Pair Production in Strong Fields: The Wigner function approach

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FILMITh 2012, Garching, 21. september 2012.

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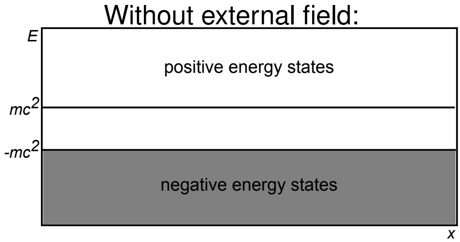
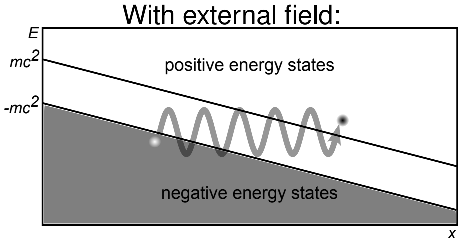
Phenomenology of pair production from vacuum:





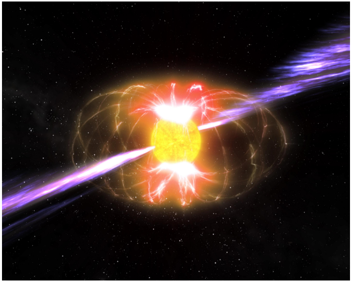
Relevant scales:

- Field strength: $\mathcal{E}_c = \frac{m^2 c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$
- Time: $t_c = \frac{\hbar}{mc^2} \approx 1 \cdot 10^{-21}\mathrm{s}$
- Frequency: $\omega_c = \frac{e\mathcal{E}}{mc} \approx 8 \cdot 10^{20}\mathrm{Hz}$ ($\mathcal{E} = \mathcal{E}_c$)
- Spatial gradient: $\partial_r = \frac{mc}{\hbar} \approx 6.6 \cdot 10^{10}\mathrm{m}^{-1}$

Motivation

Pair Production in nature:

QED pair production near massive astrophysical objects (black holes, magnetars, possible source of gamma ray bursts?)



John Rowe Animations

Pair Production in theory and experiment:

- QED pair production from vacuum was predicted half a century ago, but was not yet observed.
- Rapid development of laser technology may enable the observation in the near future.
- Pair production in strong QCD fields (Heavy Ion collisions).

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History

Some results from the early history of pair production description:

- Homogeneous static electric field. (J. Schwinger)
- Some special analytic time dependent, homogeneous electric fields. (V. S. Popov, M. S. Marinov, N. B. Narozhnyi, A. I. Nikishov, ...)

Recently a different approach is gaining attention: kinetic formulation

- Transport equation for QCD Wigner operator, later for Abelian plasmas. (D. Vasak, M. Gyulassy, H.-T. Elze)
- Equal time formulation of QED transport equations for the Wigner function (named the Dirac-Heisenberg-Wigner, DHW equations)

  (I. Bialynicki-Birula, P. Górnicki, J. Rafelski)
- Study of inhomogeneous and time dependent QED particle production. (F. Hebenstreit, R. Alkofer, H.Gies)

Dirac-Heisenberg-Wigner formalism

What is the Wigner function?

- Quantum analogue of the classical phase space distribution.

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{C}(\vec{x},\vec{s},t) = e^{-ie\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \quad (1)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle 0|\hat{C}(\vec{x},\vec{s},t)|0\rangle \mathrm{d}^3 s \quad (2)$$

Dirac-Heisenberg-Wigner formalism

- The time derivative of the Wigner function gives us the evolution of the system:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \qquad (3)$$

- Theoretical approximation: external field is classical (quantum fluctuations are neglected).

