Optimization and Limitation of Table-top Free-Electron Lasers

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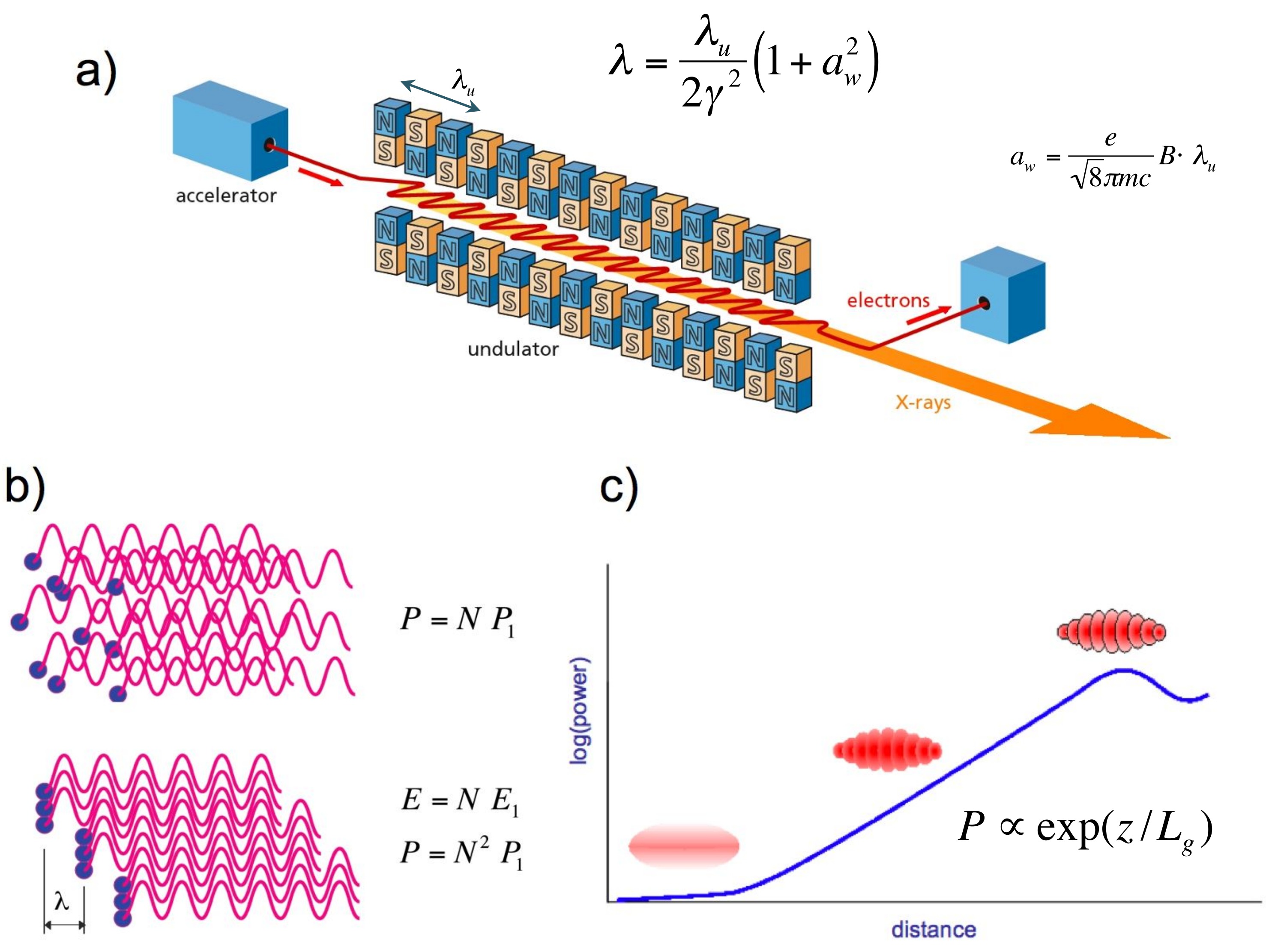
Workshop "*Frontier in Intense Laser-Matter Interaction*"
March 3rd 2010, Munich

