

An Ab-Initio approach to the dynamics of electrons and excitons in solids driven out-of-equilibrium by strong laser pulses

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Motivations and experimental evidences

The AiNEGF approach: solving the Dyson-Kadanoff equations in a Kohn-Sham basis

out-of-equilibrium electron-phonon scattering

out-of-equilibrium electron-electron scattering

silicon: intravalley scattering of photo-generated electrons

h-BN: photo-induced excitonic collapse

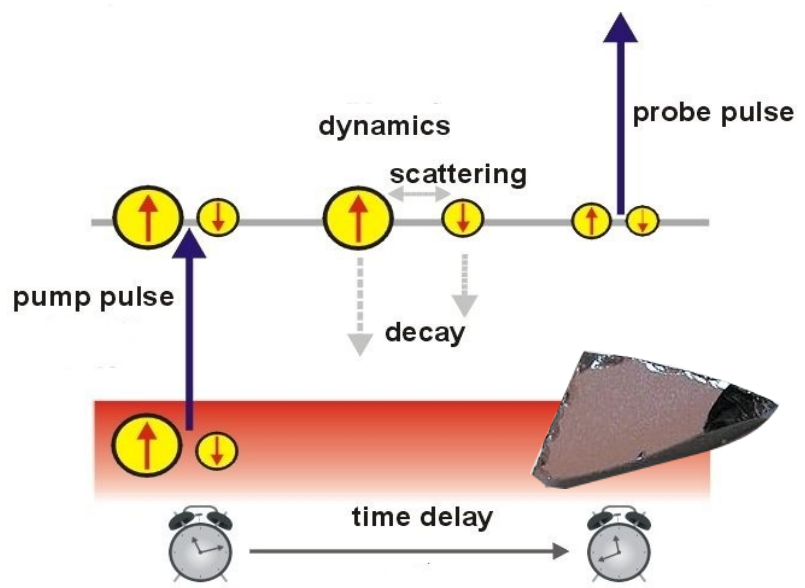
Conclusions...



*Motivations and experimental evidences*

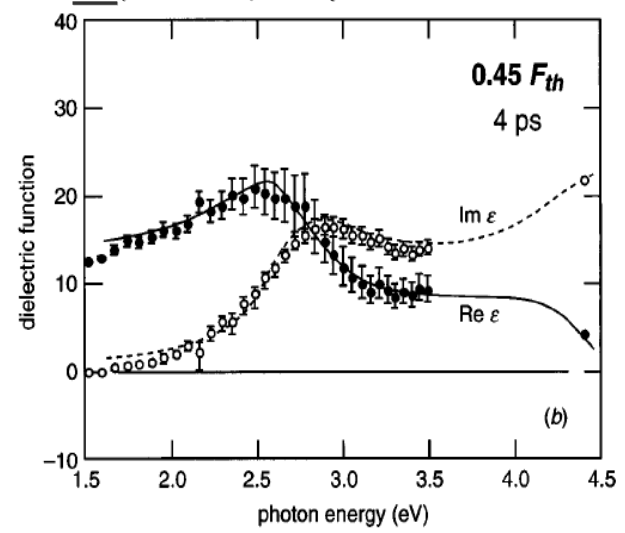
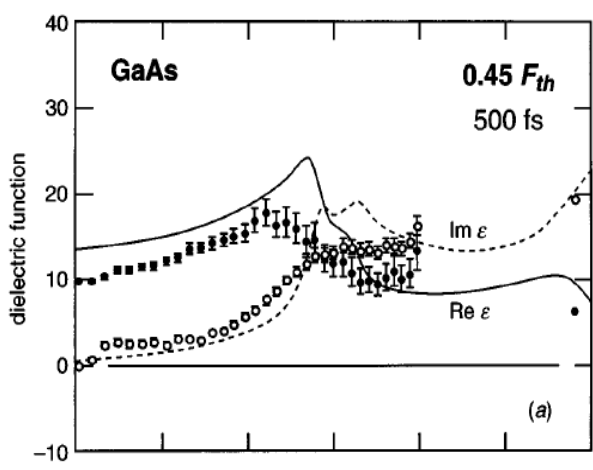


# Pump&Probe experiments

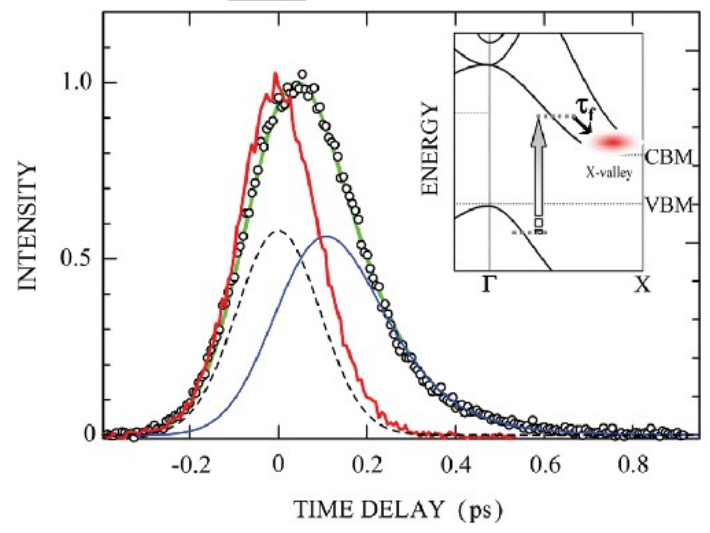


- 1 An ultra-short laser pulse pumps electrons in the conduction
- 2 The non-thermal electronic distribution relaxes via e-e and e-ph scatterings
- 3 The electronic/optical properties are probed after varying delays

L. Huang, PRL 80, 185 (1997)



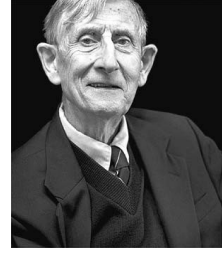
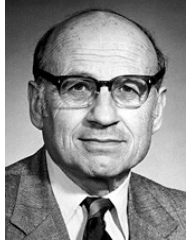
PRL 102, 087403 (2009)



The *Ab-Initio* non-equilibrium  
Green's function approach  
(*AiNEGF*)



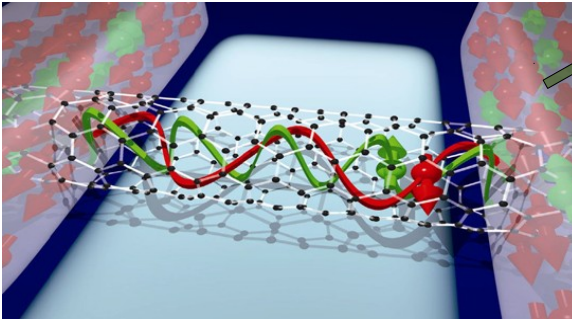
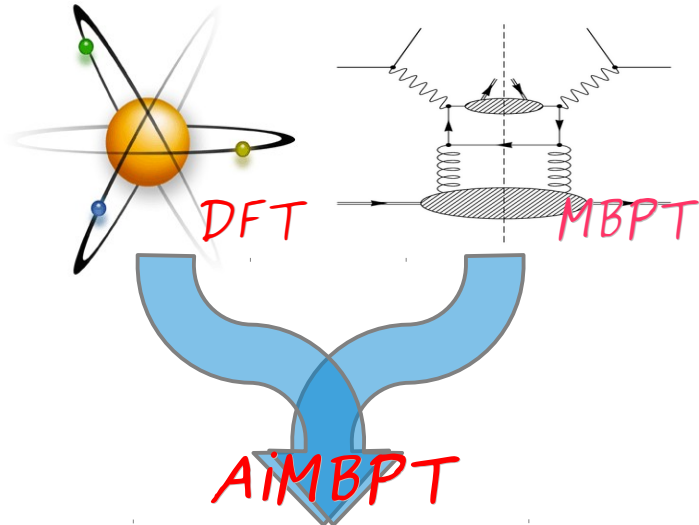
# The AiMBPT (Ab-Initio Many-Body Perturbation Theory)



*AiMBPT* is...

$$\left[ \frac{-\nabla^2}{2} + v_s(r) \right] \psi_{nk}(r) = \epsilon_{nk} \psi_{nk}(r)$$

$$v_s(r) = v_{atoms}(r) + V_{Hxc}(r)$$



$$\Sigma = \text{[Self-energy diagrams]}$$

$$G(r_1, r_2; t) \propto \langle T \{ \psi(r_1, t) \psi^+(r_2) \} \rangle$$

DFT  
MBPT

G. Onida, Rev. Mod. Phys. 2002

- ✓ Predictive
- ✓ Parameter free
- ✓ Universal
- ✓ Accurate

# The Baym-Kadanoff equation in a DFT framework (I)

Any observable is a functional of the Green's functions ( $G^<$ )



Kadanoff & Baym Equations, 1962

DFT (Ab-Initio)

Perturbations

NEGF theory

$$i \frac{\partial}{\partial t} G_{nmk}^<(t) = [H_k + U_k(t), G_k^<(t)]_{nm} + S_{nmk}^<(t)$$

Kadanoff-Baym "Statistical Mechanics" (1994)