Dirac-Heisenberg-Wigner formalism

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \quad (4)$$

The equation has the following non-local differential operators:

$$D_t = \partial_t + e\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{x}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{p}} + ... \quad (5)$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + e\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{x}} - \frac{e\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + ... \quad (6)$$

$$\vec{P} = \vec{p} + \frac{e\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + ... \quad (7)$$

Solution is expanded on irreducible 4x4 matrices:

$$W(x,p,t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \quad (8)$$

We arrive to a system for 16 unknown real functions, the DHW functions:

$$
\begin{aligned}
&D_t\mathbb{s} & & & - \quad & 2\vec{P}\cdot\vec{\mathbb{t}}_1 &= 0 \quad (9)\\
&D_t\mathbb{p} & & & + \quad & 2\vec{P}\cdot\vec{\mathbb{t}}_2 &= 2m\mathbb{a}_0 \quad (10)\\
&D_t\mathbb{v}_0 &+ \quad & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & &= 0 \quad (11)\\
&D_t\mathbb{a}_0 &+ \quad & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & &= 2m\mathbb{p} \quad (12)\\
&D_t\vec{\mathbb{v}} &+ \quad & \vec{D}_{\vec{x}}\mathbb{v}_0 &+ \quad & 2\vec{P}\times\vec{\mathbb{a}} &= -2m\vec{\mathbb{t}}_1 \quad (13)\\
&D_t\vec{\mathbb{a}} &+ \quad & \vec{D}_{\vec{x}}\mathbb{a}_0 &+ \quad & 2\vec{P}\times\vec{\mathbb{v}} &= 0 \quad (14)\\
&D_t\vec{\mathbb{t}}_1 &+ \quad & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 &+ \quad & 2\vec{P}\mathbb{s} &= 2m\vec{\mathbb{v}} \quad (15)\\
&D_t\vec{\mathbb{t}}_2 &- \quad & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 &- \quad & 2\vec{P}\mathbb{p} &= 0 \quad (16)
\end{aligned}
$$

The Quantum Kinetic limit

- A special case is when $\mathcal{B} = 0$, and $\mathcal{E}(x, y, z, t) = \mathcal{E}(t)$
- This leads to the Quantum Kinetic equation on f, u, v:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{e\mathcal{E}\varepsilon_\perp}{\omega^2} v \tag{17}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}\frac{e\mathcal{E}\varepsilon_\perp}{\omega^2}(1 - 2f) - 2\omega u \tag{18}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 2\omega v \tag{19}$$

where:

$$\omega^2(\vec{p}, t) = \varepsilon_\perp^2 + \vec{p}_{||}^2 \tag{20}$$

$$\varepsilon_\perp^2 = m^2 + \vec{p}_\perp^2 \tag{21}$$

$$\vec{p} = (\vec{q}_\perp, q_{||} - e\mathcal{A}(t)) \tag{22}$$

Illustration of similarity with ionisation

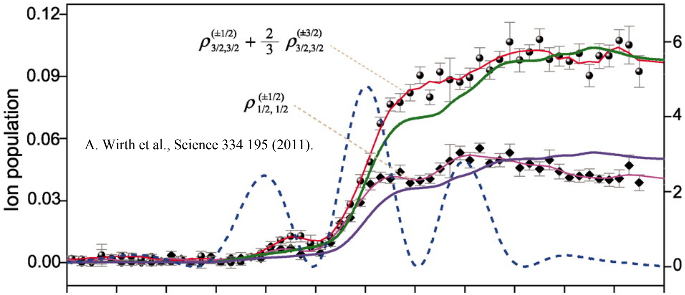
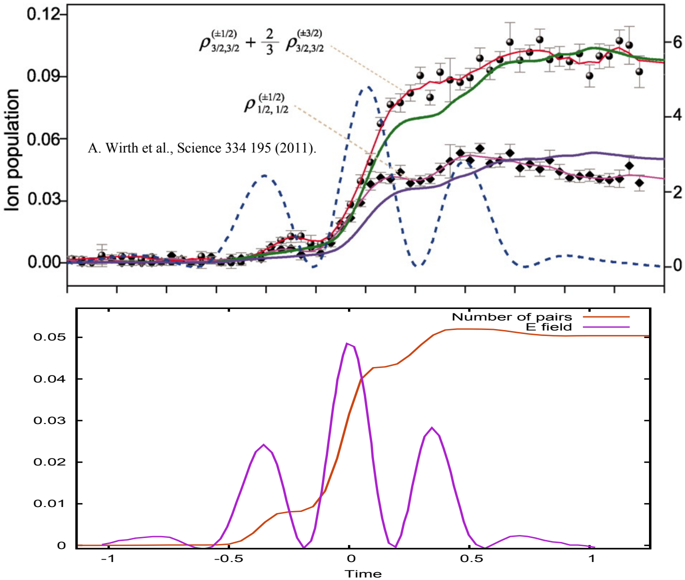