Free-Electron Lasers



$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + a_w^2\right)$$

$$a_w = \frac{e}{\sqrt{8}\pi mc} B \cdot \lambda_u$$

$$P = N\ P_1$$

$$E = N\ E_1$$

$$P = N^2\ P_1$$

$$P \propto \exp(z/L_g)$$

General Optimization

Electron Beam Parameters

- Almost everything scales with the FEL parameter:

$$\rho = \frac{1}{2\gamma}\left[\left(\frac{\lambda_u f_c a_w}{2\pi}\right)^2 \frac{I}{I_A \sigma_x \sigma_y}\right]^{\frac{1}{3}}$$

- Assuming a round beam with similar emittance and symmetric focusing, the FEL parameter scales as $(I/\varepsilon_n)^{1/3}$.
- The characteristic length is the power gain length:

$$L_g = \frac{\lambda_u}{4\sqrt{3}\pi\rho}$$

(Ideal Case)

- Increasing the current and/or reducing the emittances increase the performance and reduce the overall required length of the FEL.

Degrading Effects

	Energy Spread	Space Charge
Effect	Smears out micro bunching	Work against Coulomb field when bunching
Negligible when	$\frac{\sigma_\gamma}{\gamma} << \rho$	$k_p = \sqrt{\frac{4\pi I}{\gamma I_A A}} << 2k_u\rho\gamma$
Correction to FEL parameter	$1 - \left(\frac{\sigma_\gamma}{\rho\gamma}\right)^2$	$1 - \frac{1}{3}\left(\frac{k_p}{2k_u\rho\gamma}\right)^2$
Limitation	At lower energies	At lower energies or very high currents

Saturation Power and Brilliance

- Saturation power:

$$P_{sat} = 1.6\rho P_{beam} = 1.6\frac{mc^2}{e}\rho\gamma I$$

- For a given wavelength it favors a higher beam energy and thus a longer undulator period.

- Peak Brilliance:

$$Brill. = \frac{N_{photons}}{2\pi^3\sigma_t\left(\sigma_\omega/\omega\right)\sigma_x\sigma_{x'}\sigma_y\sigma_{y'}}$$

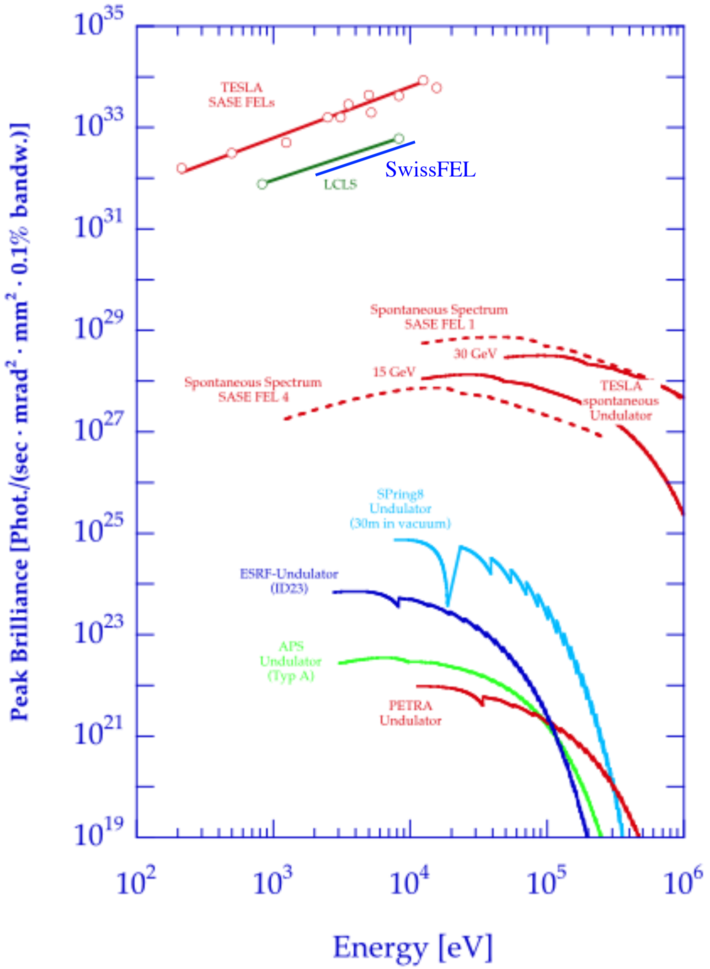
- At saturation:

$$N_{phot} = P_{sat}\sqrt{2\pi}\sigma_t/\hbar\omega$$

$$\sigma_\omega/\omega = 2\rho$$

$$\sigma_x\sigma_{x'} = c/2\omega$$ (transverse coherence)

$$Brill. \approx \frac{0.8}{e\lambda\!\!\!^{-}_e\lambda}\gamma I$$



Undulator Optimization

- Increasing aw increases the coupling between electron field and radiation field (a_w/γ), however at very large values the energy has to increase to preserve the resonant wavelength ($\gamma \sim a_w$).

$$\boxed{a_w \geq 0.8}$$

- A small undulator period reduces the overall size of the FEL, however a strong undulator field would require a fraction of the period as the gap:
  - Strong impact of undulator wakefields (~gap^{-2})
  - Strong focusing to allow full transmission through the smaller aperture

For very short periods an RF/laser wiggler is more feasible.

Optimizing the Focusing

- Transverse betatron motion delays the particle with respect to the on-axis particle. The delay scales with the focusing strength:

$$\frac{\langle v_z \rangle}{c} = 1 - \frac{1+a_w^2}{2\gamma^2} - \frac{1}{2}\left(x'^2 + y'^2\right) \approx 1 - \frac{\lambda}{\lambda_u} + \frac{\lambda}{\lambda_u}\frac{2\Delta\gamma}{\gamma_r} - \frac{\left(J_x + J_y\right)}{2\beta\gamma}$$

Action variable of β-motion

Slow β-motion

In theory a correlation could cancel last two terms!

- RMS averaging over the beam the action variables are replaced by the normalized emittance (and the energy deviation by the energy spread)
- Emittance effects are not disrupting the FEL performance if the condition is fulfill:

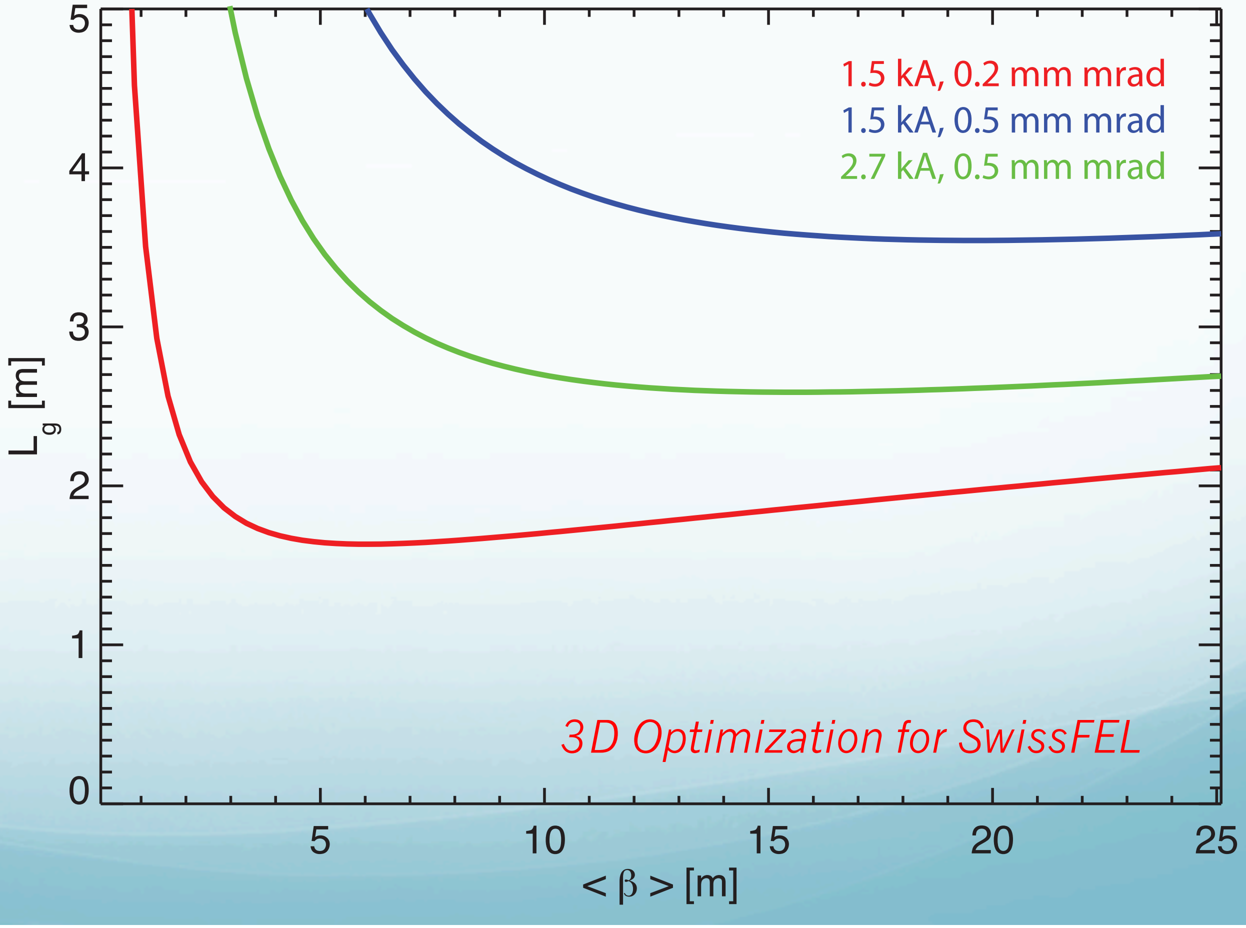
$$\boxed{\frac{\lambda_u \varepsilon_n}{2\lambda\beta\gamma} \ll \rho}$$

Optimizing the Focusing II

- Decreasing the β-function (increase focusing), increases the FEL parameter ρ.
- Too strong focusing enhances the emittance effect and increasing the FEL gain length.

From 1D Theory:

$$\beta_{opt} \approx 3\sqrt{\frac{\varepsilon_n}{\gamma}\frac{4\pi}{\lambda}L_g}$$



Transverse Coherence (2D FEL Theory)

- Diffraction Parameter:

$$B = \frac{z_R}{z_{FEL}}$$

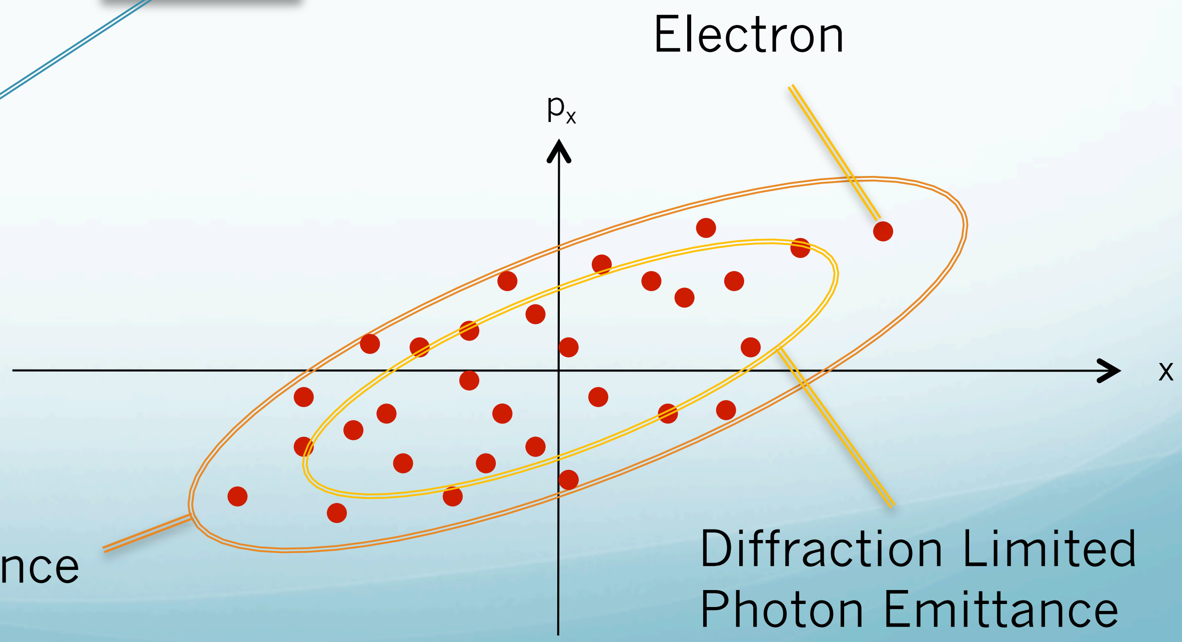
Rayleigh Length

Char. Scaling of FEL
(=$2k_u\rho$ in 1 D model)

