A Rigorous Derivation of Electromagnetic Self-Force

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Abstract

During the past century, there has been considerable discussion and analysis of the motion of a point charge, taking into account "self-force" effects due to the particle's own electromagnetic field. We analyze the issue of "particle motion" in classical electromagnetism in a rigorous and systematic way by considering a one-parameter family of solutions to the coupled Maxwell and matter equations corresponding to having a body whose charge-current density $J^a(\lambda)$ and stress-energy tensor $T_{ab}(\lambda)$ scale to zero size in an asymptotically self-similar manner about a worldline γ as $\lambda \to 0$. In this limit, the charge, q, and total mass, m, of the body go to zero, and q/m goes to a well defined limit. The Maxwell field $F_{ab}(\lambda)$ is assumed to be the retarded solution associated with $J^a(\lambda)$ plus a homogeneous solution (the "external field") that varies smoothly with λ. We prove that the worldline γ must be a solution to the Lorentz force equations of motion in the external field $F_{ab}(\lambda = 0)$. We then obtain self-force, dipole forces, and spin force as first order perturbative corrections to the center of mass motion of the body. This is our rigorous perturbative result. We also address the issue of obtaining a "self-consistent perturbative equation" associated with the perturbative result, and argue that the self-force equations of motion that have previously been written down in conjunction with the "reduction of order" procedure should provide accurate equations of motion for a sufficiently small charged body with negligible dipole moments and spin. The original (non-reduced-order) equations, on the other hand, are excluded by the rigorous perturbative result.