DFT+NEGF  $\rightarrow$  AINEGF

✓ Parameter free, predictive and accurate

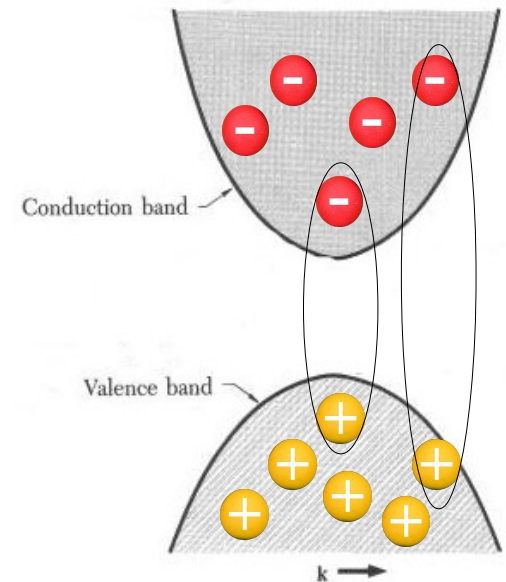
$$P(t) \propto \sum_{mnk} r_{mnk} G_{mnk}^<(t)$$

$$\chi(\omega) = \frac{P(\omega)}{E(\omega)}$$

$$N_c(t) = -i \sum_{nk \text{ empty}} G_{mnk}^<(t)$$



Marker of potential non-perturbative effects





# The Baym-Kadanoff equation in a DFT framework (II)

3

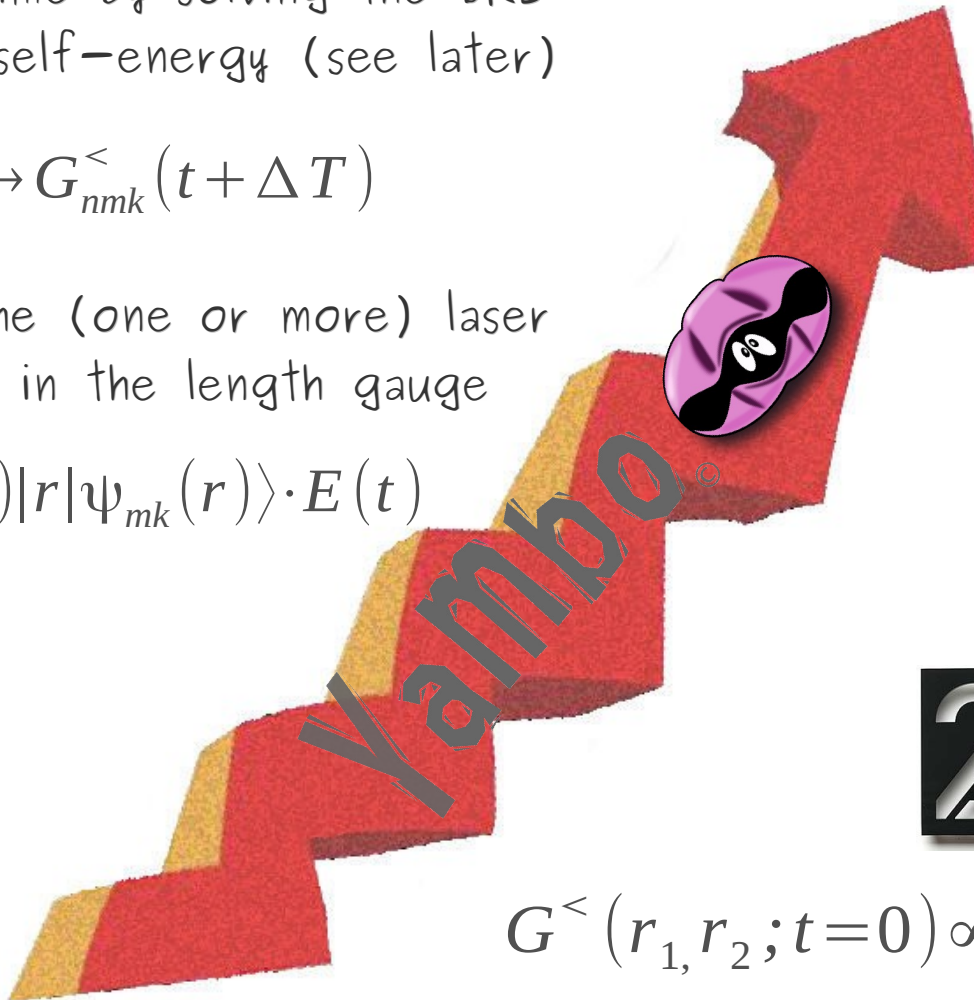
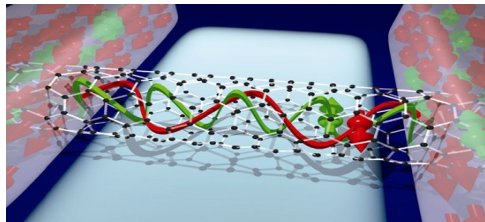
The lesser Green's function is evolved in time by solving the BKE for a given self-energy (see later)

$$G_{nmk}^<(t) \rightarrow G_{nmk}^<(t + \Delta T)$$

The interaction with the (one or more) laser fields is introduced in the length gauge

$$U_k(t) \propto \sum_{nm} \langle \psi_{nk}(r) | r | \psi_{mk}(r) \rangle \cdot E(t)$$

1



2

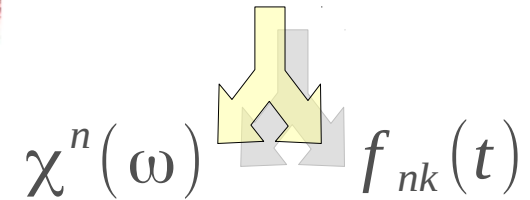
$$G^<(r_1, r_2; t=0) \propto \langle \psi(r_1, t) \psi^+(r_2) \rangle$$

Boundary conditions and lesser and retarded Green's functions are written in the KS basis

4

Post-Processing

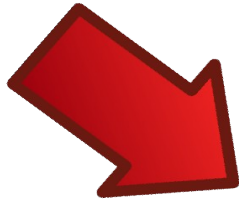
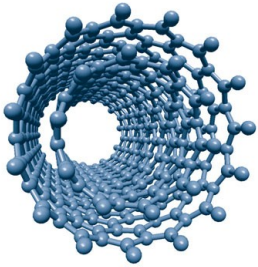
$$G_{nmk}^<(t)$$





# The Baym-Kadanoff equation in a DFT framework (III)

$$\Sigma = \text{self-energy diagram 1} + \text{self-energy diagram 2}$$



Microscopic details of the system fully and consistently taken into account

$$W(r, r'; t, t')$$
$$\psi(r, t)$$



NO ad-hoc parameters

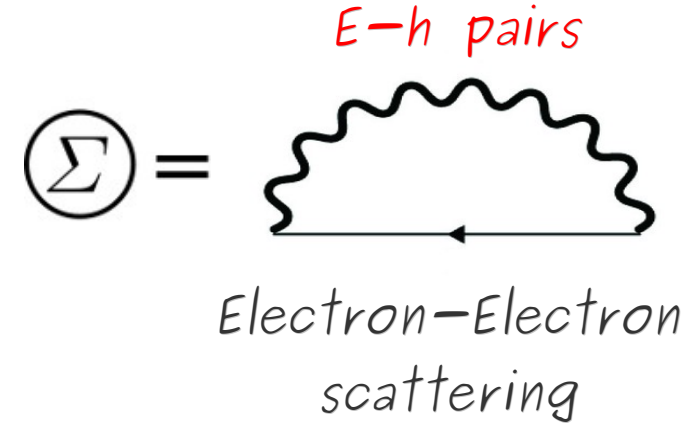
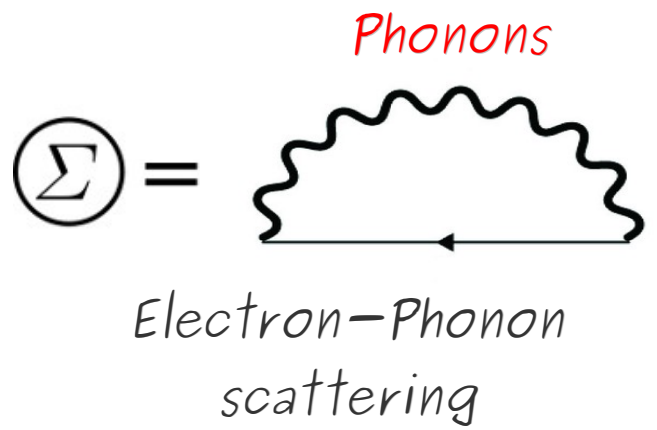
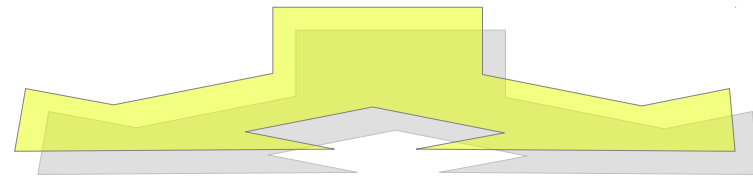


NO use of few-bands models, parabolic (effective mass) dispersion...

