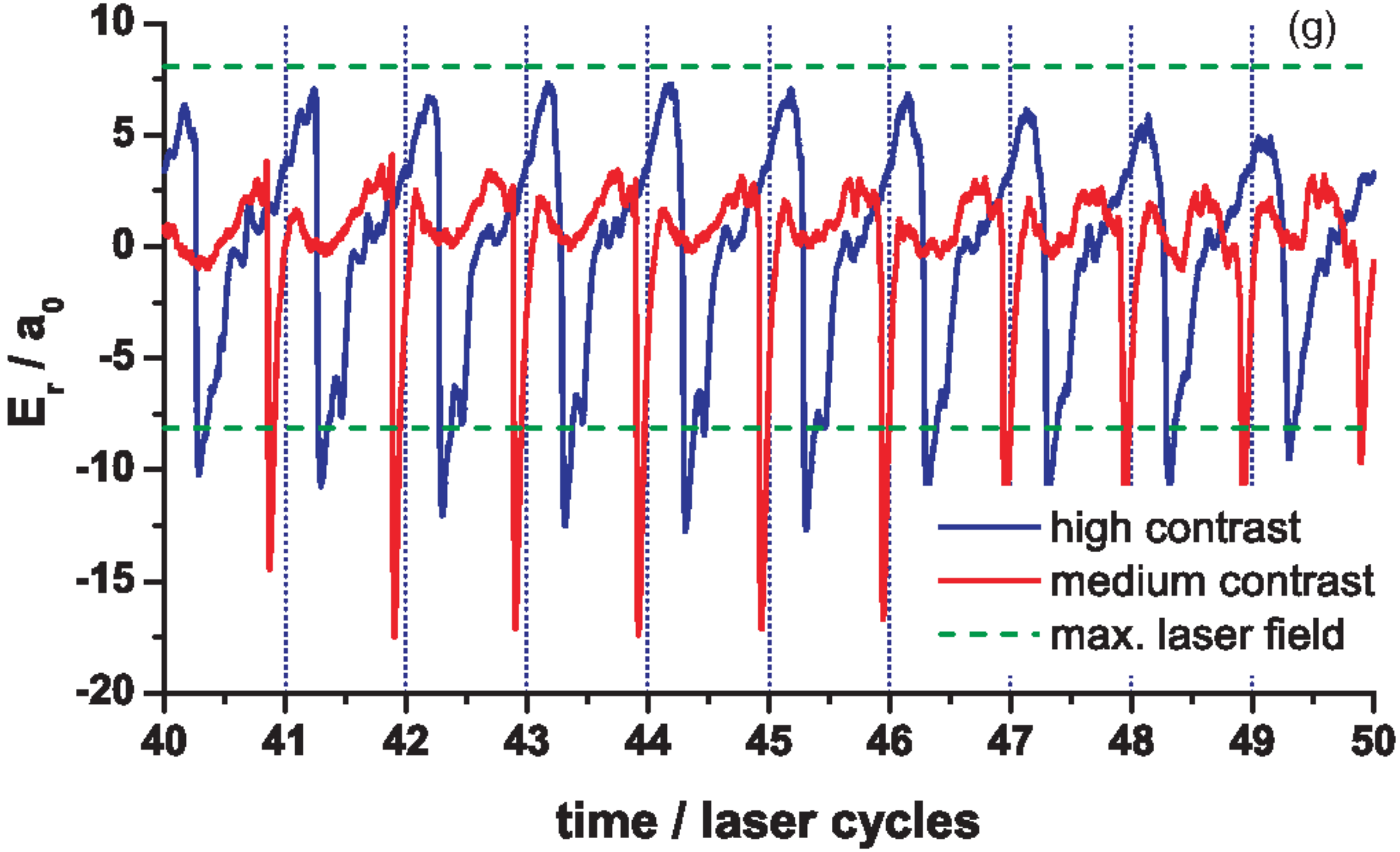
M. Behmke et al., *Phys. Rev. Lett.*, **106**, 185002 (2011)