- Assuming electron size as radiation source size:

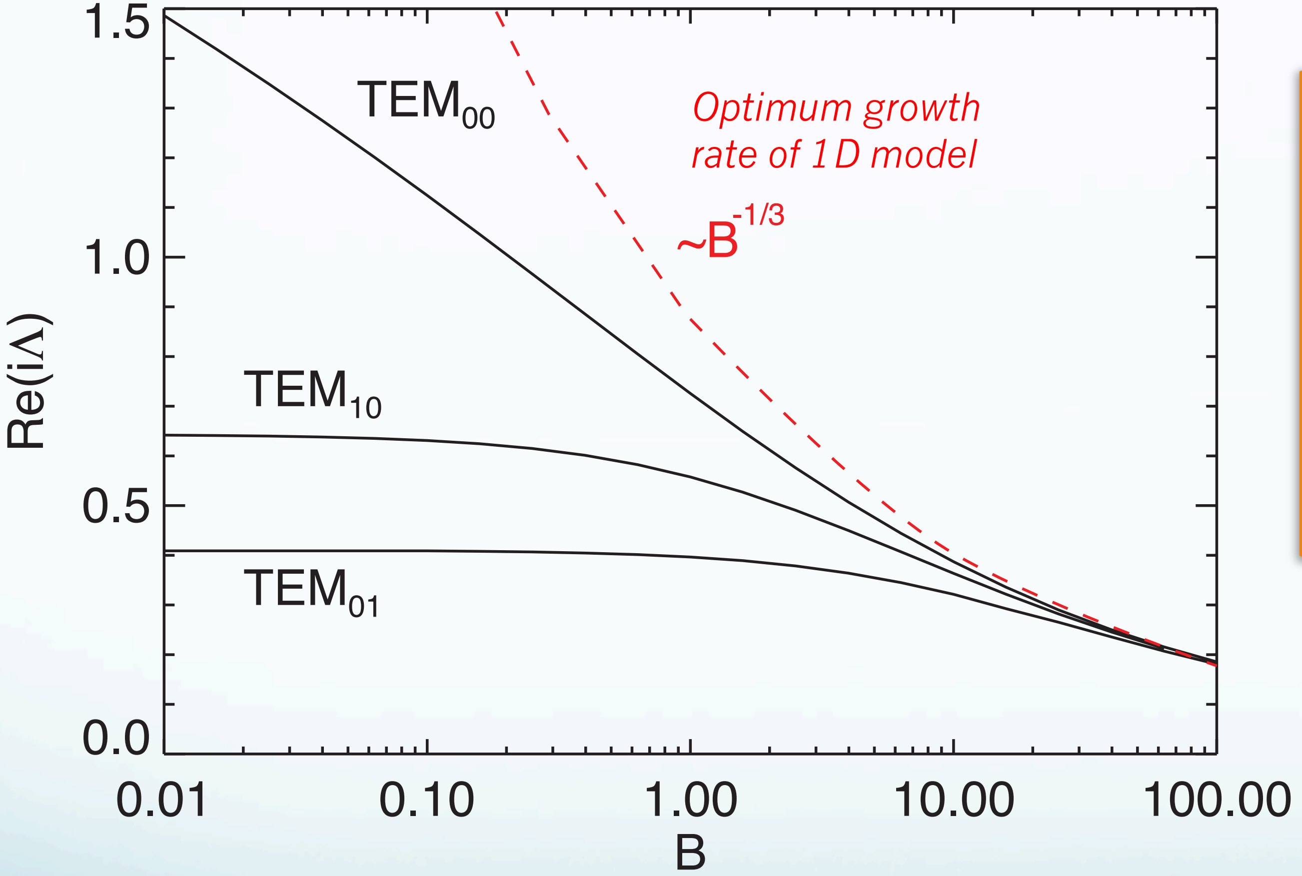
$$z_R = 2r_0^2 k = \boxed{\frac{4\pi}{\lambda}\cdot\frac{\varepsilon_n}{\gamma}}\beta$$

