Pair Production in Strong Fields: The Wigner function approach

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Abstract: The recent rapid development of laser technology renewed the interest in strong field physics especially in pair production phenomena. As the experiments are approaching the critical field necessary for e+e- pair production in crossed laser beams, the theory continues to develop more realistic models of these extreme fields. With these models we can gain insight into how the observables – such as particle spectra – depend on the field parameters. One such model is the Dirac-Heisenberg-Wigner formalism that describes the time evolution of the one-particle Wigner-function in strong external fields. We show how this versatile formalism can be used in high energy physics processes and present latest results for inhomogeneous external fields.

1. Introduction

The process of pair production has gained much interest in its half century history, since its prediction by J. Schwinger in 1951 [1]. While it is uncommon in nature (except may be in the vicinity of compact astrophysical objects), it may serve as an important test of our understanding of the vacuum structure and extreme field physics if it could be observed and tested in experiments. With the advent of high intensity and ultra-short laser technology, the realization of e+e- pair production has been appearing among the scientific goals of the next generation laser facilities, for example the Extreme Light Infrastructure (ELI) project [2]. On the other hand, high energy physics was historically evolved from the processes created in particle colliders, such as the Large Hadron Collider (LHC) at CERN. These facilities can also create high energy density on very short timescales, most prominently the Quark-Gluon Plasma (QGP). The proper description of these highly transient phenomena remains a challenge for the theoretical physicists.

Motivated by the need of a quantum transport equation for transient strong field processes, the Dirac-Heisenberg-Wigner (DHW) formalism for QED was developed and proven to contain many earlier studied models as limiting cases (the Schwinger-model, Vlasov-equation, etc.) [3, 4]. Also, this formalism can be easily generalized to non-Abelian cases and thus can be used to describe QCD processes [5].

Since the DHW formalism results in a system of coupled partial-differential equations it can only be solved analytically for very simplified field configurations. For more realistic calculations numerical methods are necessary but these methods should be carefully verified against known analytical solutions.

2. QED pair production in inhomogeneous external fields

For many years pair production models were only capable for describing homogeneous field configurations. Within the framework of the DHW formalism inhomogeneity can be easily taken into account. For an external electric field with an inhomogeneity transverse to its direction the equation system turns out to be 16 coupled equations in at 3 dimensions (1 time + 1 transverse spatial [z] + 1 longitudinal momentum [p_x]). To have a reference to compare the results against, we consider the field:

$$E(t,z) = E_0 \exp\left(-\frac{z^2}{dz^2}\right) \mathrm{sech}^2\left(\frac{t}{\tau}\right) \tag{1}$$

since for the $dz \to \infty$ limit there exist an analytical solution for the asymptotical particle distribution. Note, that we set $\hbar = c = e = m = 1$, the field amplitudes are measured in critical field units, $E_{cr} = \frac{m^2 c^3}{e\hbar}$, time in $\frac{\hbar}{mc^2}$ and distance in $\frac{\hbar}{mc}$.

It is expected from estimations [4], that inhomogeneities become important for late times (long pulses) and for large momenta, but to know their magnitude the solution of the DHW equations is needed. We set up a framework for numerically investigating the asymptotic pair density, focusing on the longitudinal momentum distribution (at $z = 0$) and calculate it for different values of dz and τ. Figure 1. shows our results for $E_0 = 0.5, \tau = 0.5$. For low values of dz there is a small enhancement but even for $dz = 5$, the reference result of $dz \to \infty$ is almost recovered. However, the picture changes completely if we consider a longer pulse: $E_0 = 0.5, \tau = 2.0$, as seen on Figure 2. For even this slightly longer pulse, the z gradient sharply increases the particle density with respect to the reference solution. The fact that particle density may increase for longer pulses is also the opposite of what is known for homogeneous electric fields.

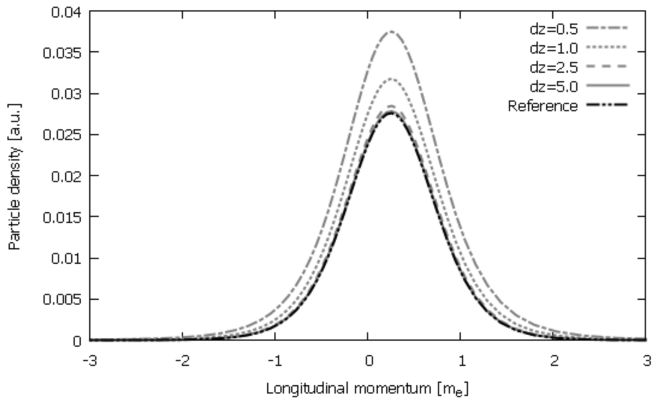

Fig. 1. Asymptotic pair density (for unit phase-space volume) calculated with field parameters $E_0 = 0.5, \tau = 0.5$, and different z widths. The reference is the analytical $dz \to \infty$ limit.

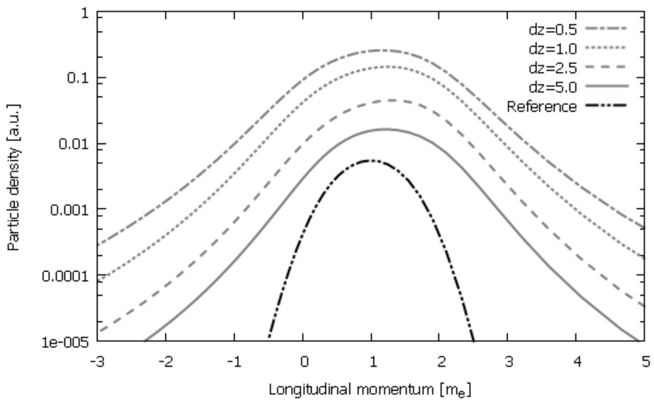

Fig. 2. Asymptotic pair density (for unit phase-space volume) calculated with field parameters $E_0 = 0.5, \tau = 2.0$, and different z widths. The reference is the analytical $dz \to \infty$ limit.

3. Summary

We developed numerical tools to investigate pair production within the DHW formalism to be used in high energy physics calculations. One application is the modeling of more realistic laser pulses that was not possible before. We studied e+e- pair production in inhomogeneous external electric fields, near the Schwinger and Compton limit. The inhomogeneity tends to increase the asymptotic particle density with respect to the homogeneous one; moreover this enhancement may get larger for longer pulses. This could be an important effect to take into account when setting the pulse parameters for laser facilities aiming for the observation of vacuum decay.

4. References

[1] J. S. Schwinger, "On Gauge Invariance and Vacuum Polarization", Phys. Rev. **82**, 664, (1951).
[2] J.-P. Chambaret, "Extreme Light Infrastructure: architecture and major challenges", Proc. SPIE 7721 1D (2010).
[3] I. Bialynicki-Birula, P. Górnicki, and J. Rafelski, "Phase-space structure of the Dirac Vacuum", Phys. Rev. **D44**, 1825, (1991).
[4] F. Hebenstreit, R. Alkofer, and H. Gies, "Schwinger pair production in space and time-dependent electric fields", Phys. Rev. **D82**, 105026, (2010).
[5] V. V. Skokov, and P. Lévai, "Transverse and longitudinal momentum spectra of fermions produced in strong SU(2) fields", Phys. Rev. **D78**, 054004, (2008).

5. Acknowledgement

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