

# Radiation back-reaction on electrons in relativistically strong and QED-strong fields and ion acceleration

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# Motivation

- We develop the model which allows us:
  - to take into account the radiation back-reaction on electrons in QED-strong fields, applicable to particle-in-cell (PIC) scheme,
  - and to find the angular and frequency distributions of the radiated high-energy photons.
- The model allows to simulate the laser-plasma interaction at intensities of  $10^{22}$ - $10^{25}$  W/cm<sup>2</sup>.
- We apply the model for simulation of ion acceleration in bulk targets at super-high intensity.

# Outline

- An account of the radiation back-reaction is important for intensities  $10^{22}$ - $10^{25}$  W/cm<sup>2</sup> soon available with Extreme Light Infrastructure (ELI), as it influences strongly the laser-plasma interaction.
- Radiation effect stabilizes the laser piston, cooling electrons and leading to more efficient ion acceleration.
- Laser piston apart from quasi-stationary behavior possess non-stationary effects, limiting the model applicability.
- Linear polarization in comparison with circular polarization leads to higher radiation losses in the laser piston.
- QED-strong field affects the radiation spectrum.

# Model description

# Account of the radiation back-reaction in the model

QED is not compatible with the traditional approach to the radiation force in classical electrodynamics (LAD equation and its known approximations).

An alternative equation of motion for a radiating electron with respect to *proper time* has been suggested:

$$\frac{dp^i}{d\tau} = \frac{e}{c} F^{ik} \frac{dx_k}{d\tau} - \frac{I_{QED} p^i}{mc^2}$$

$$\frac{dx^i}{d\tau} = \frac{p^i}{m} + \tau_0 \frac{I_{QED}}{I_{cl}} \frac{eF^{ik} p_k}{m^2 c}$$

Here :  $I_{QED}$  is used instead of  $I_{cl}$

The derivation of these equations, a way to solve them applicable for PIC scheme, and a means of integrating the emission are described in Refs. [1,2]

[1] I.V. Sokolov, JETP **109**, 207 (2009);

[2] I.V. Sokolov, *et al*, PoP **16**, 093115 (2009)

# Account of QED effect in the model

At  $\chi > 1$  the following inequalities hold:

(1) Electric field acting on the particle in the co-moving frame exceeds  $(2/3)E_S$ ,

That is, this field is QED-strong.

$$E_S = m_e c^2 / |e| \hat{\lambda}_C$$

(2) Formally calculated within the classical theory the typical energy of

emitted photons  $\hbar\omega_c$  exceeds the electron energy  $\varepsilon mc^2$  so that:

$$\hbar\omega_c > \varepsilon mc^2$$

That is, the electron recoil should be accounted for; and

(3) The radiation loss rate, calculated in the classical theory exceeds the

"Comptonian radiation loss rate":  $I_{cl} > I_C$

$$I_{cl}(E_0) = 2e^4 E_0^2 / (3m_e^2 c^3)$$

$$I_C = I_{cl} \left( \frac{2E_S}{3} \right)$$

$$\chi = \sqrt{\frac{I_{cl}}{I_C}}$$

If  $\chi > 1$  the actual radiation loss rate differs from  $I_{cl}$

It should be *re-calculated* self-consistently

(see Figure :  $I_{QED}(I_{cl})$  below).

$$I_C = \frac{8}{27} \frac{e^2 c}{\hat{\lambda}_C^2}$$

# Emission spectra

$$Q(r_0, \chi) = \frac{r_0 \left( \int_{r_\chi}^{\infty} K_{\frac{5}{3}}(y) dy + r_0 r_\chi \chi^2 K_{\frac{2}{3}}(r_\chi) \right)}{\int_0^{\infty} dr_0 r_0 \left( \int_{r_\chi}^{\infty} K_{\frac{5}{3}}(y) dy + r_0 r_\chi \chi^2 K_{\frac{2}{3}}(r_\chi) \right)}$$

$$r_\chi = \frac{r_0}{1 - \chi r_0}$$

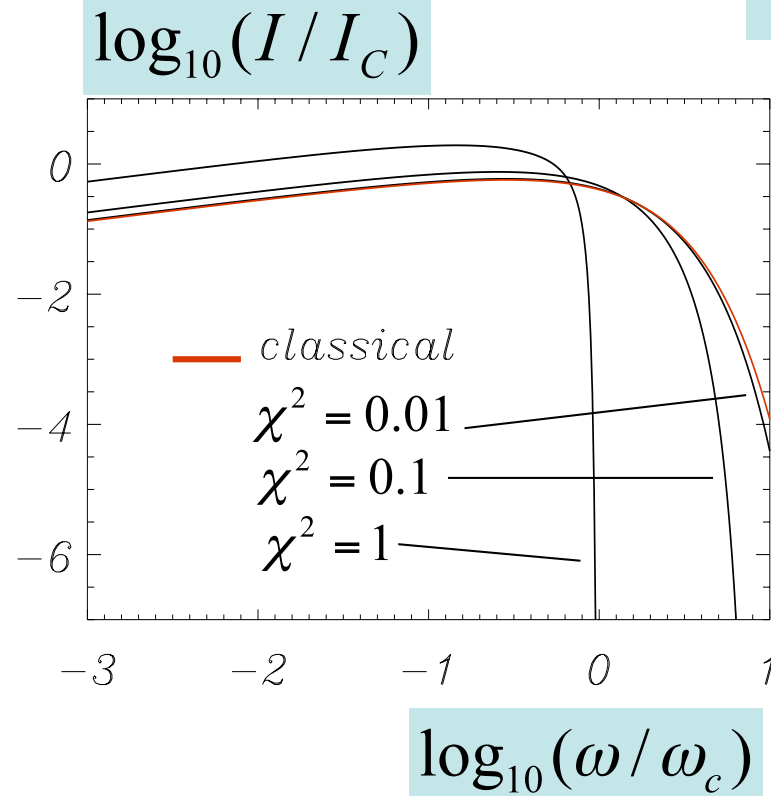
$$r_0 = \frac{\omega}{\omega_c}$$

For various values of  $\chi$ :

$$\chi = \sqrt{\frac{I_{cl}}{I_C}}$$

$$I_C = \frac{8 e^2 c}{27 \hat{\lambda}_C^2}$$

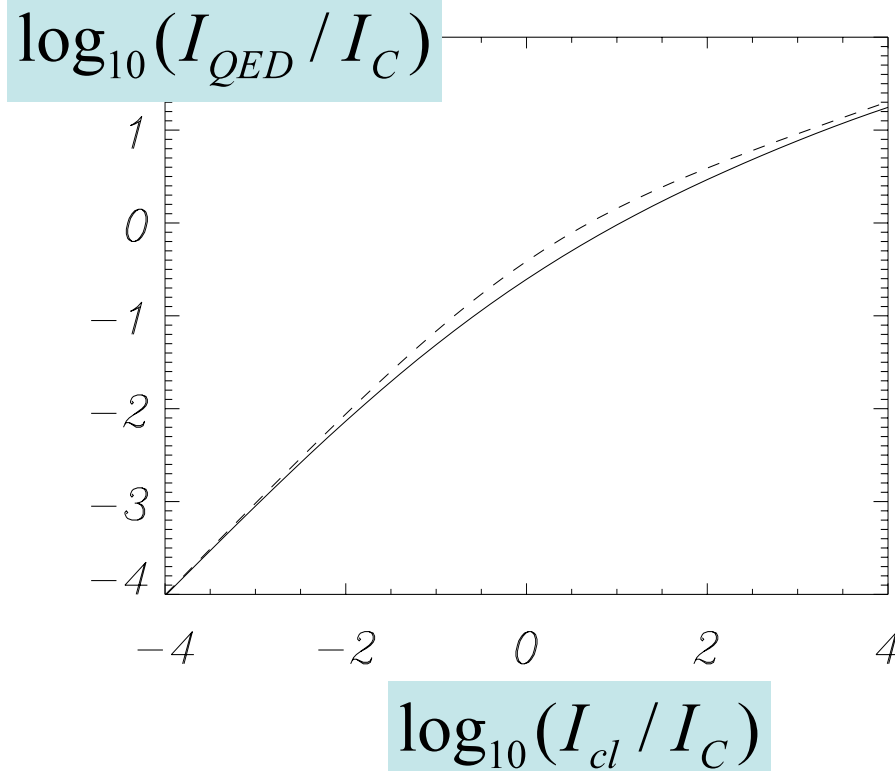
$$\omega_c = \epsilon c \chi / \hat{\lambda}_C$$