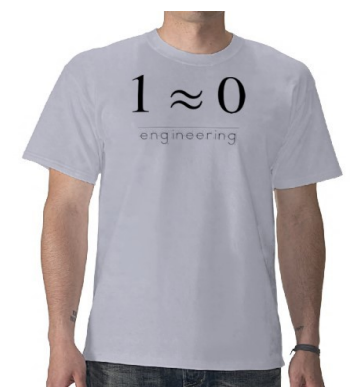
# The out-of-equilibrium kernel

$$i \frac{\partial}{\partial t} G_{nmk}^<(t) = [H_k + U_k(t), G_k^<(t)]_{nm} + S_{nmk}^<(t)$$

$$S(t) = \int_{-\infty}^t d\tau [\Sigma^>(t, \tau) G^<(t, \tau) + G^<(t, \tau) \Sigma^>(t, \tau) - \Sigma^<(t, \tau) G^>(t, \tau) - G^>(t, \tau) \Sigma^<(t, \tau)]$$



+ Massive approximations  
 + number-crunching techniques



The NEGF kernel:  
Electron-phonon and  
electron-electron scatterings  
from an Ab-Initio perspective





# The electron-phonon out-of-equilibrium kernel (I)

$$i \frac{\partial}{\partial t} G^<(t, t) = [h^{DFT} + \Sigma_s(t), G^<(t, t)] + S(t)$$

$$S(t) = \int_{-\infty}^t d\tau \left[ \Sigma^>(t, \tau) G^<(t, \tau) + G^<(t, \tau) \Sigma^>(t, \tau) - \Sigma^<(t, \tau) G^>(t, \tau) - G^>(t, \tau) \Sigma^<(t, \tau) \right]$$

Kadanoff-Baym "Statistical Mechanics" (1994)

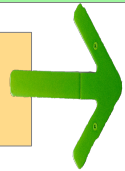
$$\Sigma_{nn'k}^<(t, \tau) = \frac{i}{N} \sum_{mm'q} \overline{g_{nmk}^{q\lambda}} g_{n'm'k}^{q\lambda} D_{q\lambda}^<(t, \tau) G_{mm'k-q}^<(t, \tau) \quad +$$

BKE



$$G^<(t, \tau) \approx i \left[ G^r(t-\tau) G^<(\tau) - G^<(t) G^r(t-\tau) \right] \quad +$$

Quasi-equilibrium & Complete Collisions Approx



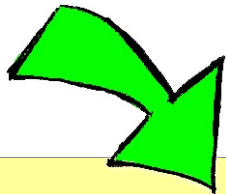
$$G_{nmk}^<(T) \approx \delta_{nm} f_{nk}(T) \quad +$$

$G^r$  is polaronic  
(no short-time correction)



$$G_{nk}^r(T) = -i e^{-i\varepsilon_{nk}(\beta)t - \Gamma_{nk}(\beta)t} \theta(t) \quad =$$

HOLE  
Lifetime



ELECTRON  
Lifetime

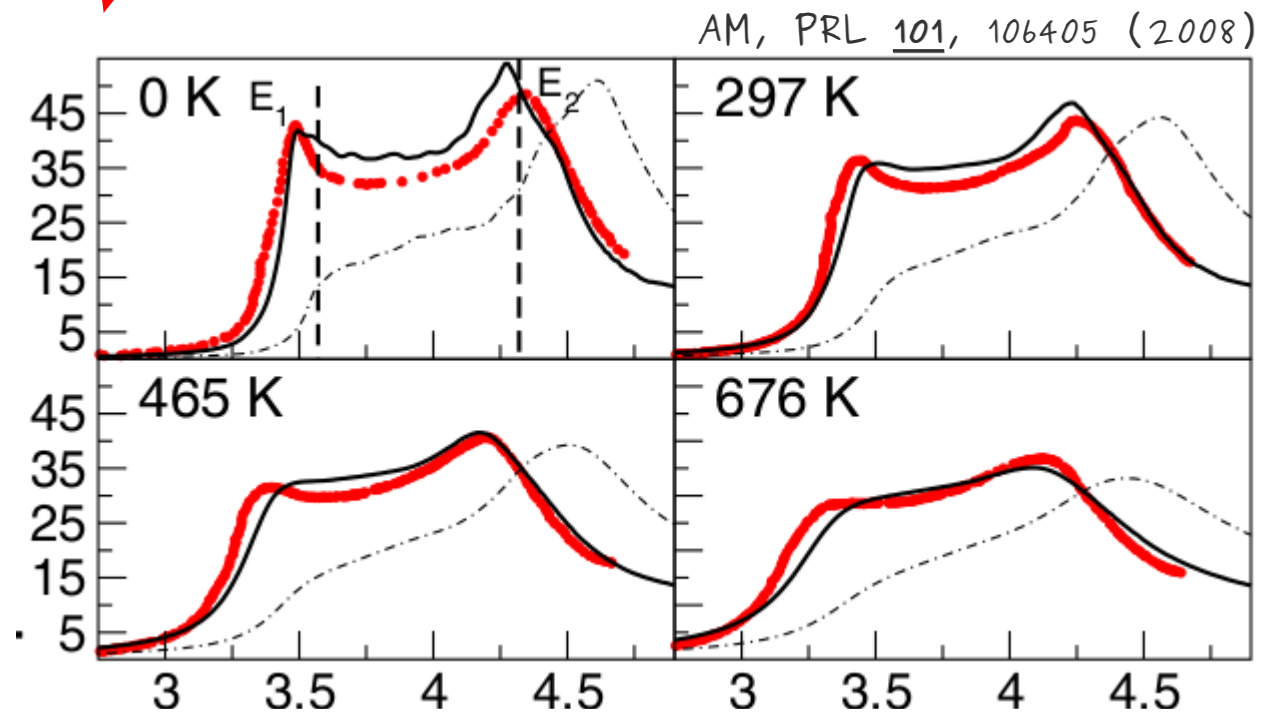
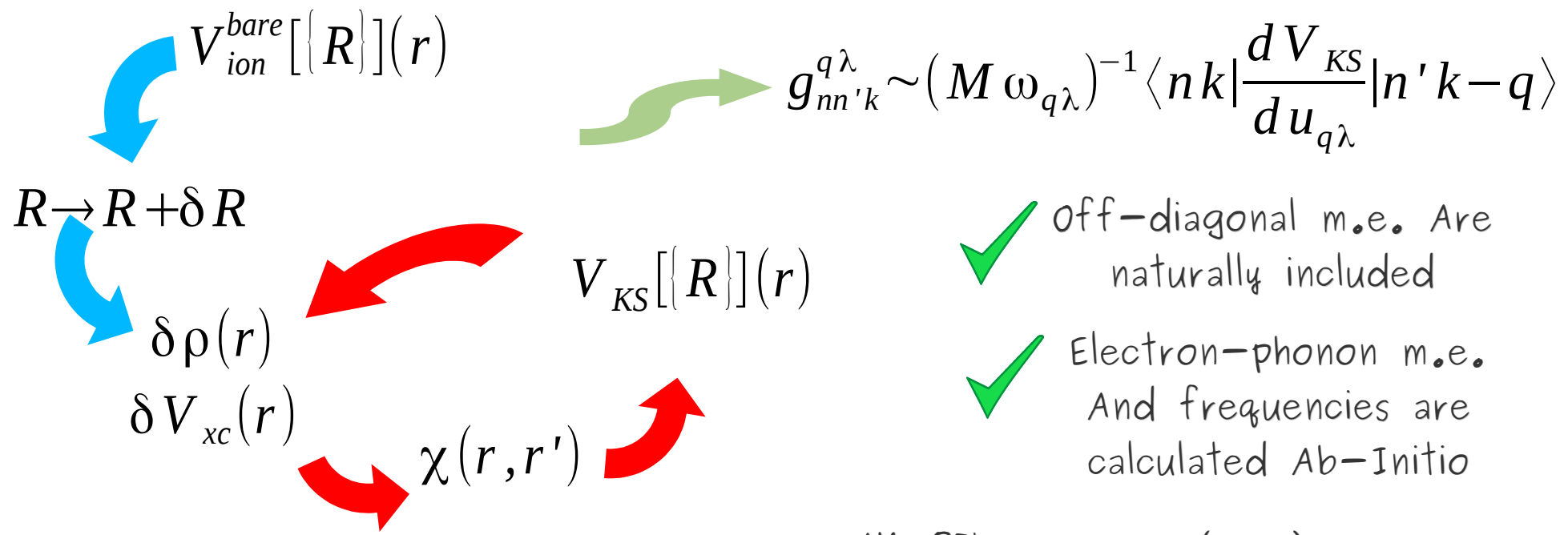


THERMAL  
Lifetime

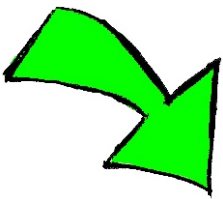



$$\partial_t f_{nk}(T) = \gamma_{nk}^{(h, e-p)}(T) (1 - f_{nk}(T)) - \gamma_{nk}^{(e, e-p)}(T) f_{nk}(T) - \gamma_{nk}^{(th)}(\beta, T)$$