Illustration of similarity with ionisation



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Influence of external field parameters

What is needed for experimentalists?
$\Rightarrow$ Realistic laser fields!
But they are complicated, even an idealised one may look like:

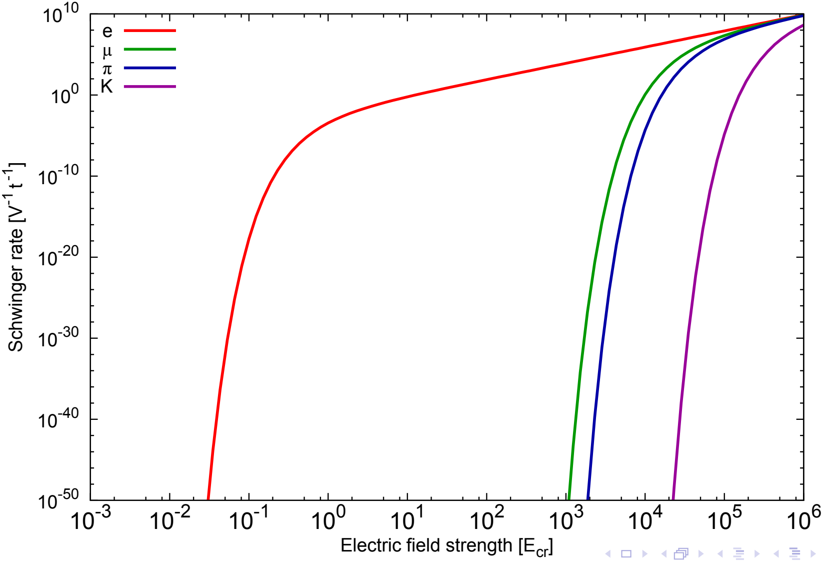
$$\mathcal{E}(t,x,y,z) = E_0 e^{\left(-\frac{x^2}{\Delta x^2}-\frac{y^2}{\Delta y^2}-\frac{(t-z)^2}{\Delta z^2}\right)} \times \cos\left(\phi + \omega(t-z) + \frac{c}{2}(t-z)^2\right). \quad (23)$$

Many-many parameters... we need to understand their influence on particle creation...

Try to learn from even simpler models!

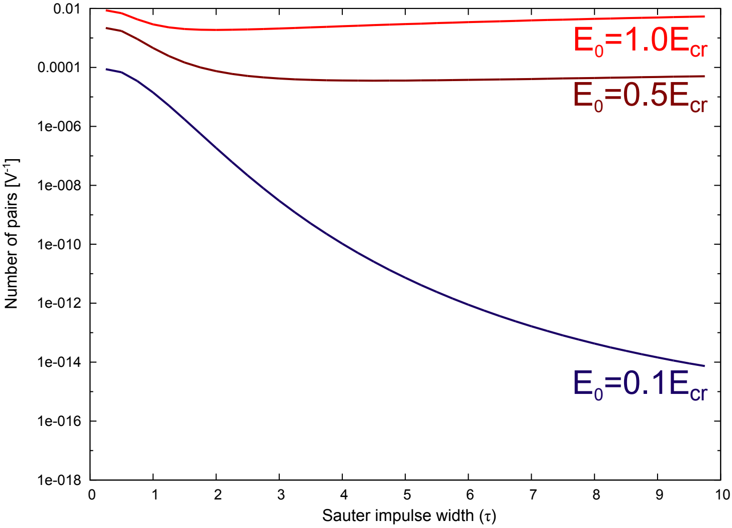
Field strength dependence

Schwinger result (constant electric field): $n \simeq \frac{q^2 E^2}{4\pi^3} \exp\left(-\frac{m^2 \pi}{qE}\right)$