Constraint for emittance to be smaller than photon emittance for all electrons to contribute on the emission process



Transverse Coherence (2D FEL Theory)

- Growth rates for FEL eigenmodes (r,ϕ-decomposition):



Optimum:

$$B \approx 7$$

$$\frac{\varepsilon_n}{\gamma} \approx \frac{\lambda}{2\pi}$$

Note: fundmental FEL Eigenmode has intrinsic wavefront curvarture and thus correspond to a larger photon emittance

Increased gain length due to strong diffraction

Mode competition and reduced coherence

SASE and Partial Coherence

- Spontaneous radiation as seed couples to many modes.

- Mode content visible in fluctuation of instantaneous power:

$$\zeta = \frac{\left\langle \left(P - \langle P \rangle\right)^2 \right\rangle}{\langle P \rangle^2} = \frac{1}{M_T}$$

- Agrees (surprisingly) well with standard definition of coherence:

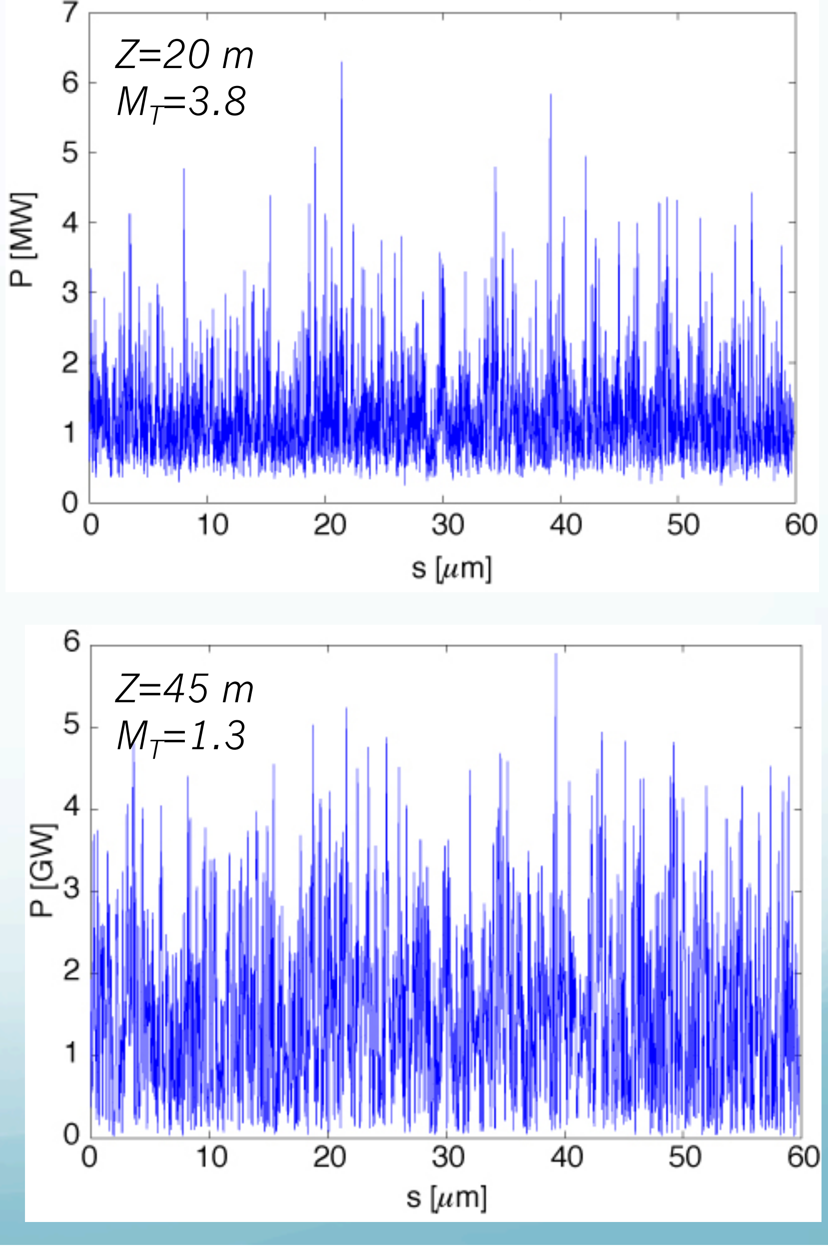
$$\zeta = \frac{\iint \left|\mu_{12}(r_1,r_2)\right|^2 \left\langle I(r_1) I(r_2) \right\rangle dr_1 dr_2}{\left[\int \langle I(r) \rangle dr\right]^2}$$

with the mutual intensity function:

$$\mu_{12}(r_1,r_2) = \frac{\left\langle E(r_1) E^*(r_2) \right\rangle}{\left[\left\langle |E(r_1)|^2 \right\rangle \left\langle |E(r_2)|^2 \right\rangle\right]^{\frac{1}{2}}}$$

$\mu_{12}=1$: full coherence between r_1 and r_2

SASE Profiles (SwissFEL)



Case Study:
High Current Beam
Soft X-ray FEL
Micro-Undulator

Electron Beam Parameters

Energy	1 GeV
Current	10 kA
Emittance	1 mm mrad
Energy spread	5 MeV

- Minimum wavelength (emittance constraint):

$\lambda > 1$ **nm**

- Micro undulator:

$a_w = 0.7, \lambda_u = 5$ **mm**

- Field and Gap Estimate (planar hybrid undulator):

B=2.1 T, g = 0.5 mm

(gap can be increased with kryogenic undulators)

FEL Performance (Ming Xie Model)

- Strongly effected by the energy spread and sub-sequentially by the emittance. The simple 1D FEL parameter is about 2% but reduces by factor of 20 in the 3D model.

Effective FEL Parameter	0.1%
Gain Length	0.23 m
Saturation Length	4.8 m
Saturation Power	9 GW
Bandwidth	0.23 % FWHM

- Requires 5 m long micro undulator.
- Performance goes down (less power, reduced coherence) for:
  - Shorter wavelength / higher energy
  - Larger energy spread