# Density Functional Perturbation Theory [S. Baroni, Rev. Mod. Phys. 73, 515 (2001)]



# The electron-phonon out-of-equilibrium kernel (II)

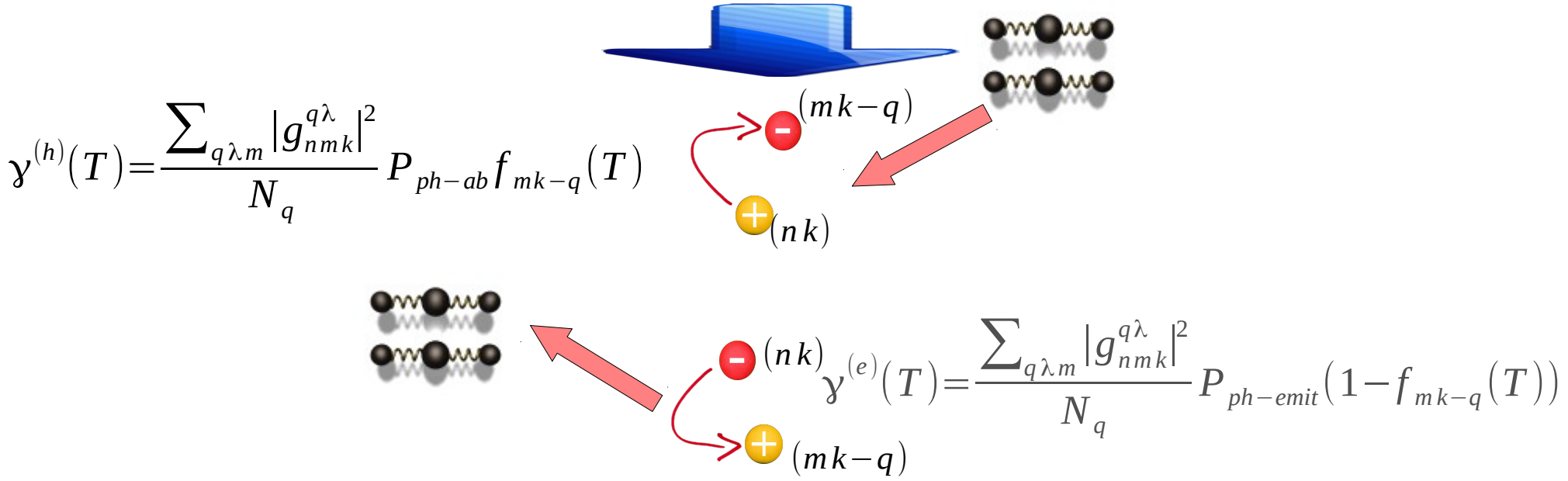
HOLE Lifetime 

ELECTRON Lifetime 

THERMAL Lifetime 

$$\partial_t f_{nk}(T) = \gamma_{nk}^{(h,e-p)}(T)(1-f_{nk}(T)) - \gamma_{nk}^{(e,e-p)}(T)f_{nk}(T) - \gamma_{nk}^{(th)}(\beta, T)$$

$$P_{ph-emit} \equiv \frac{\Gamma_{nk} + \Gamma_{mk-q}}{(\Gamma_{nk} + \Gamma_{mk-q})^2 + (\epsilon_{mk-q} - \epsilon_{nk} + \omega_{q\lambda})^2} \quad P_{ph-ab} \equiv \frac{\Gamma_{nk} + \Gamma_{mk-q}}{(\Gamma_{nk} + \Gamma_{mk-q})^2 + (\epsilon_{mk-q} - \epsilon_{nk} - \omega_{q\lambda})^2}$$



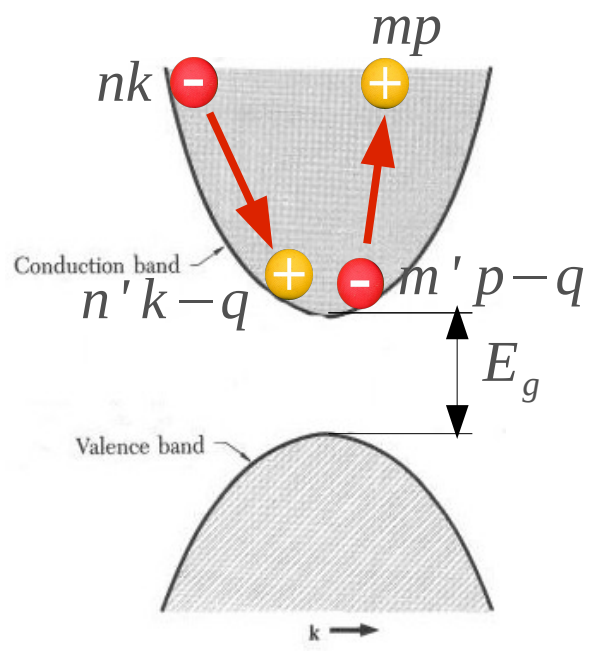
The thermal lifetime includes both kinds of scatterings weighted by the phonon occupation. It is vanishing at zero temperature



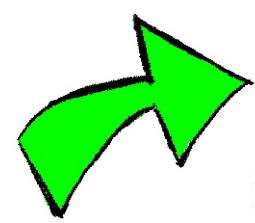
# Electron-Electron scattering in the GW approximation

$$\partial_t f_{nk}(T) = (\gamma_{nk}^{(h,e-p)}(T) - \gamma_{nk}^{(e,e-e)}(T)(1-f_{nk}(T)) - \gamma_{nk}^{(e,e-p)} + \gamma_{nk}^{(e,e-e)}(T)f_{nk}(T) - \gamma_{nk}^{(th)}(\beta, T))$$

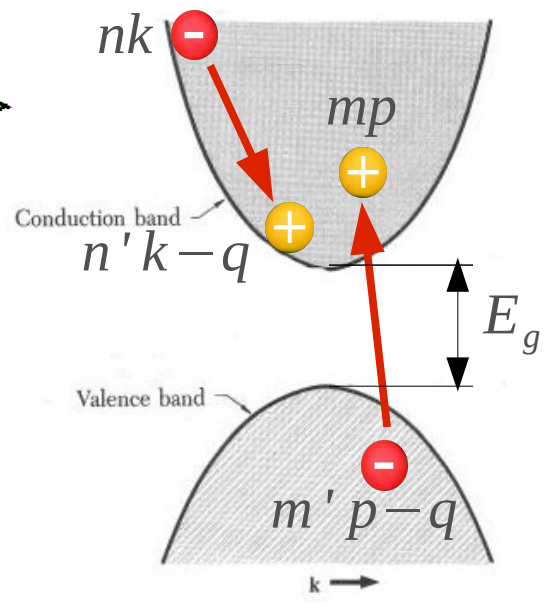
$$\gamma_{nk}^{(e,e-e)}(T) \propto \sum_{qp} \sum_{n'mm'} |W_{nn'k,mm'p}^q|^2 \left[ \frac{4\Gamma}{(\epsilon_{n'k-q} + \epsilon_{mp} - \epsilon_{m'p-q} - \epsilon_{nk})^2 + 16\Gamma^2} \right] (1-f_{n'k-q}(T))(1-f_{mp}(T))f_{m'p-q}(T)$$



Intraband scattering. It takes contributions ONLY from the photo-excited electrons. Its strength goes with the carrier density.



In the e-p case this process is ALWAYS non zero as the Debye energy is much smaller than the gap



Dominant process in the low carriers density. It takes contributions also from the unperturbed electrons. But it is zero whenever

$$\epsilon_{nk} \leq 2E_g$$

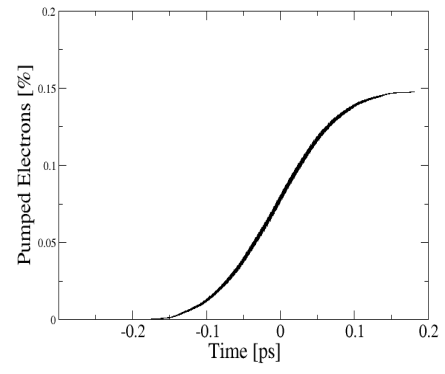
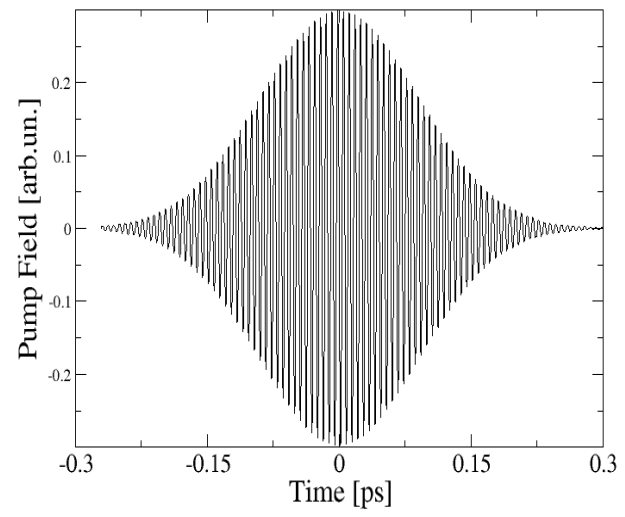
*Ultrafast Carrier Relaxation in Si:  
Intravalley Scattering and Energy  
Relaxation of photoexcited  
Electrons*



Phonon induced electronic decay in Bulk Silicon [A. Marini, in preparation]