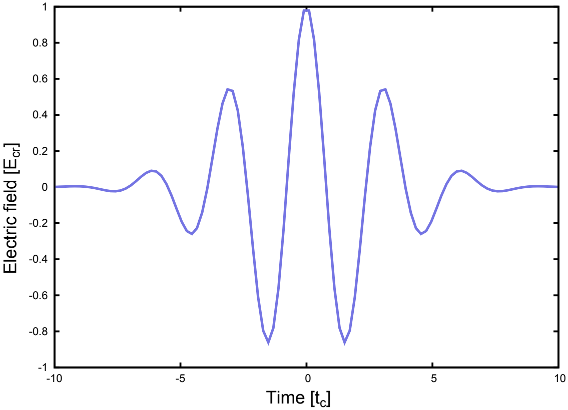
Pulse width dependence

Sauter field: $\mathcal{E}(t) = E_0 \mathrm{sech}^2\left(\frac{t}{\tau}\right)$

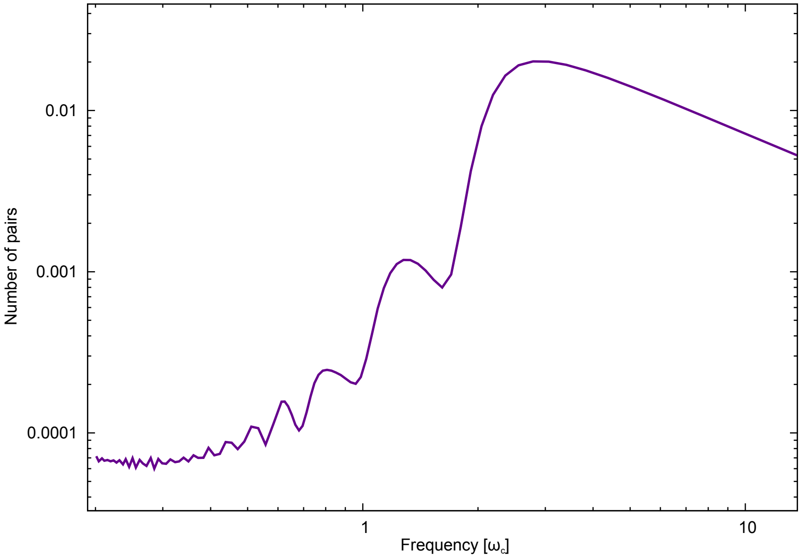


Frequency dependence: few cycle laser pulse

$$\mathcal{E}(t) = \mathcal{E}_0 \mathrm{e}^{-\left(\frac{t}{\tau}\right)^2} \cos\left(\phi + \omega t\right)$$



Frequency dependence: few cycle laser pulse



The peaks mark the position of multi-photon resonances.

Inhomogenity

- So far only homogeneous, time dependent fields...

  They can be calculated at most by the Quantum Kinetic equations.

- Inhomogeneity requires the solution of all the 16 DHW equations, in at least 3 dimensions!

  An analytically unmanageable and numerically demanding problem!

Dirac-Heisenberg-Wigner equations

$$
\begin{aligned}
&D_t\mathbb{s} & & & - \quad & 2\vec{P}\cdot\vec{\mathbb{t}}_1 &=0\\
&D_t\mathbb{p} & & & + \quad & 2\vec{P}\cdot\vec{\mathbb{t}}_2 &=2m\mathbb{a}_0\\
&D_t\mathbb{v}_0 &+ \quad & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & &=0\\
&D_t\mathbb{a}_0 &+ \quad & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & &=2m\mathbb{p}\\
&D_t\vec{\mathbb{v}} &+ \quad & \vec{D}_{\vec{x}}\mathbb{v}_0 &+ \quad & 2\vec{P}\times\vec{\mathbb{a}} &=-2m\vec{\mathbb{t}}_1\\
&D_t\vec{\mathbb{a}} &+ \quad & \vec{D}_{\vec{x}}\mathbb{a}_0 &+ \quad & 2\vec{P}\times\vec{\mathbb{v}} &=0\\
&D_t\vec{\mathbb{t}}_1 &+ \quad & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 &+ \quad & 2\vec{P}\mathbb{s} &=2m\vec{\mathbb{v}}\\
&D_t\vec{\mathbb{t}}_2 &- \quad & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 &- \quad & 2\vec{P}\mathbb{p} &=0
\end{aligned}
$$

Dirac-Heisenberg-Wigner equations

- The DHW equations are known since 20 years, but despite their potential in the description of pair production, they are difficult to solve.
- Problem arises from the high dimensionality, and high derivatives in momentum and coordinate space etc.