# Emitted radiated power

$$\frac{dI}{d\Omega dr_0} = I_{cl} \delta\left(\Omega - \frac{\vec{p}}{p}\right) \left(\frac{I_{QED}}{I_{cl}}\right) Q(r_0, \chi)$$

$$\frac{I_{QED}}{I_{cl}} = \frac{9\sqrt{3}}{8\pi} \int_0^\infty dr_0 r_0 \left( \int_{r_\chi}^\infty K_{\frac{5}{3}}(y) dy + r_0 r_\chi \chi^2 K_{\frac{2}{3}}(r_\chi) \right)$$



Interpolation formula  
(dashed line):

$$I_{QED} = \frac{I_{cl}}{\left(1 + 1.04\sqrt{I_{cl} / I_C}\right)^{4/3}}$$



# A signature of the radiated photons in the polarization plane

3D PIC simulation parameters:

Laser pulse: 30-fs linearly polarized

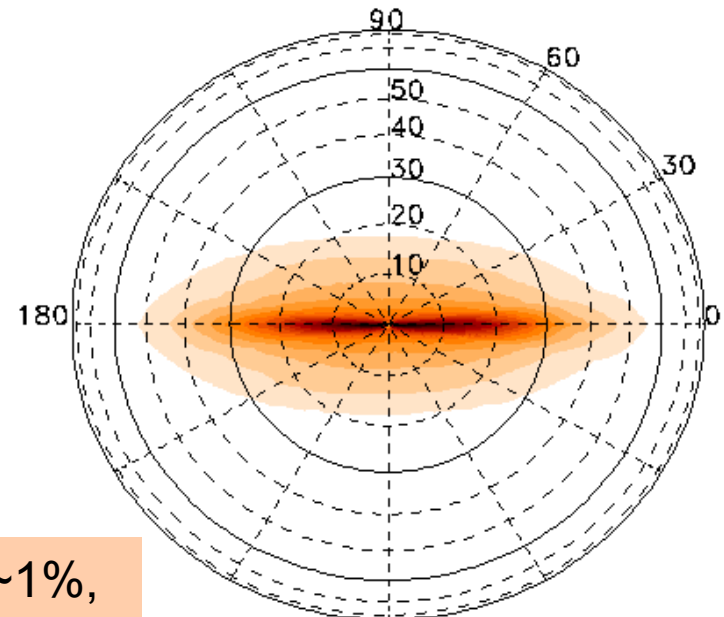
$$\lambda = 0.8 \mu\text{m}$$

*linear polarization*

Plasma:  $n_0 = 3n_{cr}$

$$L = 10\lambda$$

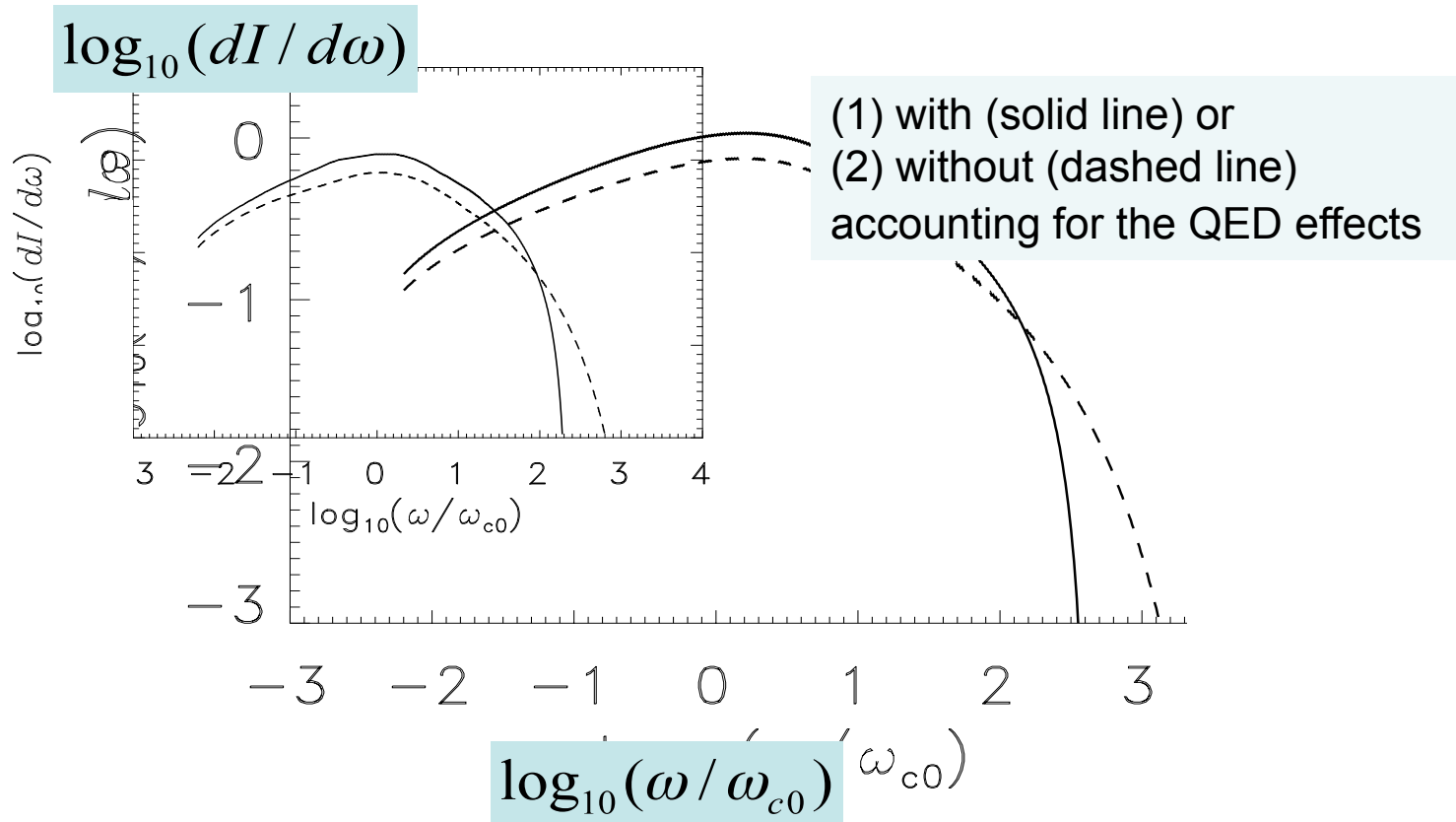
Angular distribution of backward  
scattered radiation above 150 keV



Backward scattered high-frequency radiation  $\sim 1\%$ ,  
or 0.26J; above 150keV – 0.24J.

# Validating QED effects in the model

Emission spectrum for 600 MeV electrons interacting with 30-fs laser pulse of intensity  $2 \times 10^{22}$  W/cm<sup>2</sup>



Here:  $\hbar\omega_{c0} = 1.1$  MeV for  $\lambda = 0.8$   $\mu$ m

We see that the physically absurd prediction (see the dashed curve) that the maximum photon energy exceeds 1 GeV is eliminated by the QED effects.

# Summary for the model description

In a wide range of applications, even including the case of very strong laser fields with essential QED effects, a quasi-classical electron motion may be successfully described within a new radiation force approximation, conserving energy and momentum.

The necessary corrections in the radiation force and the emission spectra to account for the QED effects are parameterized by the *sole* parameter:  $I_{cl}$

The formulation of radiation spectrum and pattern are conveniently set in terms of delta-functions enabling efficient computation.

Application of the described  
model to laser-plasma interaction  
and efficient ion acceleration

# Ion acceleration with high intensity laser pulses

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Fast ions can find many applications in fusion, industry and medicine: **low ratio current/energy flux, simple ballistic transport, high absorption efficiency**  
but one needs an efficient and compact ion accelerator to the energies  $> 100$  MeV.