Violation of ROM boundary condition in the D`Arcturus experiment

M. Behmke et al., *Phys. Rev. Lett.*, **106**, 185002 (2011)



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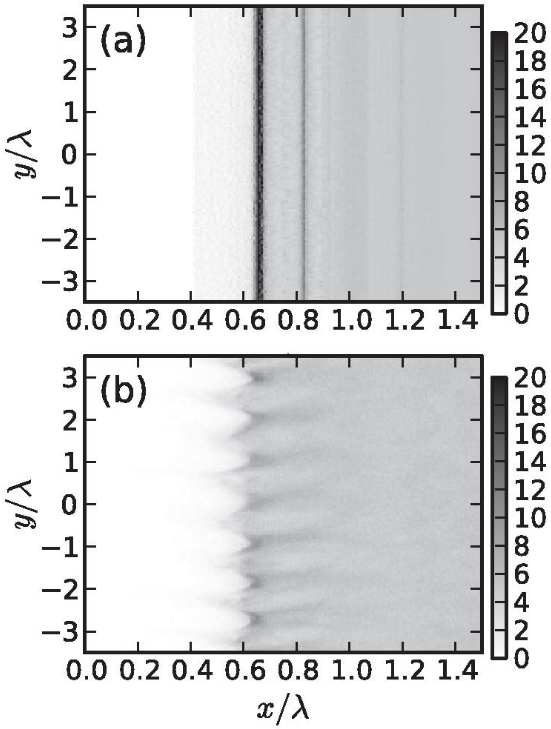
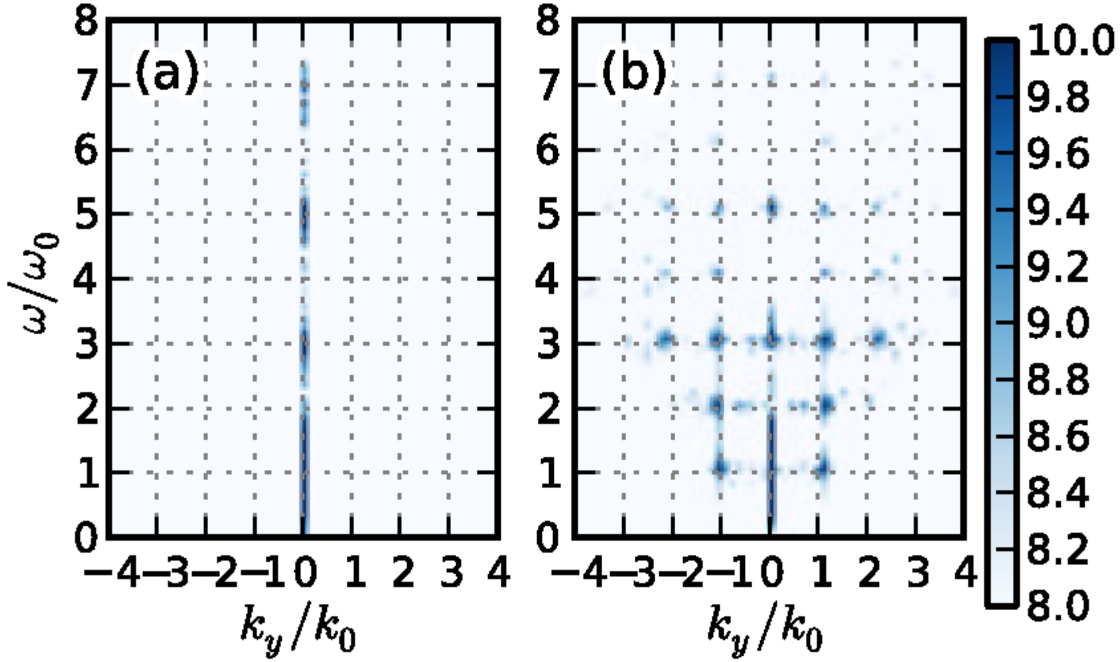
2D surface dynamics

D. an der Bruegge et al., *Phys. Rev. Lett.*, **108**, 125002 (2012)

$$ck_{\mathrm{SPW}} = \sqrt{\frac{\omega_p^2 - \omega_{\mathrm{SPW}}^2}{\omega_p^2 - 2\omega_{\mathrm{SPW}}^2}}\,\omega_{\mathrm{SPW}}$$





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Summary

- The HHG spectrum is a power.
  The exponent is
  p=8/3 for BGP spectrum
  p=6/5 for CSE spectrum

- Spectral modulations encode the femtosecond plasma surface dynamics

- 2D surface dynamics may lead to sideband emissions at selected angles