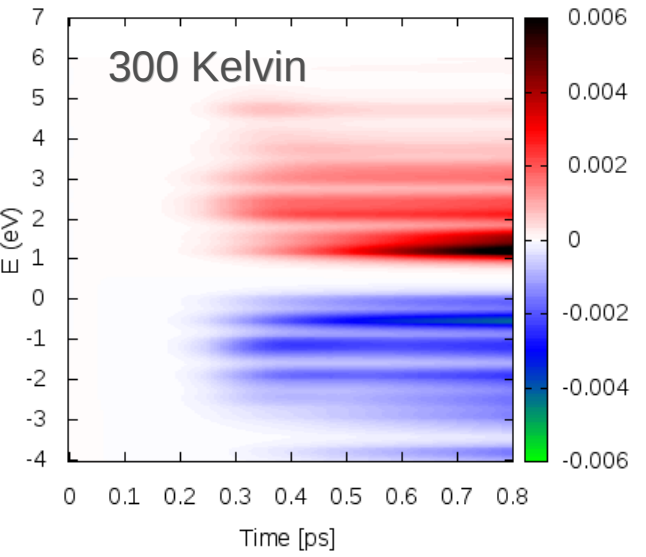
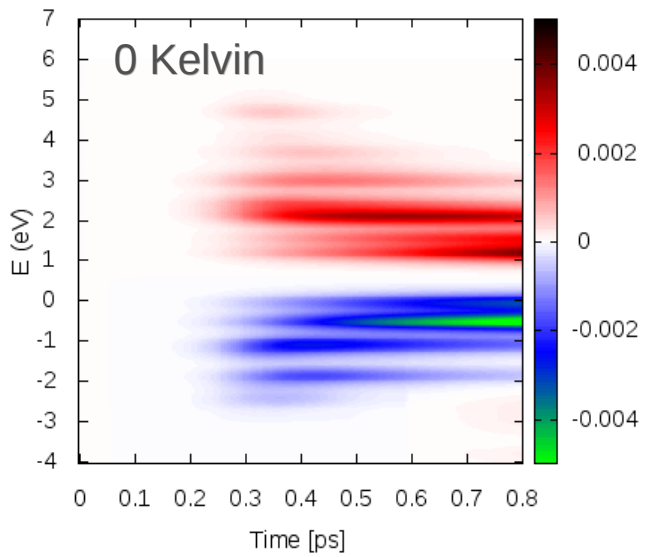
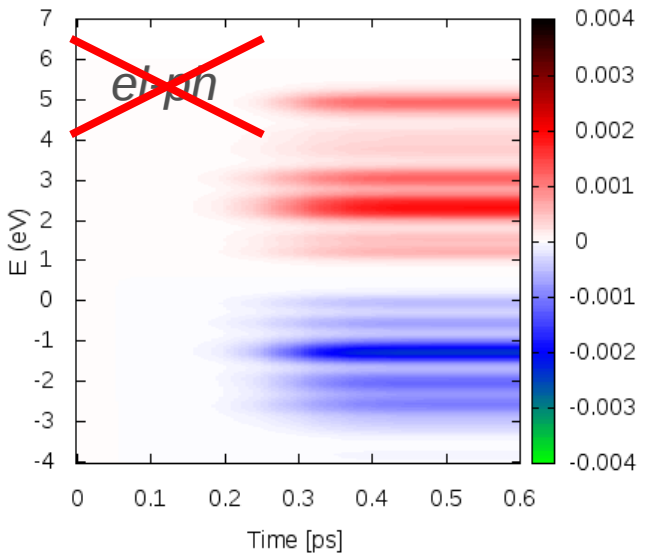
$$\partial_t f_{nk}(T) = \gamma^{(h)}(1 - f_{nk}(T)) - \gamma^{(e)}(T)f_{nk}(T) - \gamma^{(th)}(\beta, T)$$

Pump field is "quasi harmonic"  
with frequency 2.03 eV and fluence  $0.004 \frac{J}{cm^2}$



← Pumped electrons

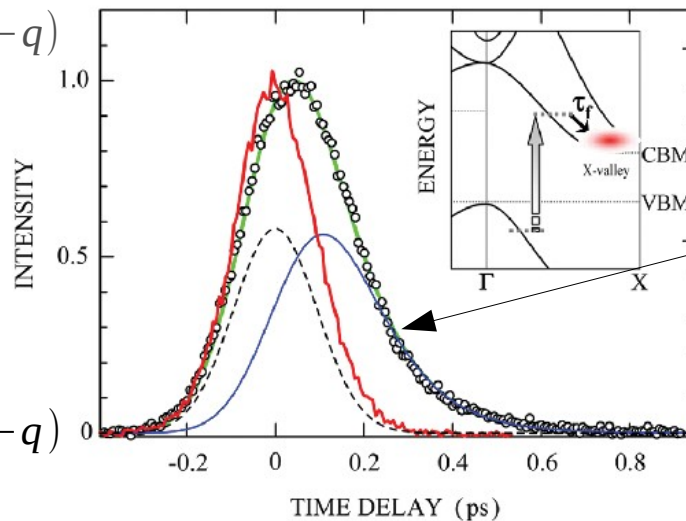
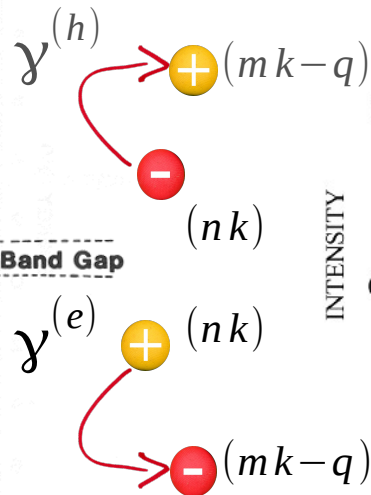
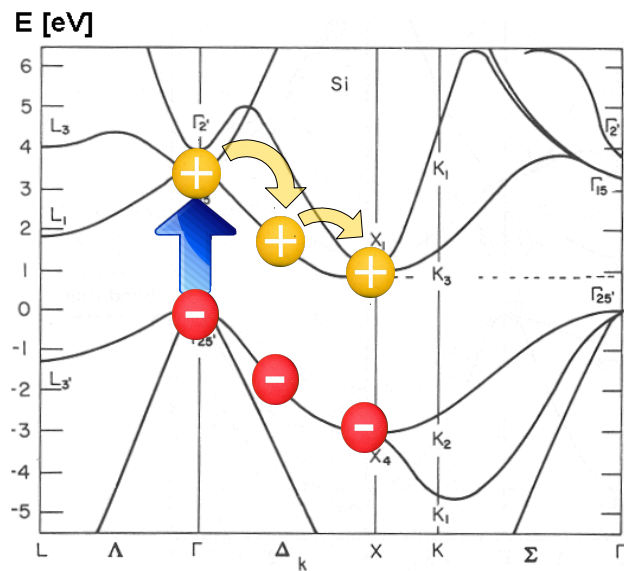
Time-dependent Density-of-states





# Intra-valley scattering in Bulk silicon

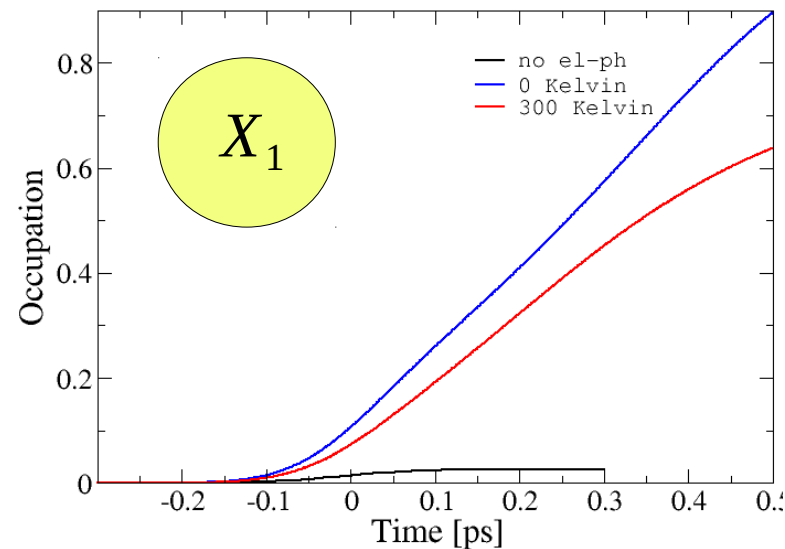
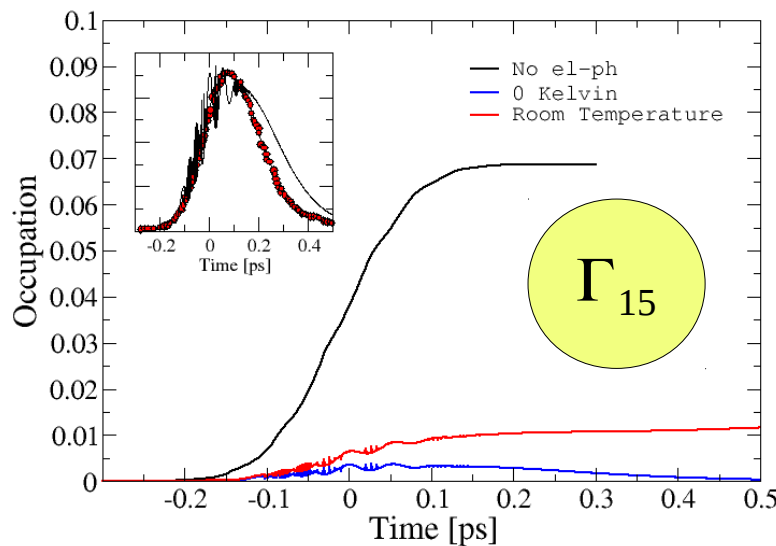
$$\partial_t f_{nk}(T) = \gamma^{(h)}(1 - f_{nk}(T)) - \gamma^{(e)}(T)f_{nk}(T) - \gamma^{(th)}(\beta, T)$$



$$\tau_f^{\text{MBPT}} = 18 \text{ fs}$$



Relaxation is a **CASCADE** process.  
Standard MBPT does not apply!



h-BN: photo-induced  
excitonic collapse



# The Method

The TD-BSE

$$f_{nk}(t) = -iG_{nk}^<(t)$$

L.X. Benedict PRB 63, 075202 (2001)

F.X. Gamscasse PRL 77, 5429 (1996)