Ponderomotive ion acceleration in bulk targets: requires **cold electrons, high quality laser pulse**, less restrictions on the target, could be very efficient

Radiation friction effects at **super-high intensities** can have a positive role in **electron cooling**

# Ion acceleration by the laser piston: the piston velocity

Conservation of the momentum (pressure) in the piston reference frame: **stationary propagation**

$$2 \frac{I_{\text{las}}}{c} \frac{1 - \beta_f}{1 + \beta_f} = 2 \rho_0 \gamma_f^2 \beta_f^2 c^2$$

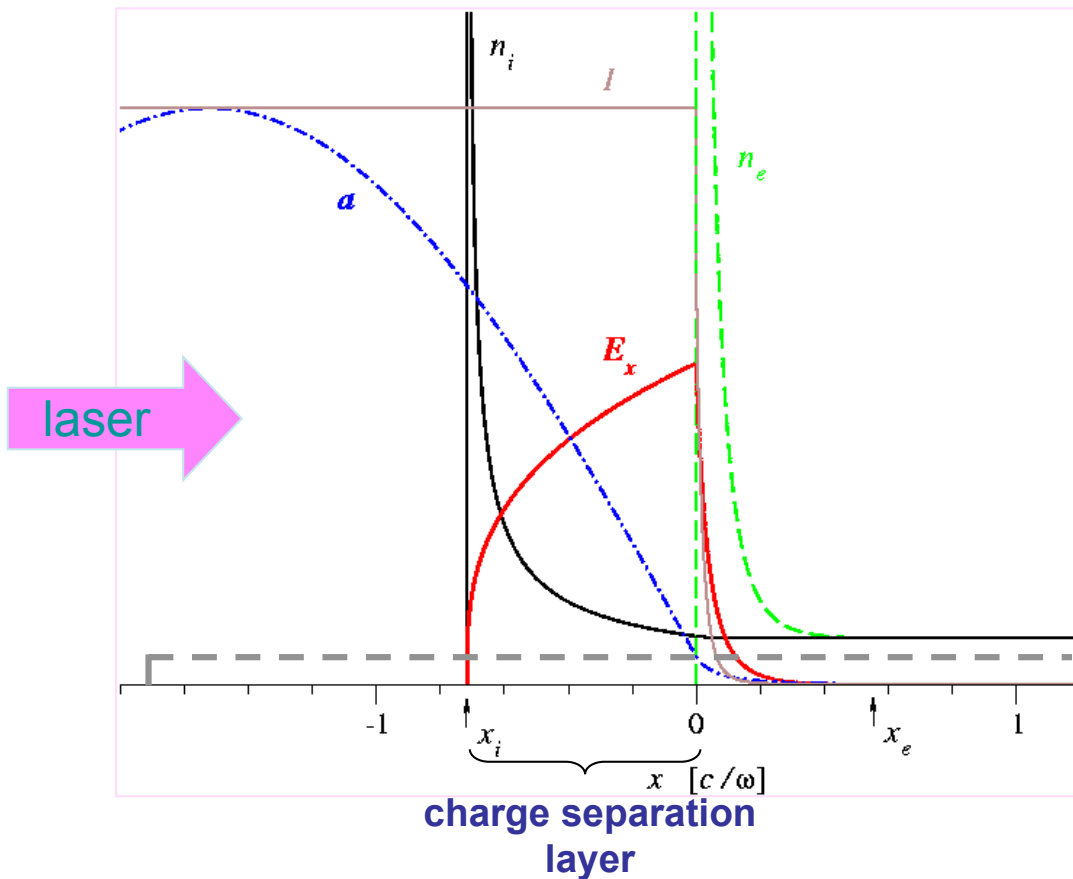
The piston velocity

$$\beta_f = \frac{\sqrt{I_{\text{las}}}}{\sqrt{I_{\text{las}}} + \sqrt{\rho c^3}}$$

Ion energy and the efficiency of ion acceleration are defined by the piston velocity

$$\varepsilon_i = 2 m_i c^2 \beta_f^2 \gamma_f^2$$

$$1 - R = \frac{2 \beta_f}{1 + \beta_f}$$



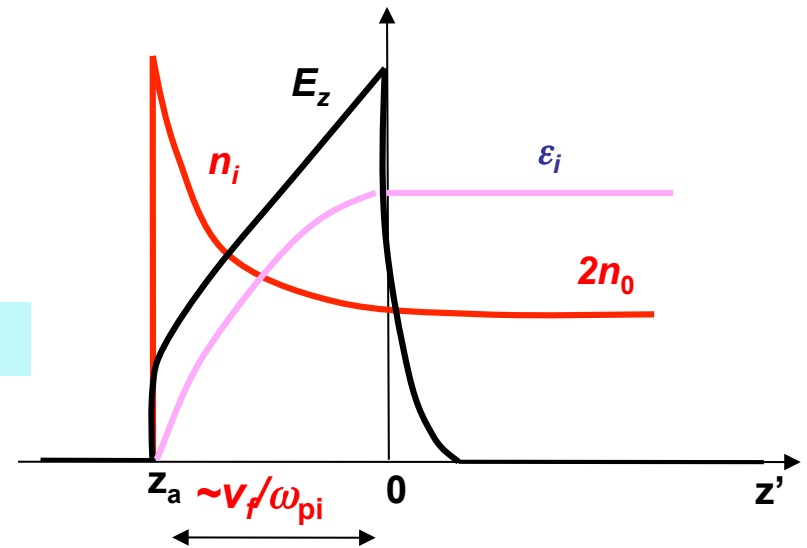
# Structure of the charge separation layer: electrostatic field and ion density distribution

The electrostatic field profile in the charge separation layer follows from the Poisson equation ( $n_e = 0$ )

$$\frac{d^2 \Phi'}{dz'^2} = - \frac{Z e n_i'}{\epsilon_0}$$

and the ion energy and mass conservation in the piston reference frame:

The first integral defines the electric field strength:



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# Thickness of the ion charge separation layer

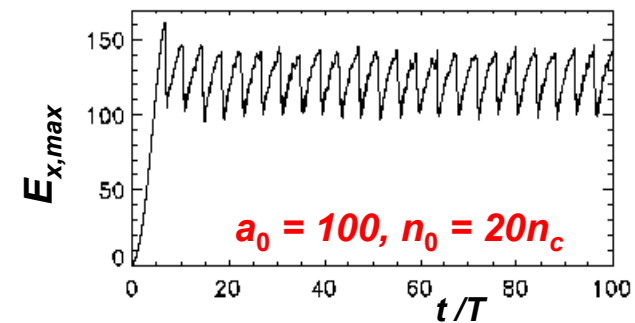
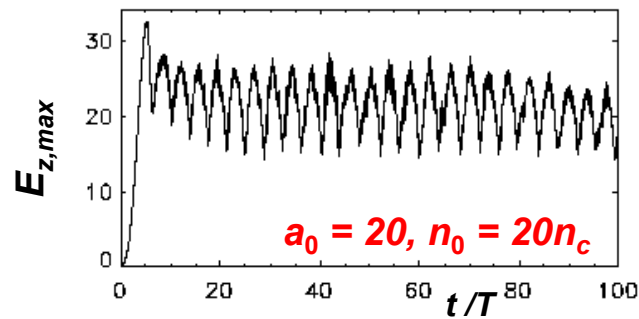
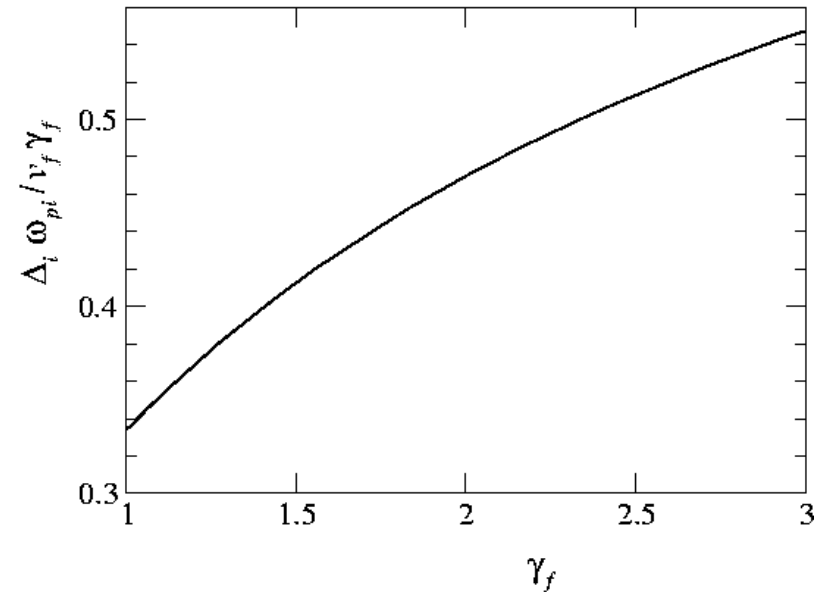
The thickness of the ion charge separation layer is proportional to the piston velocity (if  $\beta_f \ll 1$ )

$$\Delta z_i \cong \frac{a_0 c}{3\omega_0} \frac{n_c}{n_0}$$

The time of ion circulation is independent on the laser intensity

$$\Delta t_i \cong 2\gamma_f / \omega_{pi}$$

This time of ion circulation coincides exactly with the period of piston velocity oscillations found in the PIC simulations