We combined pseudo-spectral methods with finite difference and characteristics and were able to create a reliable numeric solver to evolve the DHW equations.

Influence of inhomogeneity

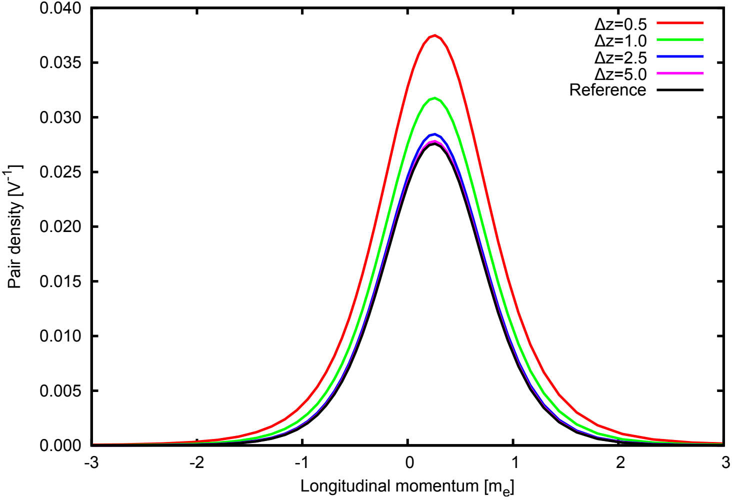
Consider the modified Sauter field:
$\vec{\mathcal{E}}(t,z) = \vec{e}_x E_0 \exp\left(-\frac{z^2}{\Delta z^2}\right) \mathrm{sech}^2\left(\frac{t}{\tau}\right)$.

Clearly $\Delta z \longrightarrow \infty$ recovers the known analytic result.

Let's fix $E_0 = 0.5 E_{cr}$ and investigate the interplay of τ and Δz.