Standard BSE  
(at each time)

$$H_{IJ}^{BSE}(t) = E_I \delta_{IJ} + f_I^{eh}(t) (2V_{IJ} - W_{IJ}(t))$$

Time-resolved  
Probe Absorption

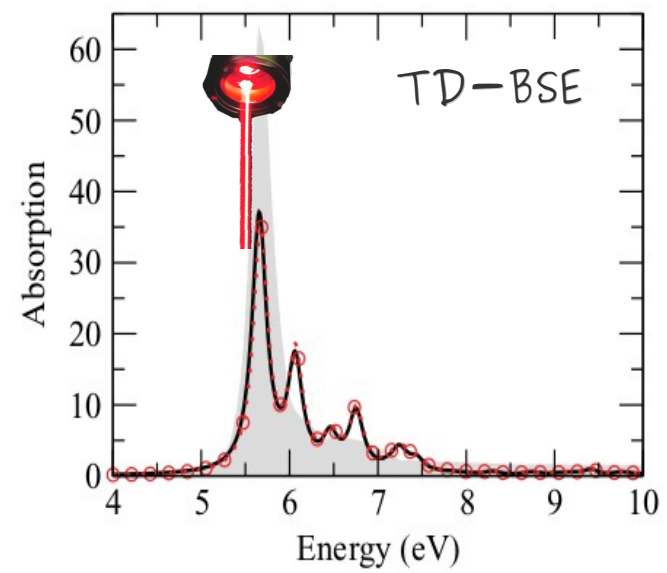
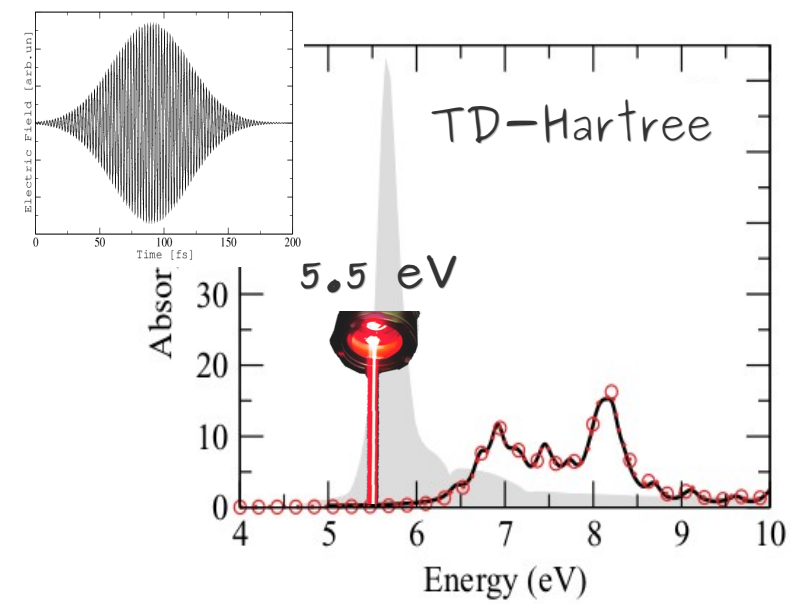
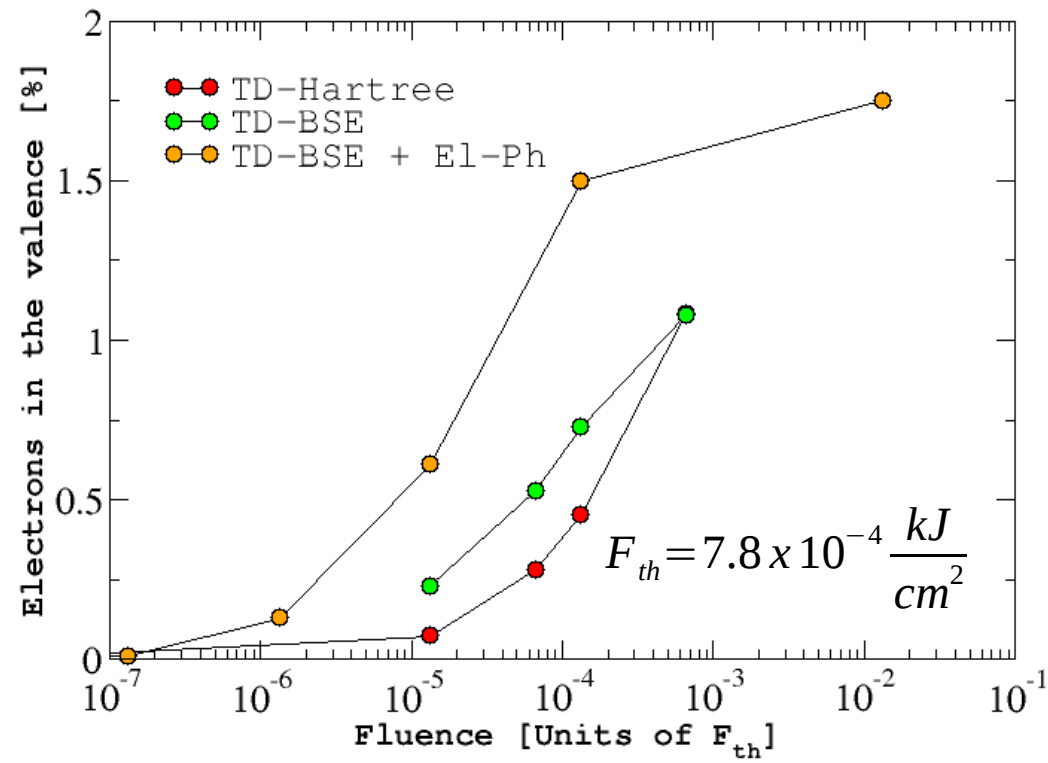
K. F. Berggren  
PRB 24, 1971 (1981)

Standard GW  
(at each time)

$$\Sigma(t) = iG(t)W(t)$$

Band-gap  
renormalization

# Exciton collapse by Pauli blocking + intra-band screening in h-BN (I)

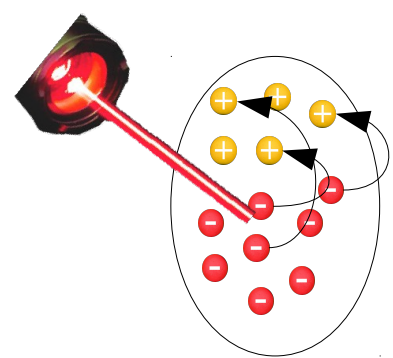
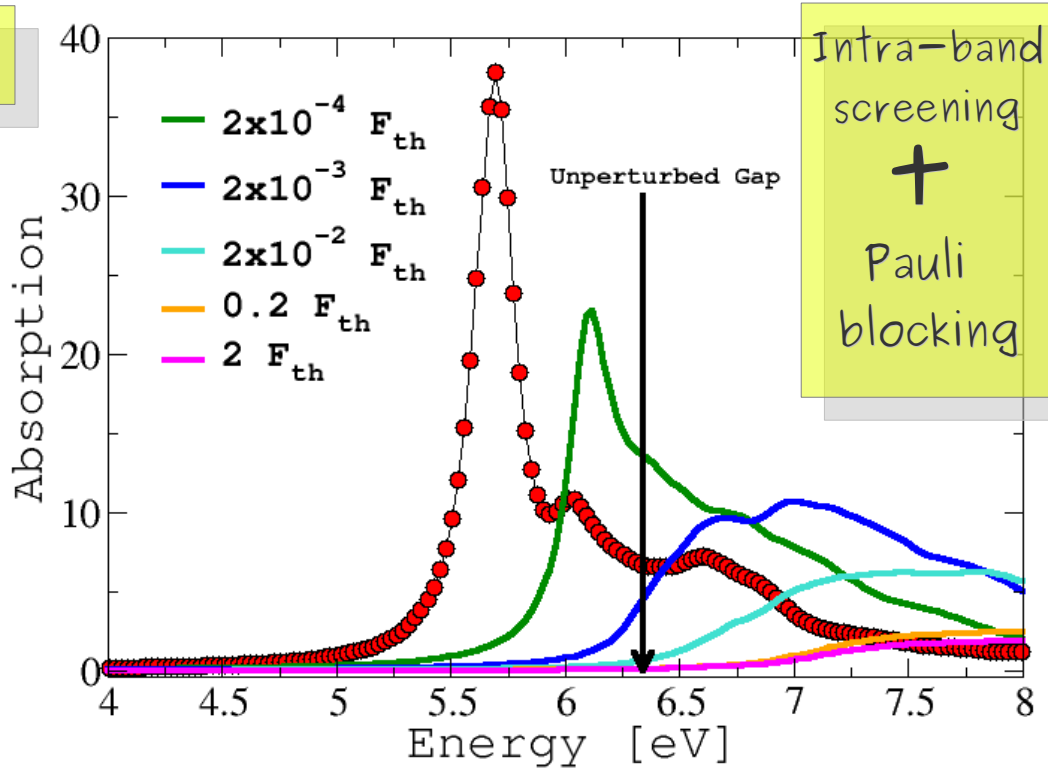
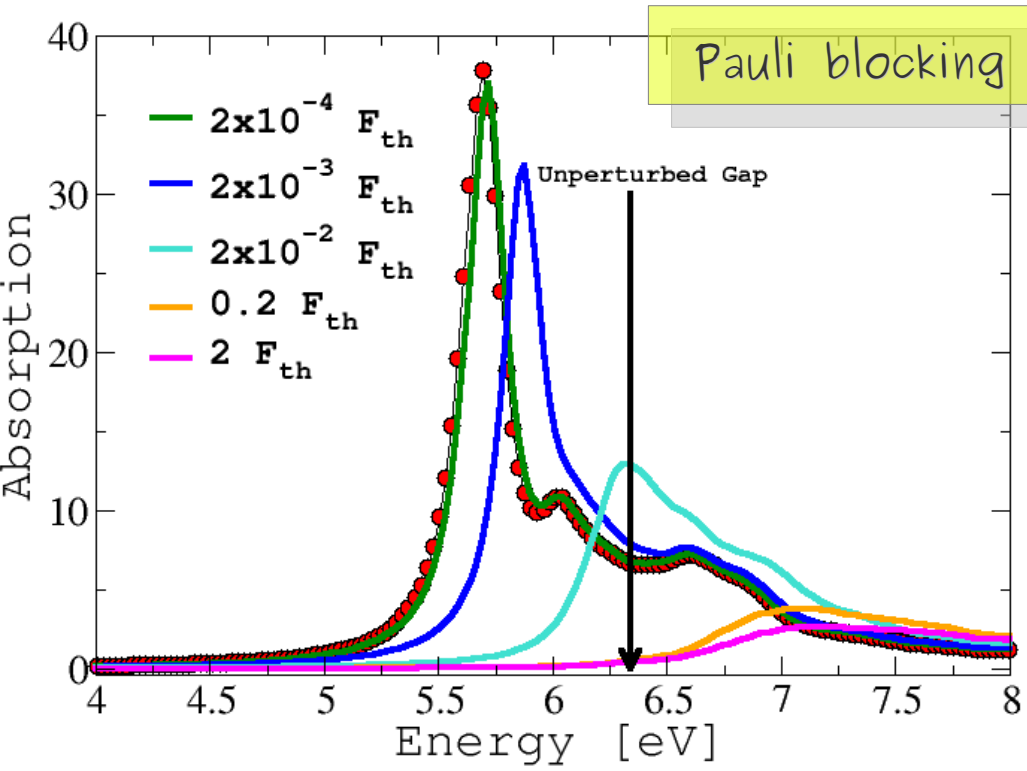