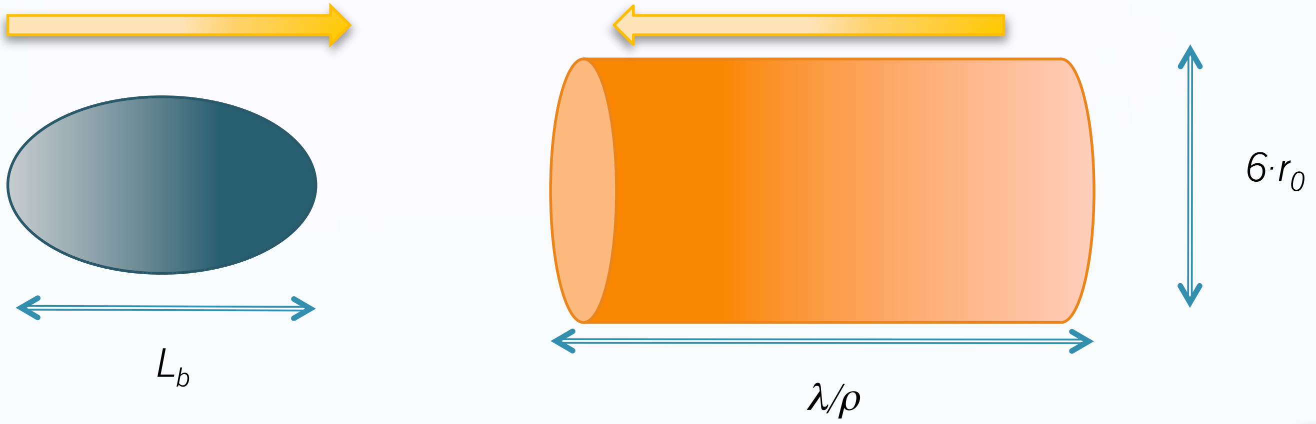
Case Study:
Ultralow Emittance Beam
Hard X-ray FEL
Laser Wiggler

The Advantage of Laser Wigglers

- Counter-propagating laser fields have the same impact on the electron beam as an undulator field. (in electron rest frame, the undulator field becomes an EM wave).
- The period can be reduced significantly while keeping a strong field (a ~1)
- Avoids the boundary problems of a magnetic undulator, which are:
  - Physical aperture between the poles
  - Strong wakefields within the undulator.
- Tunability of the a-value with the pulse energy.

Requirement for the Laser Field.

- Minimum interaction volume:



- Within the interaction volume the field stability over the interaction time L_b/c has to be:

$$\frac{\Delta a}{a} \leq \rho$$

- The absolute minimum Rayleigh length is (Mode stacking):

$$z_R = \frac{L_b}{\sqrt{8\rho}}$$

- Fundamental transverse mode only:

$$z_R \approx \frac{kr_0^2}{\rho}$$

Breaking the Angstrom Barrier...

- Goal: 1 Ångstrom radiation, using Ti:Saph laser wiggler
- Electron beam: $E = a \cdot 50$ MeV, $\varepsilon_n = a \cdot 10^{-9}$ m **(!!!)**
- Beam source is most likely limited to current < 100 A.
- Beta-function about 10 cm for tight spot of $r_0=1\ \mu$m.

Huge penalties

$$\rho = \frac{1}{2\gamma}\left[\left(\frac{\lambda_u f_c a_w}{2\pi}\right)^2 \frac{I}{I_A \sigma_x \sigma_y}\right]^{\frac{1}{3}} \approx 10^{-4}$$

- Additional problems with coherence and energy spread.

Making Laser (based) Wiggler Realizable...

- Relaxed condition for longer period (~100 μm):
  - THz Radiation (doubtful)
  - Laser Beatwave Plasma Wiggler (channeled, but very sensitive to spot size, requires even lower emittance)

- Work with higher field strength (a>>1) to relax demands on electron beam emittance and allow higher beam currents.

- Active control of FEL parameter ρ:
  - Too small → field stability/energy spread requirements
  - Too high → reduced coherence

Summery

- Beam emittance defines the achievable wavelength:
  - ➟ Small emittance value
  - ➟ Lower beam energy
  - ➟ Shorter period length
  - ➟ Shorter saturation length.
- Coherence seems to be the most limiting factor when going to shorter wavelength.
- With current e-sources, 1 nm seems reasonable but 1 Å requires significant improvement in beam quality
- Saturation power and peak brightness will be lower than X-ray FEL facilities (XFEL, LCLS)
- Micro undulators seem to be the preferred choice, laser wiggler have many technical issues, which needs to be resolved.