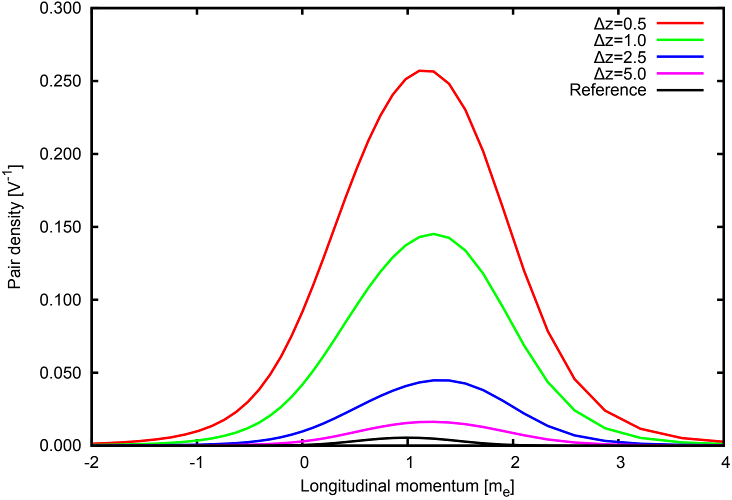
Influence of inhomogeneity

Short pulse: $\tau = 0.5$



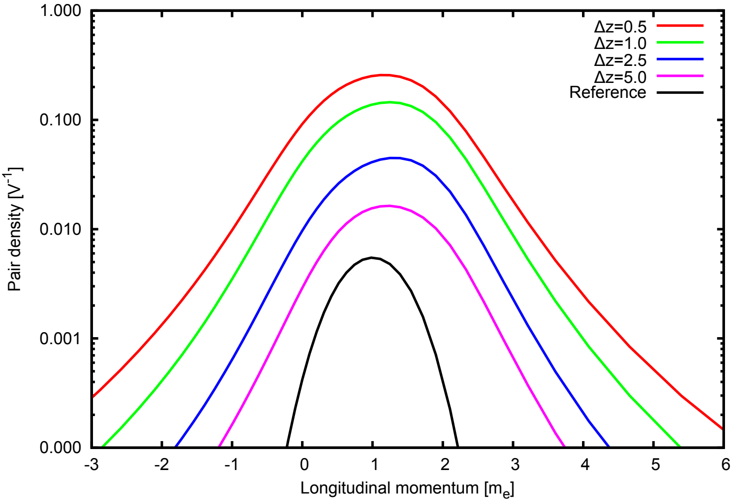
Influence of inhomogenity

Longer pulse: $\tau = 2.0$

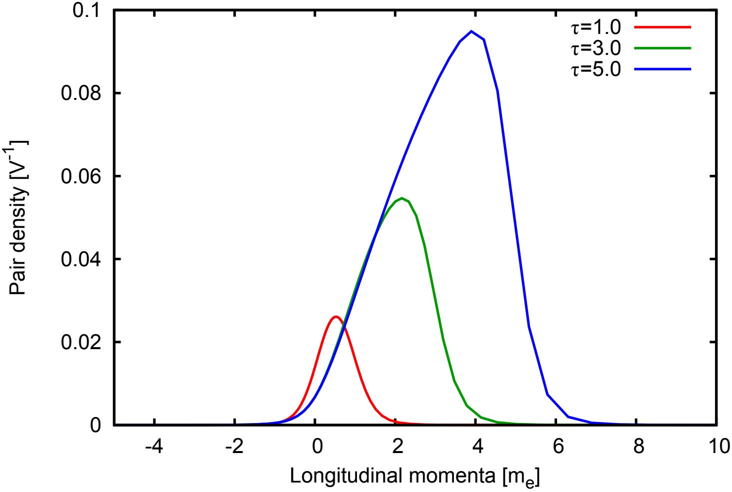


Influence of inhomogeneity

Longer pulse: $\tau = 2.0$ on log-scale

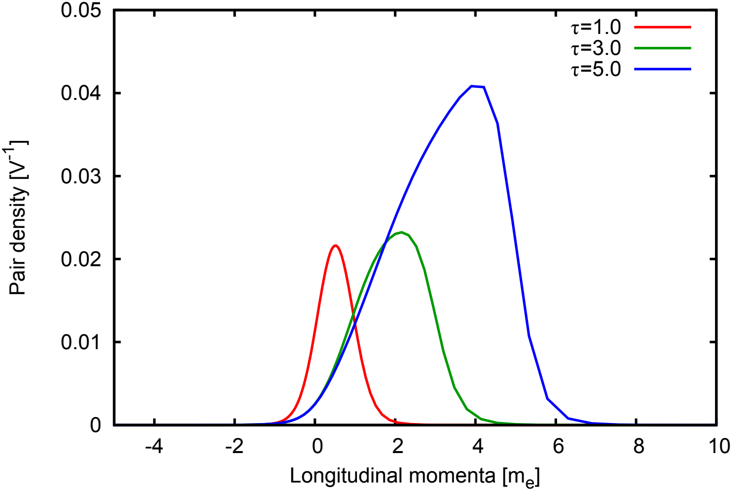


Pulse width dependence at $\Delta z = 3.0$



Influence of inhomogeneity

Pulse width dependence at $\Delta z = 5.0$



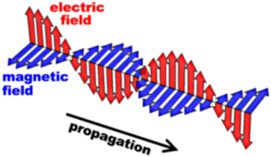
Influence of inhomogeneity

Summary of τ, Δz interplay:

- Larger gradients increase particle production.
- As the system evolves for longer times, even smaller gradients can increase density considerably.

- Spatial gradients can counter balance or even overcome the decreasing effect of pulse widening.
  Important factor in planning future laser experiment parameters (effect of focal area)

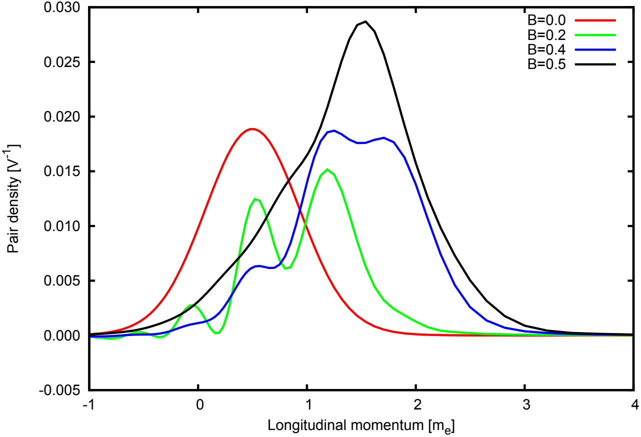
Influence of magnetic field



- Realistic laser fields has magnetic component also (again, many parameters to understand...).
- DHW equations can include magnetic field too, but dimensionality may increase.
- Magnetic field is important, because it is related to the first quantum correction to the momentum!