**!** Crucial use of ab-initio polaronic dephasing for a correct pumping dynamics

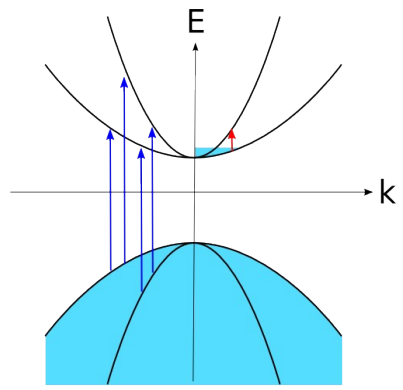
**✓** The TD-BSE enhances the carriers excitation when the pulse is resonant with the exciton

**✓** The El-Ph interaction empties on-the-fly the pumped states

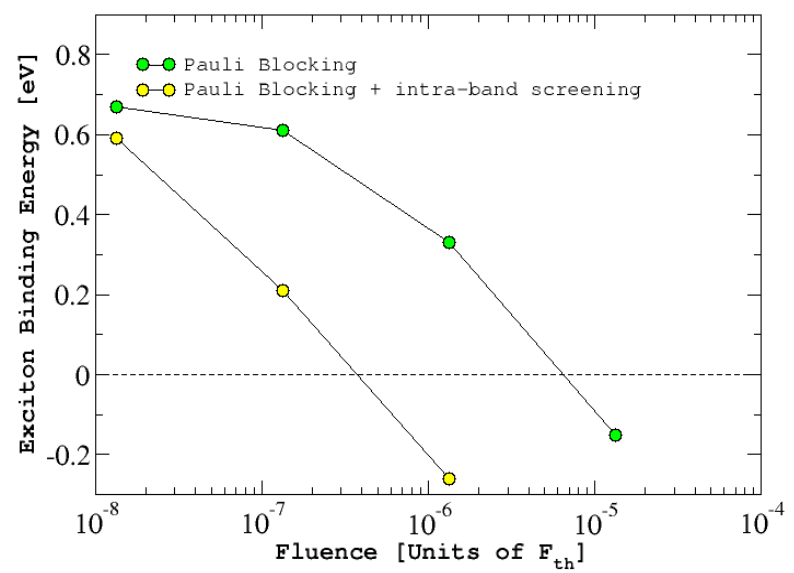
# Exciton collapse by Pauli blocking + intra-band screening in h-BN (II)



Pauli blocking



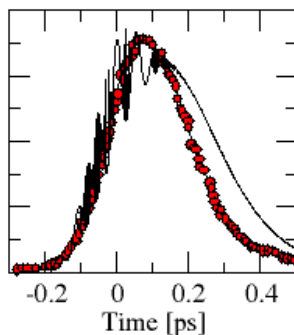
Intra-band screening



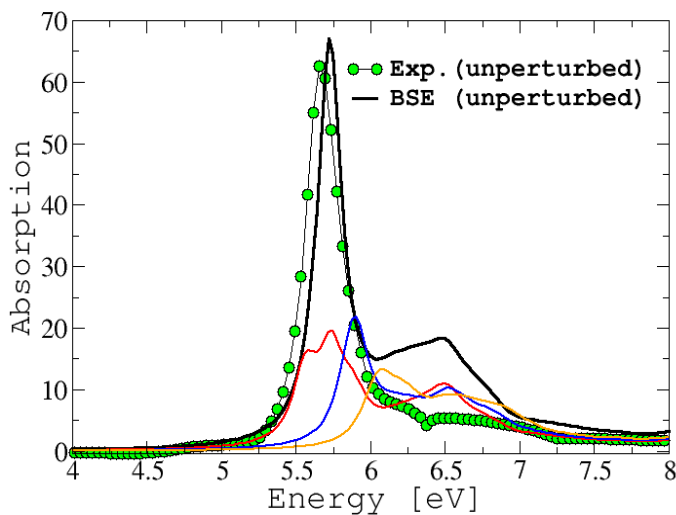


# Conclusions...

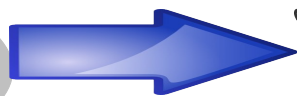
Time-dependent Density-of-states in excellent agreement with experimental results



The modifications induced by the pump in the probe absorption can be moderate (GaAs) or even so strong to collapse the excitonic state (hBN)

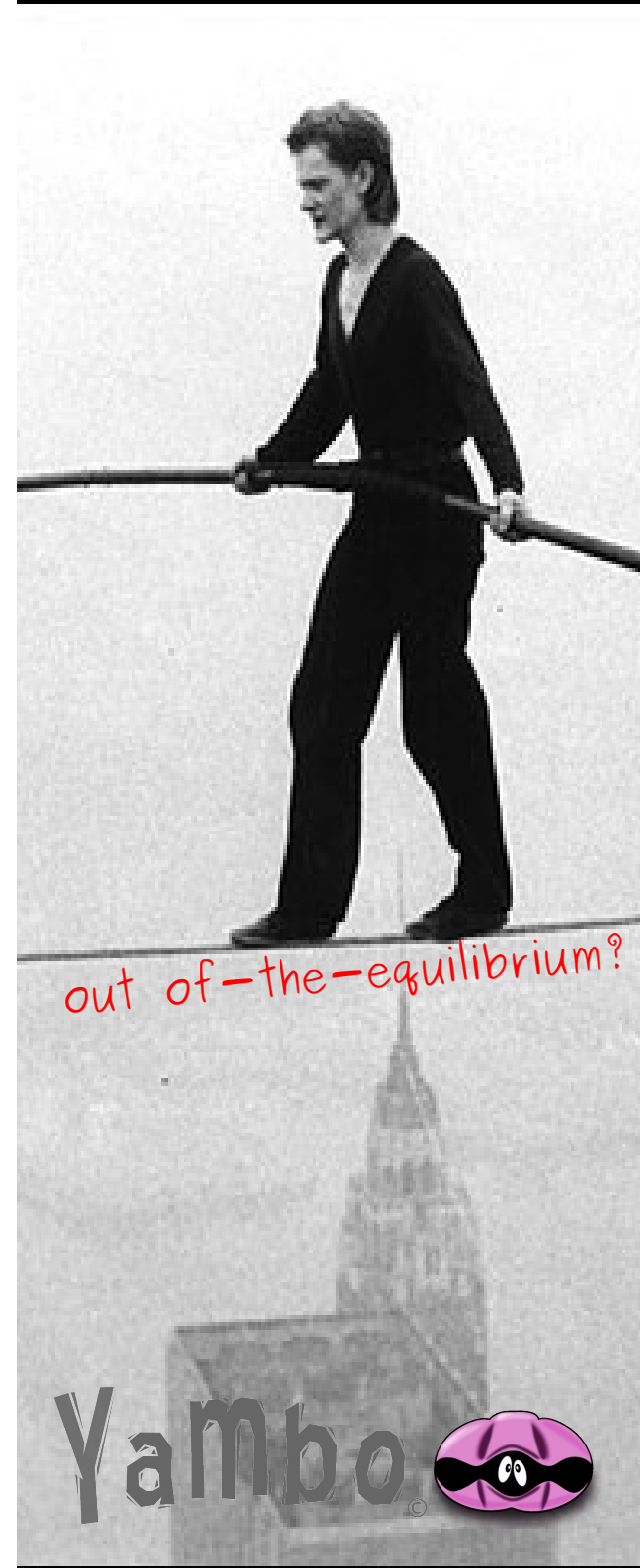


**AINEGF**



Several potential applications...