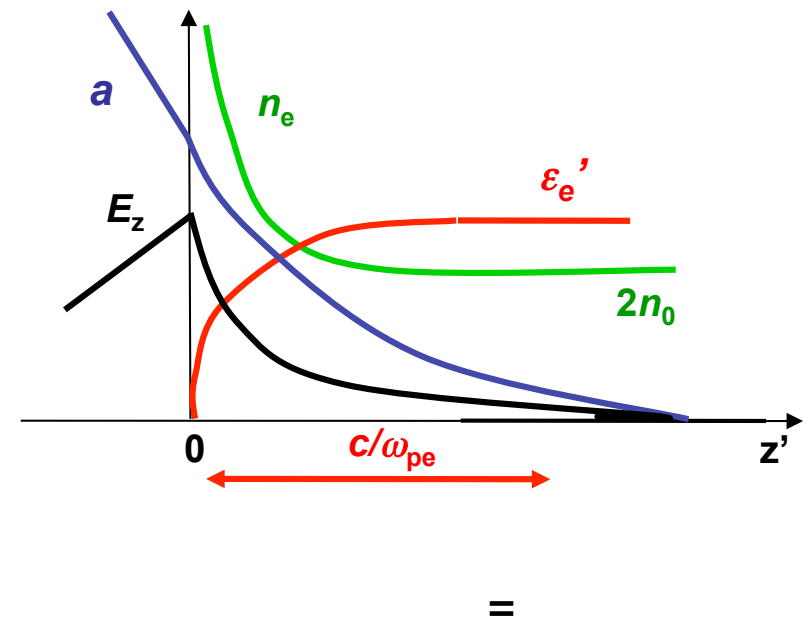


# Structure of the charge separation layer: laser field and electron density distribution

The electrostatic field profile in the charge separation layer follows from the Poisson equation

$$\frac{d^2\Phi'}{dz'^2} = -e \frac{Zn'_i - n'_e}{\epsilon_0}$$

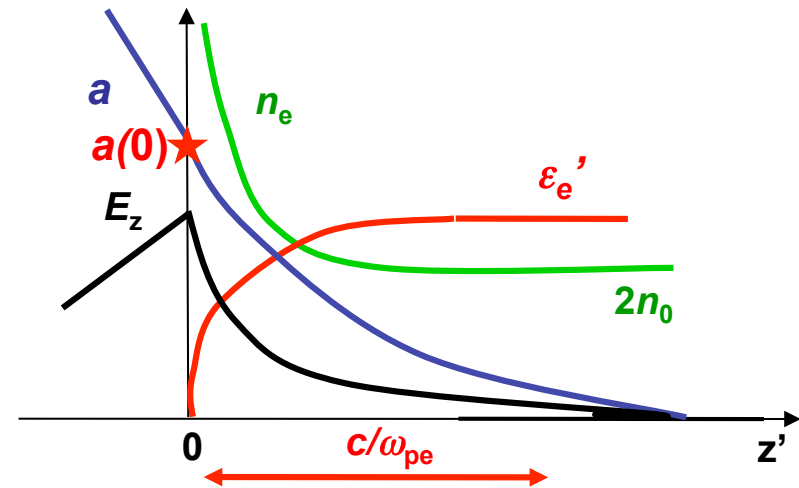
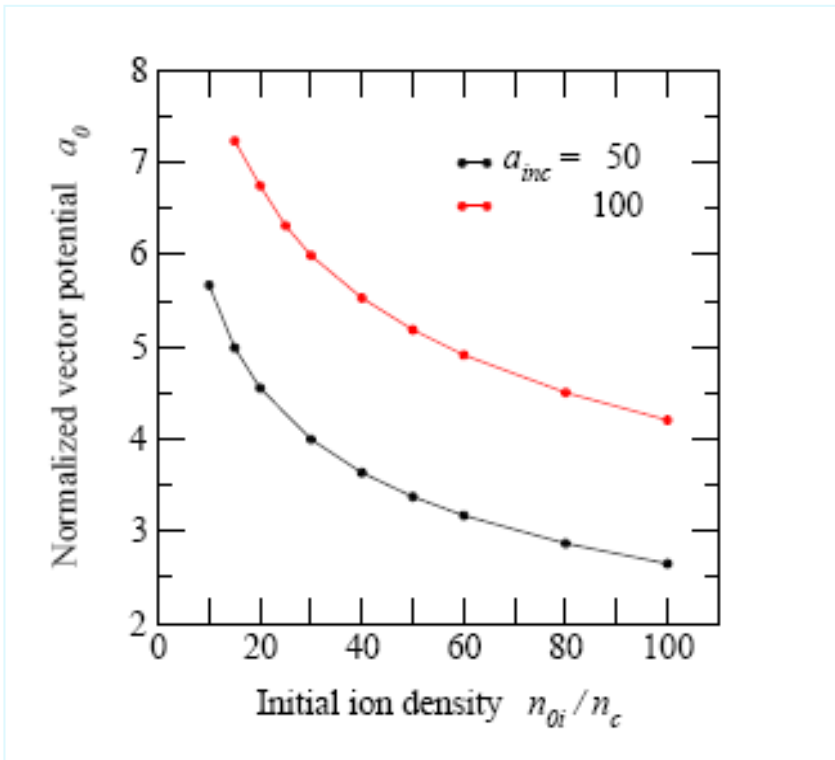
and the electron energy and mass conservation in the piston reference frame:



The electron layer thickness  $c/\omega_{pe}$  the laser field

# Laser amplitude in the electron charge separation layer

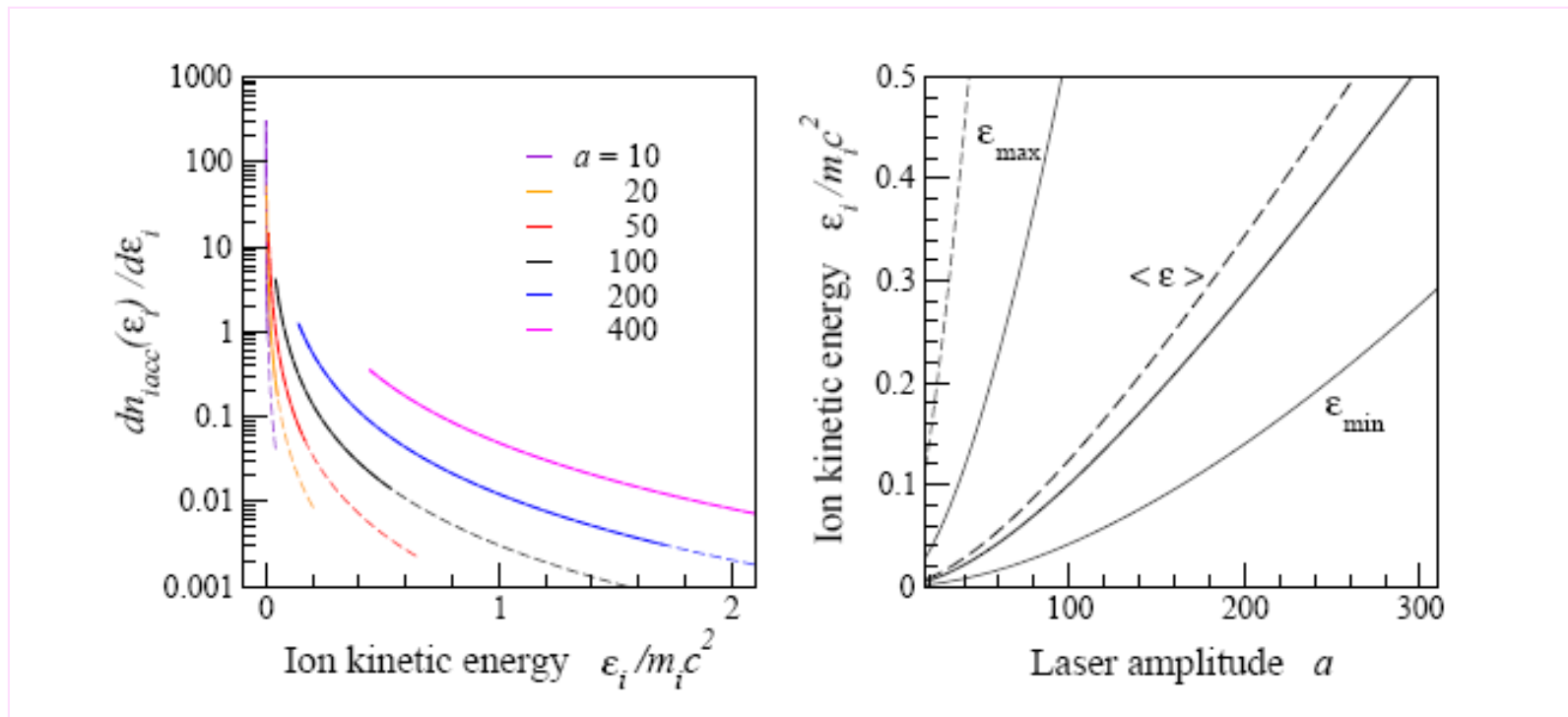
Laser amplitude on the board of the electron charge separation layer  $a(0)$  is adjusted self-consistently in such a way that the ponderomotive force is equal to the electric force:  $F_{pf} = e E_{z \max}$   
 It decreases slowly with the plasma density