Influence of homogeneous magnetic field

Let's modify the Sauter field ($E_0 = 0.5, \tau = 1.0$) with a constant magnetic field $\mathcal{B} = B$!



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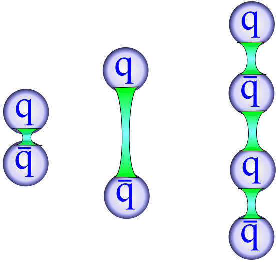
QCD pair production



Quark potential is linear with separation: if a $q - \bar{q}$ pair is separating, the interaction creates more and more quark pairs until energy is depleted.

This process is modelled by color ropes/strings that fragment into final particles.

Success in describing particle spectra in ultra-relativistic nucleus-nucleus collisions!

Outlook to the non-Abelien case

The particle production from these extreme strong color fields can be calculated by the evolution of the non-Abelian Wigner function: ($SU(2)$, homogeneous external field)

$$D_t W = -\frac{g}{8}\frac{\partial}{\partial p_i}\left(4\{W, F_{0i}\} + 2\{F_{i\nu}, [W, \gamma^0\gamma^\nu]\} - [F_{i\nu}, \{W, \gamma^0\gamma^\nu\}]\right) + ip_i\{\gamma^0\gamma^i, W\} - im[\gamma^0, W] + ig[A_i, [\gamma^0\gamma^i, W]] \quad (24)$$

A_μ: color four-potential,

$F_{\mu\nu}$: color field tensor

Particle spectra is very similar to the Abelian case!

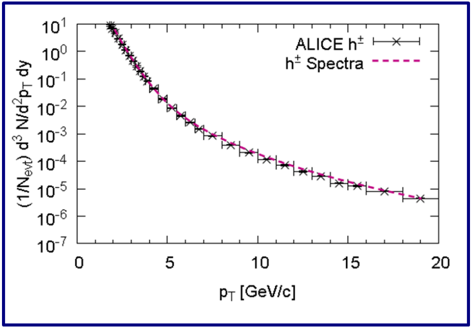
V.V. Skokov, P. Lévai, Phys. Rev. D71 054004 (2008).

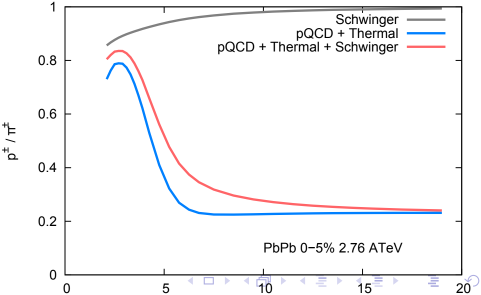
Combined framework for heavy ion collisions

- We built a complex framework to describe heavy ion collisions especially at LHC energies.

  P. Lévai, D. Berényi, A. Pásztor, V. V. Skokov, J. Phys. G38 124155 (2011)

- Multiple processes should be included, one of them is the pair production from strong color fields modelled by the Quantum Kinetic equation.
- Parameters of the external field are fitted to describe unidentified particle spectra.
- Predictions can be given on identified particle spectra and ratios.





Summary

- So far pair production was calculated in spatially homogeneous external fields only.
- The solution of the DHW system in time dependent and inhomogeneous and even magnetic external fields is now possible.
- We pointed out some effects that need more investigation in the near future to aid laser facilities in choosing the optimal parameters for the observation of vacuum decay.
- The Wigner function formalism is a versatile tool for describing and connecting different areas of high energy physics, from lasers to heavy ion collisions.

Supporters: OTKA Grants No. 77816, No. 104260, No. 106119 and US DoE contract DE-AC02-98CH10886.