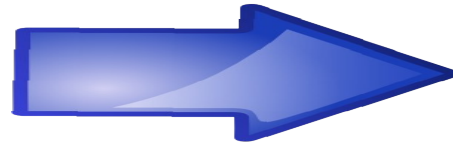
Yambo: an ab initio tool for excited state calculations, A. Marini, C. Hogan, M. Grüning, D. Varsano, Comp. Phys. Comm. 180, 1392 (2009).



Yambo

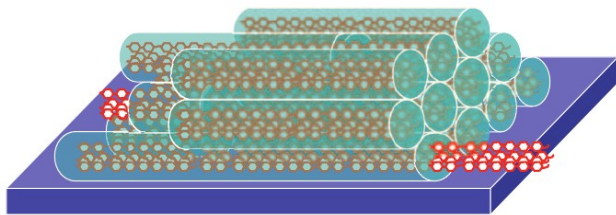
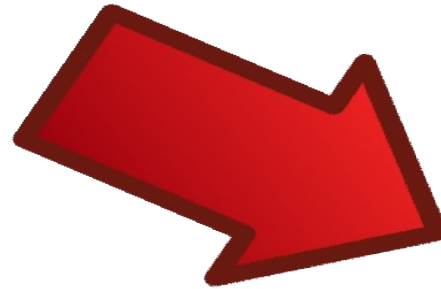
# Potential AiNEGF applications...

# AiNEGF

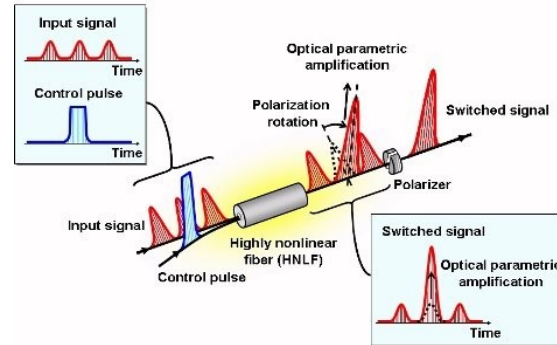


FLASH II.

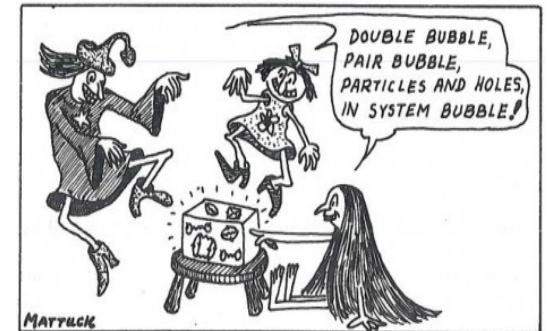
Saturation Phenomena  
X-ray induced transparency in Al  
(Nagler. Nature, 2009)



High-gain materials  
Amplified light emission in  
aligned polymers (I.B.  
Martini. Nature, 2007)

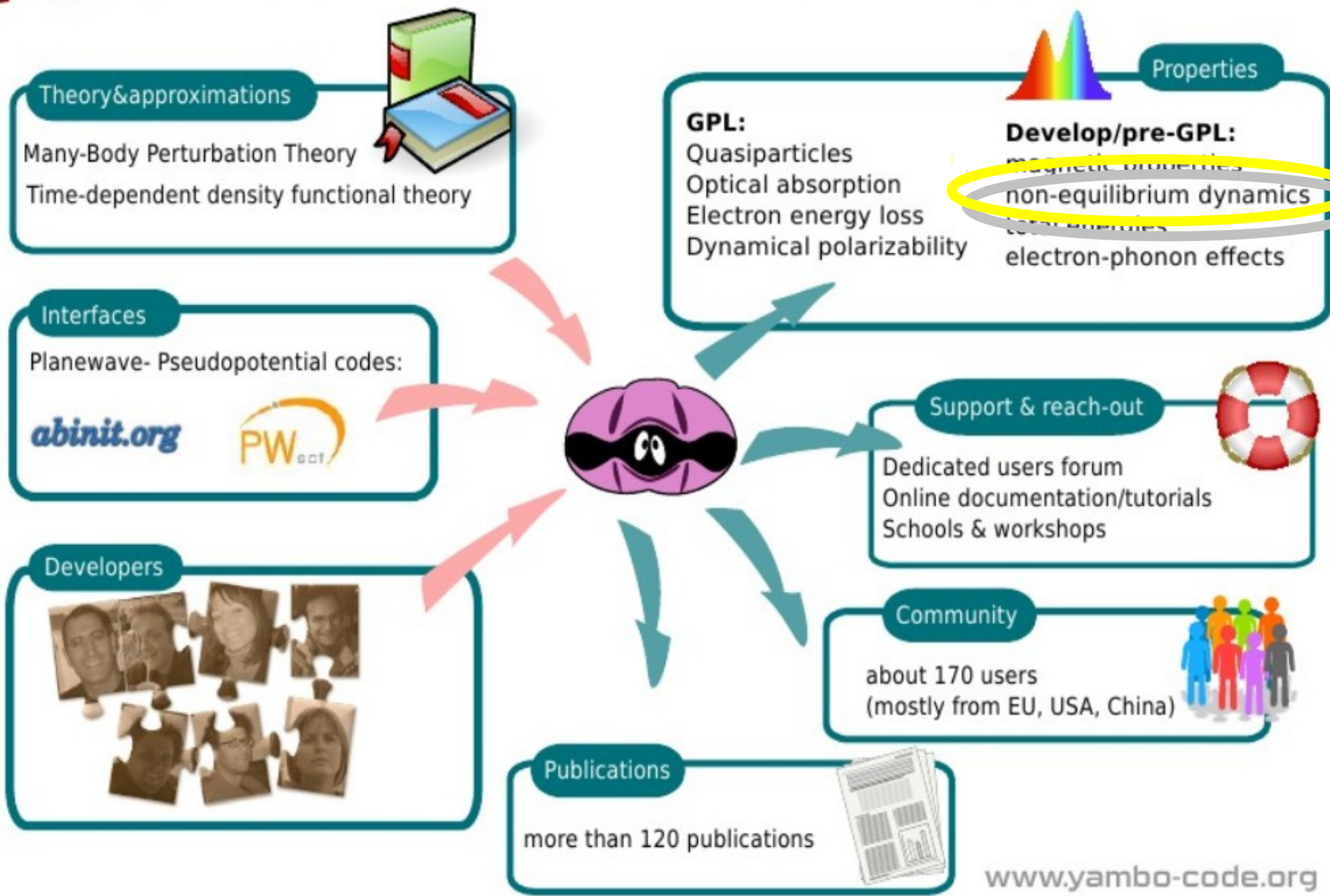


Nano optical devices  
Single-molecule (Hwang. Nature, 2009)  
and Carbon Nanotube (Tans, Nature,  
1998) optical transistors



Severe  
testing-ground for  
Many-Body theories

# Yambo: an ab-initio tool for excited state calculations





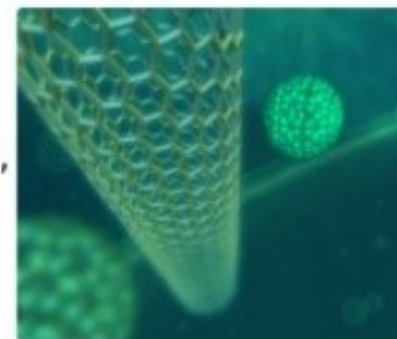
# Yambo: an ab-initio tool for R&D

## Material Science



applications to  
e.g. photovoltaics,  
lithium batteries,  
microelectronics

## Nano Science



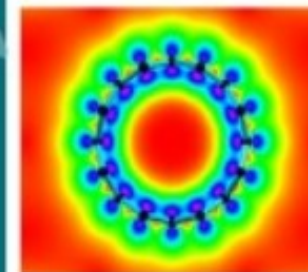
applications to  
e.g. nanophotonics,  
nanoelectronics

## Biology



studies of  
photoactive  
molecules  
and molecular  
complexes

## Physics




fundamental  
understanding  
of physical  
processes




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WIKIPEDIA  
The Free Encyclopedia  
Yambo@Wiki



The Fortran Cafe



The Bethe-Salpeter-Equation (BSE) wine



Yambo road & bar

### GW quasiparticle calculations in condensed matter physics and nanoscience

Location : CECAM-HQ-EPFL, Lausanne, Switzerland  
April 16, 2012 - April 20, 2012

- Paolo Umari  
University of Padova, Italy
- Andrea Marini  
National Research Council, Italy
- Angel Rubio  
University of the Basque Country, San Sebastian, Spain
- Feliciano Giustino  
University of Oxford, United Kingdom

**Lectures**

**Intro to the GW method**

Feliciano Giustino  
Department of Materials, University of Oxford

$$\Sigma(\omega) = i \int G_0(\omega - \omega') W(\omega')$$


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**Beyond one-shot  $G_0W_0$ : from partial to full self-consistency**


Fabio Caruso  
Potsdam Institute, Berlin

GW quasiparticle calculations in condensed matter physics and nanoscience

19th April 2012, Lausanne

**The linear response regime**

http://www.yambo-code.org/events





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GW without using unoccupied states

Paolo Umari  
University of Padua, Italy

**GW & ARPES**

Feliciano Giustino  
Department of Materials, University of Oxford




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GW calculations: general implementations and common approximations

Daniele Varsano  
Department of Physics  
University of Rome "La Sapienza"