A very tight balance between the ponderomotive potential and the electrostatic field can make the electron confinement unstable

# Ion energy spectrum in inhomogeneous plasma

Ions are mono-energetic in a homogeneous plasma, in an **exponential density profile** the ions are a power spectrum



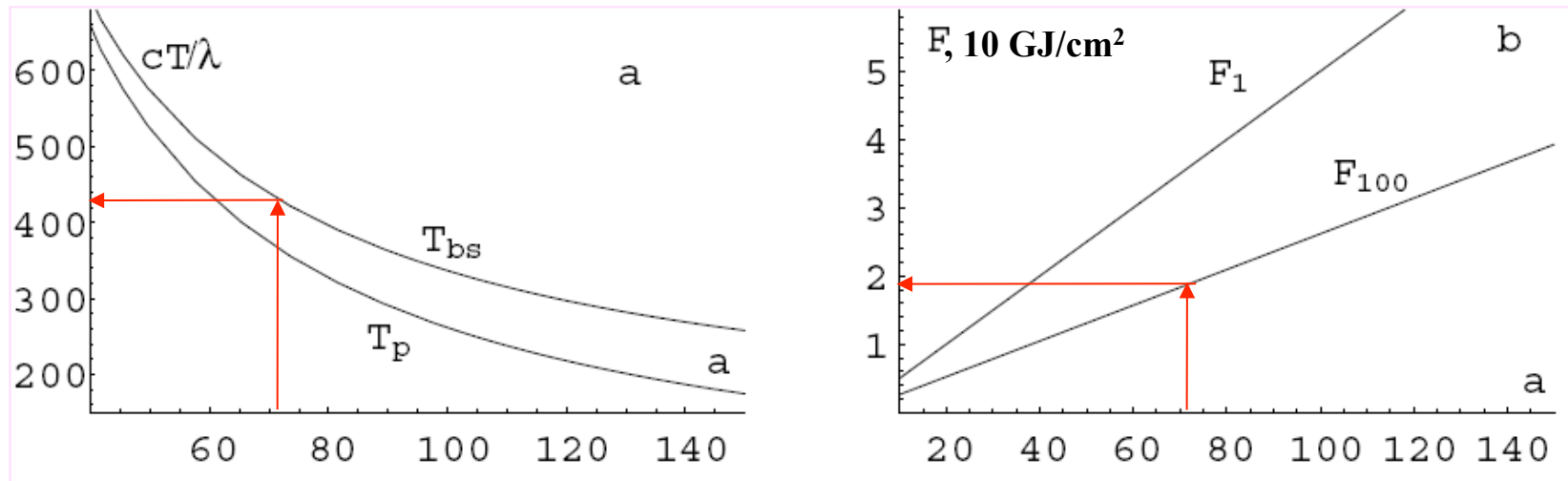
Deuteron spectra in a plasma with the density increasing from 1 to  $100n_c$

$$\frac{dN_i}{d\epsilon} = \frac{I_{\text{las}} L}{2m_i^2 c^5} \frac{1}{\beta_f^4 \gamma_f^6 (1 + \beta_f)}$$

$$\langle \epsilon_i \rangle = \frac{4I_{\text{las}}}{n_{i\text{max}} c} \ln \frac{\beta_{f\text{min}}}{\beta_{f\text{max}}}$$

# Time of hole boring and laser fluence

Analytical formulas provide the scalings for design of the fast ignition parameters



$F_{100} = I_{inc} T_p$  is the laser fluence needed for accelerate ions from the density increasing from 1 to  $100n_c$  over the length of  $100\lambda$ ,  $F_1$  is the same for the density range 0.1 to  $1n_c$  over the length of  $1000\lambda$

$$F_{las} = \sqrt{m_i c I_{las}} \int \frac{L dn_i}{\sqrt{n_i}} \quad F_i \approx \int L \varepsilon_i(n_i) dn_i$$

# Example of 1D PIC simulation

$$a_0 = 20, \quad I_0 = 1.6 \times 10^{21} \text{ W/cm}^2, \quad n_0 = 20 n_c$$

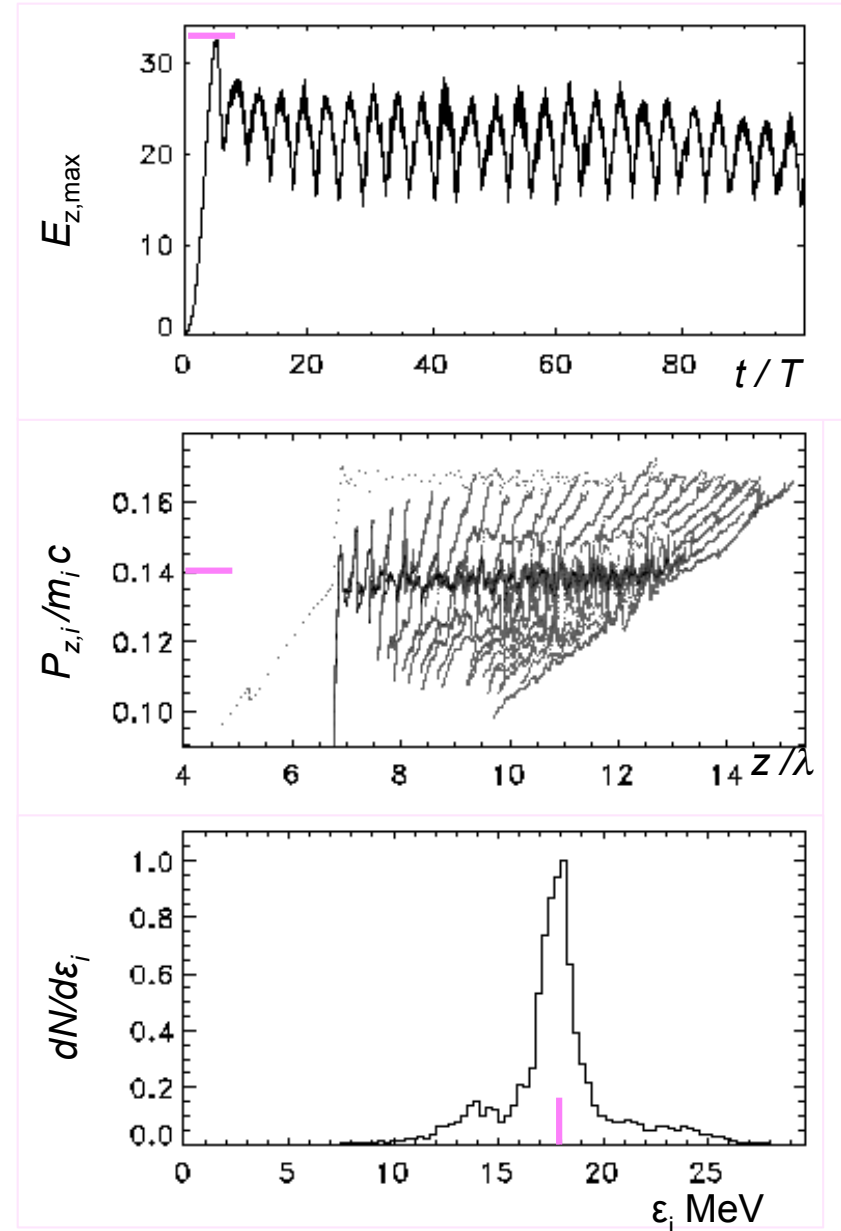
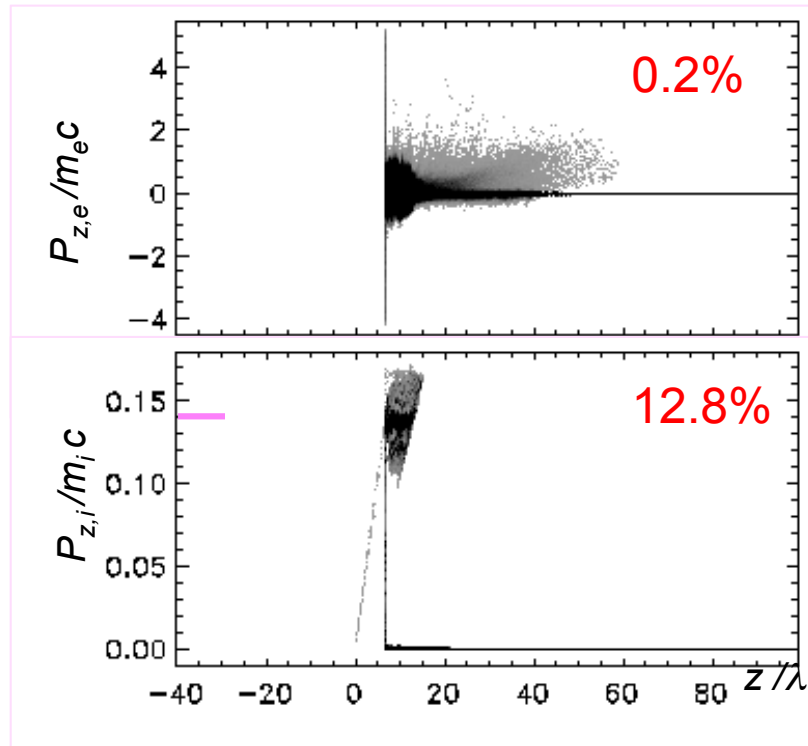
Estimates:

$$1 - R = 0.129$$

$$\beta_f = 0.07 \quad T_b = 93T$$

$$eE_z / m_e \omega_0 c = 37$$

$$p_i / m_i c = 0.14 \quad \epsilon_i = 18 \text{ MeV}$$



# 1D & 2D PIC simulations of ion acceleration & hole boring

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## Laser pulse:

$$a_{\text{inc}} = 100$$

circularly polarized

$$I_{\text{inc}} = 4 \times 10^{22} \text{ W/cm}^2$$

$$\tau = 188 \lambda/c$$

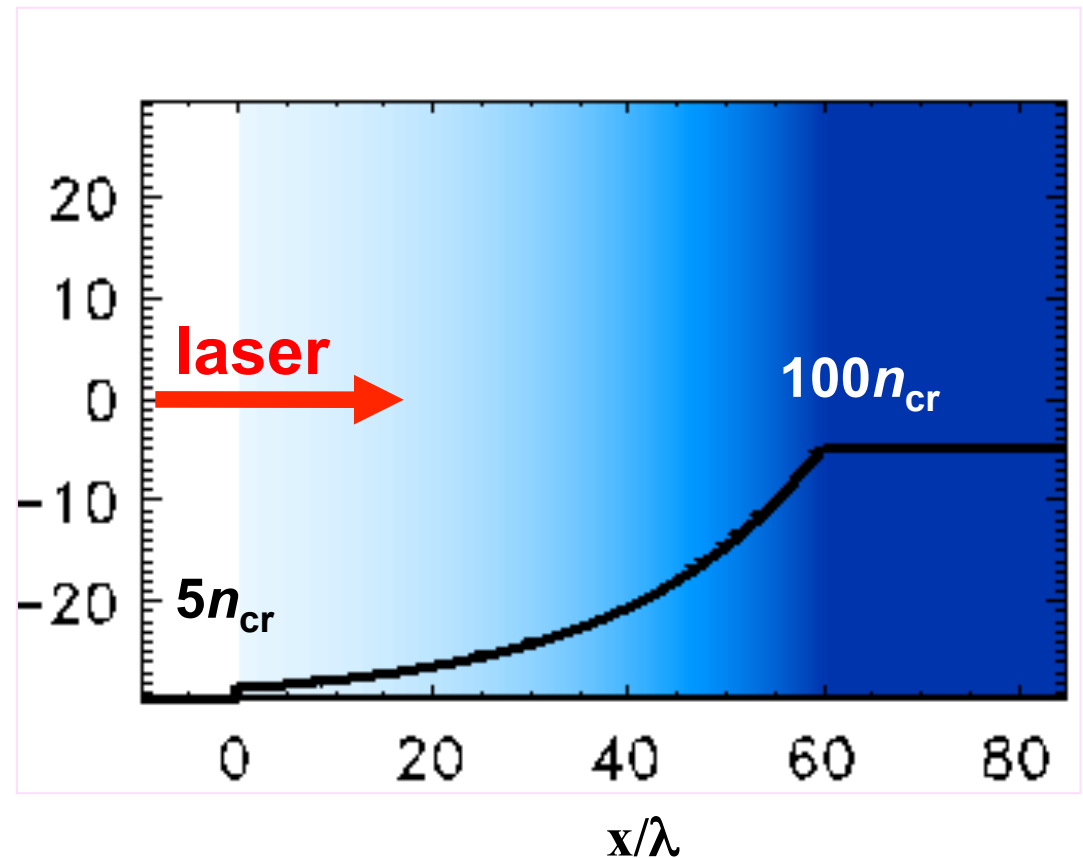
2D:  $d/\lambda = 20$  flat-top  
with exp. wings

## Plasma: deuterium

exponential profile

$$L_p/\lambda = 60; L/\lambda = 20$$

$$n_0/n_{\text{cr}} = [5-100]$$

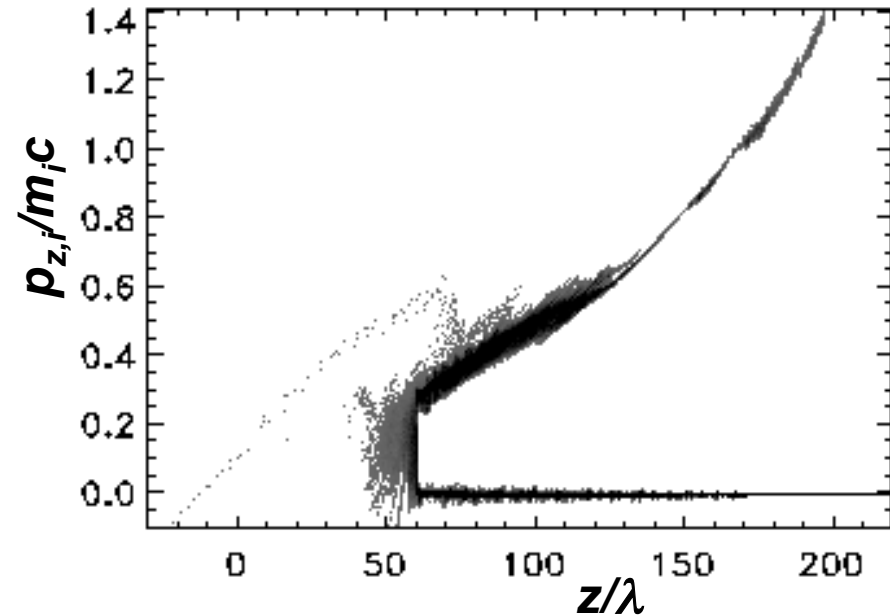
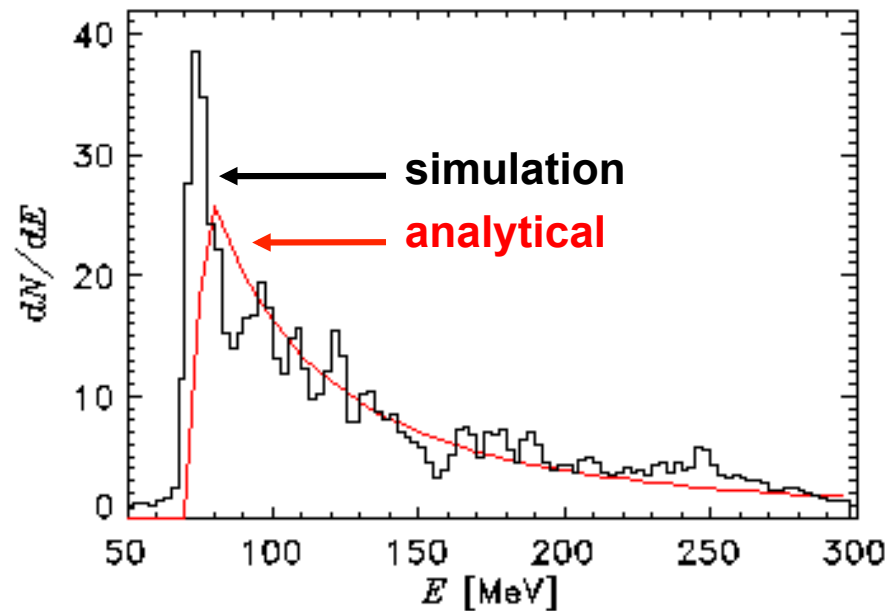


# 1D PIC simulation – ion energy distribution

$t = 250 \lambda/c$  end of acceleration

Laser fluence: 20 GJ/cm<sup>2</sup>  
Ions: 5.4 GJ/cm<sup>2</sup> (27%)  
Electrons: ~1%  
High energy photons: ~ 10%

The ions are gaining the main part of laser energy, the electrons remain cold due to the radiation losses



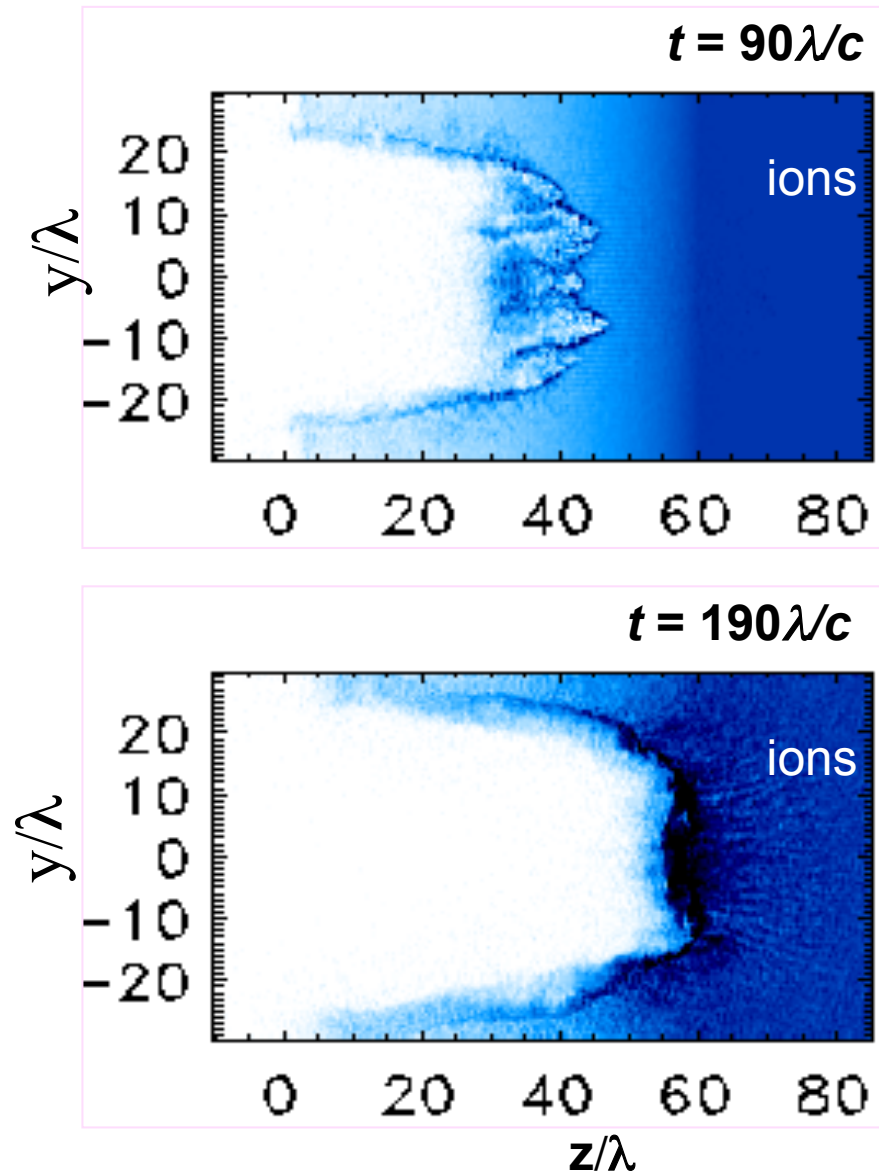
## 2D PIC simulation – channel formation

### Flat-top laser intensity profile

Ion density distribution demonstrates an efficient hole boring in the plasma, a clean and a stable channel

Filamentation is strongly suppressed due to radiation losses

Velocity of hole boring is in agreement with the 1D model



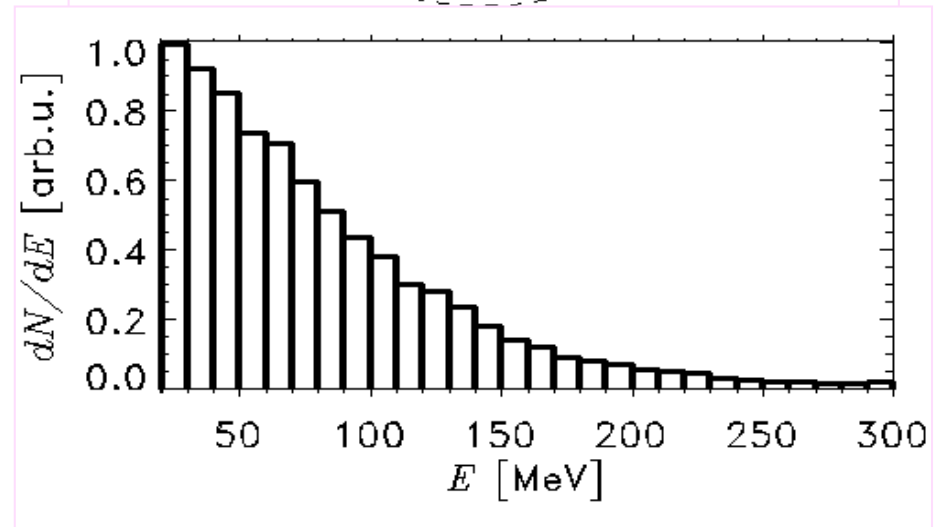
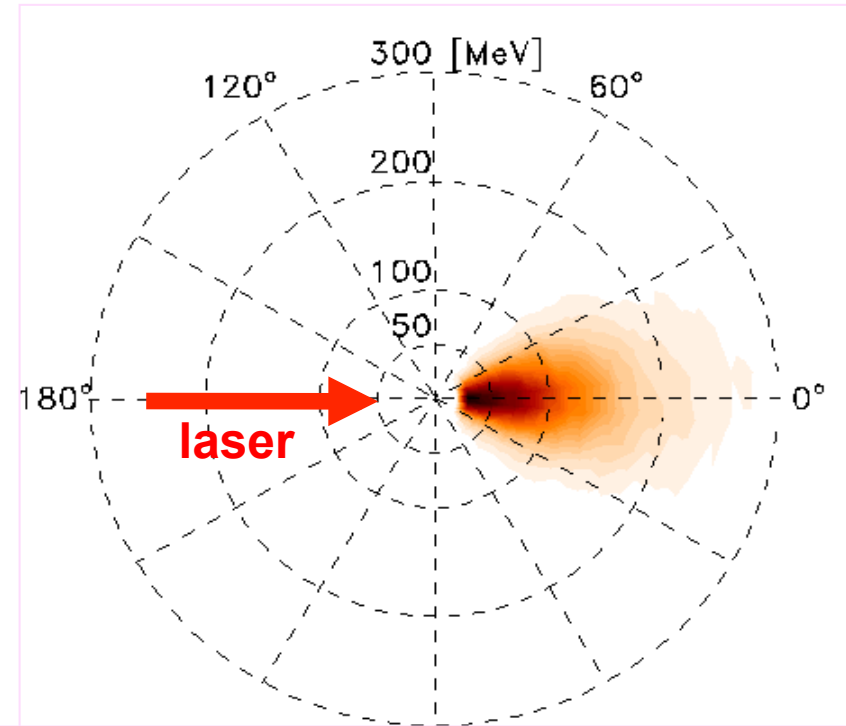


## 2D PIC simulation – ion energy distribution and angular spread

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Angular distribution of ions vs energy at the final instant at  $|y/\lambda| < 10$  shows a narrow peak in forward direction

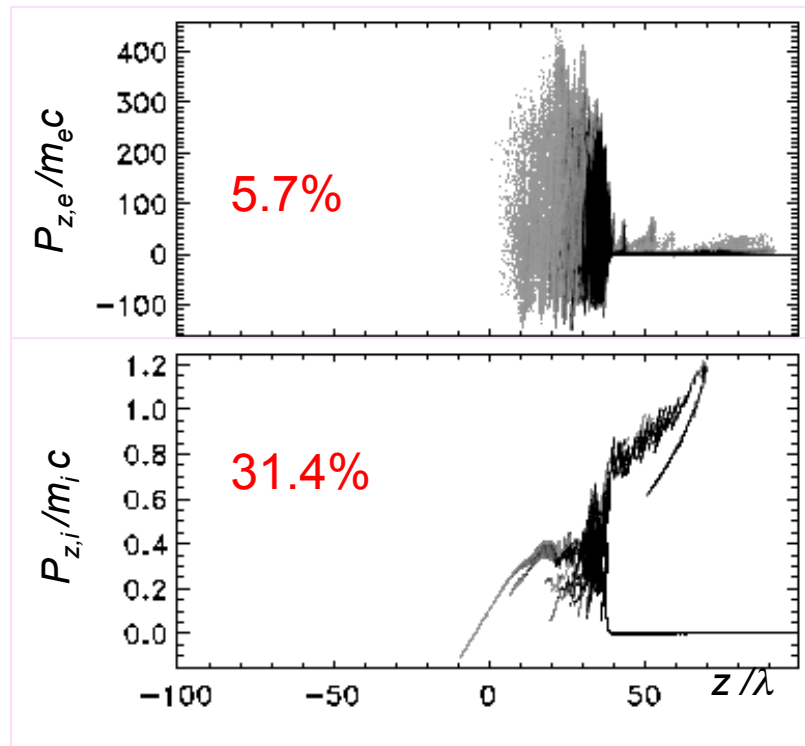
Energy distribution in the central part (a cone of  $6^\circ$ ) agrees well with 1D PIC simulations and analytical model



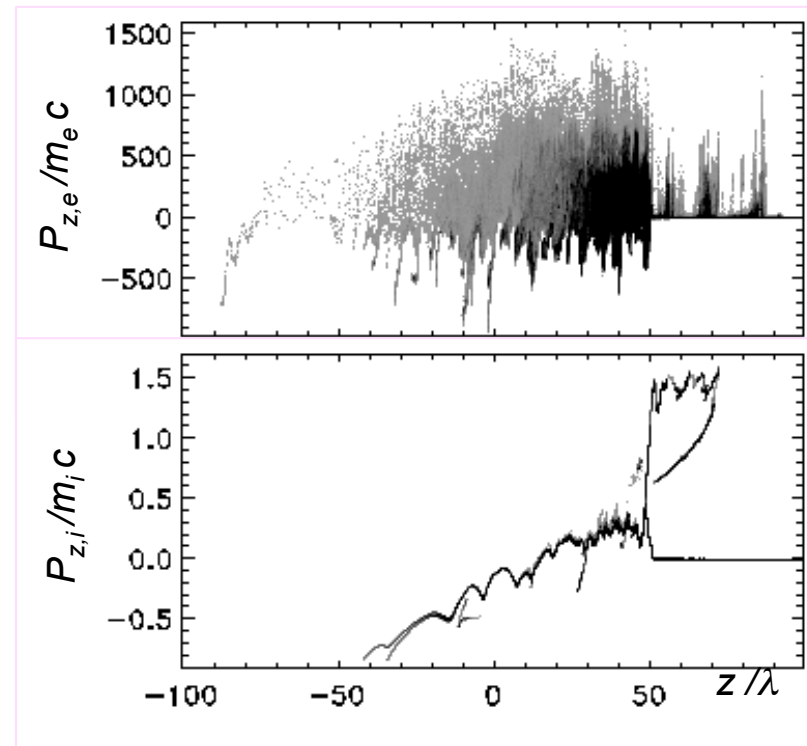
# 1D PIC simulations with/without radiation reaction

$a_0 = 100$ , circular polarization,  $t = 100T$ , plasma:  $n_0 = 10n_c$ ,  $m_i = 2m_p$

radiation losses : 43%



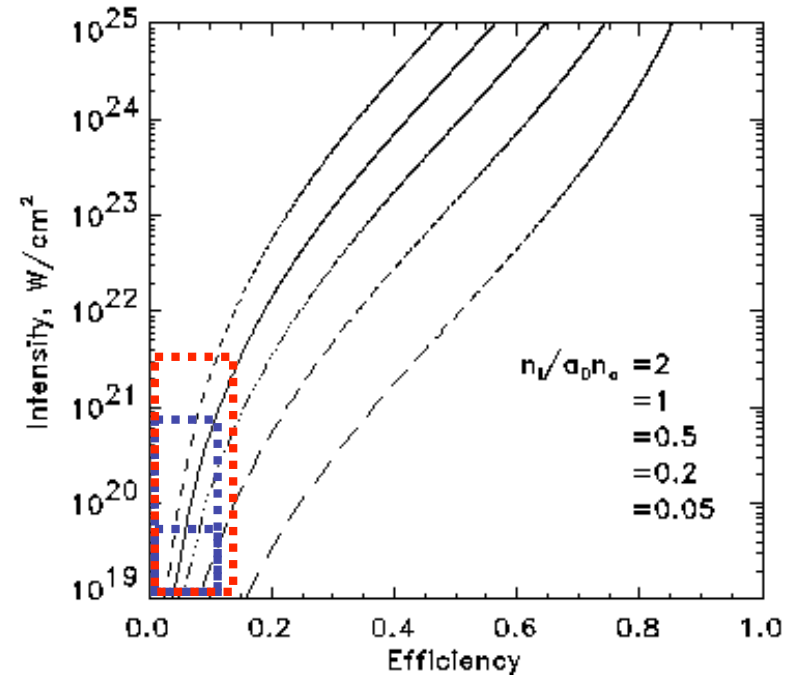
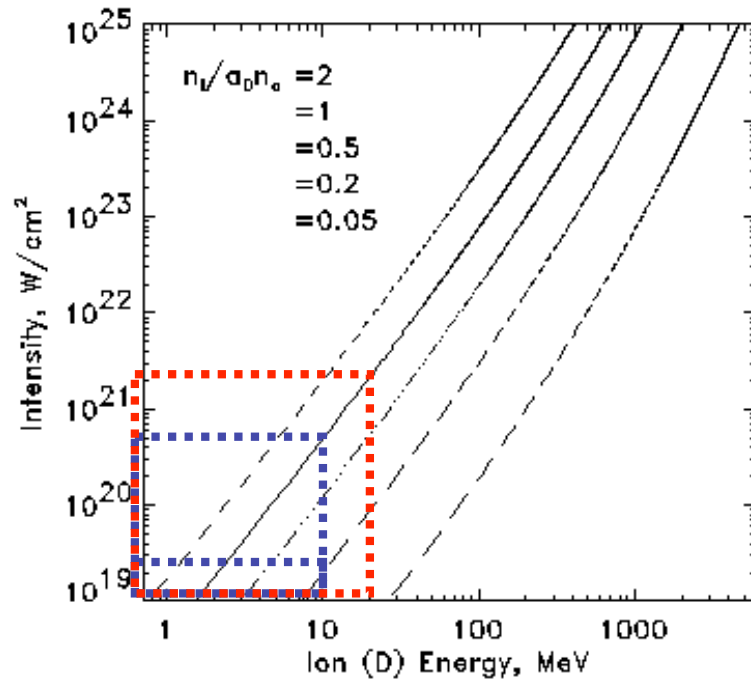
without radiation losses



Electrons escape the ponderomotive potential if the ratio  $a_0/(n_0/n_c)$  becomes too large.

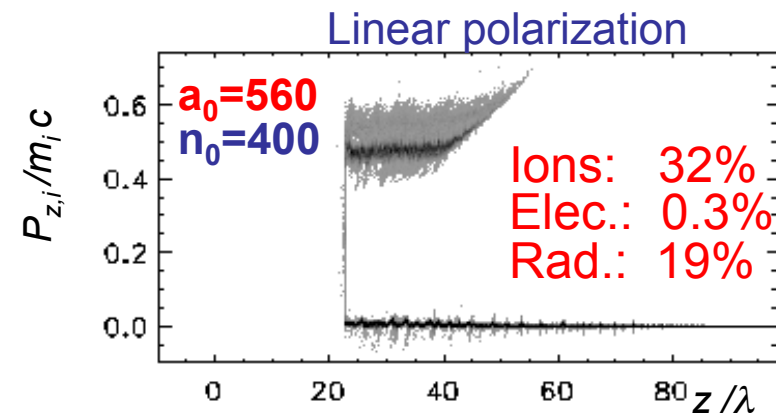
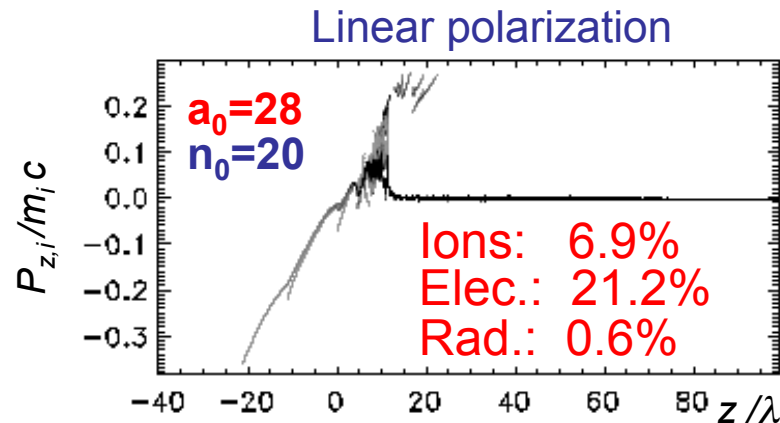
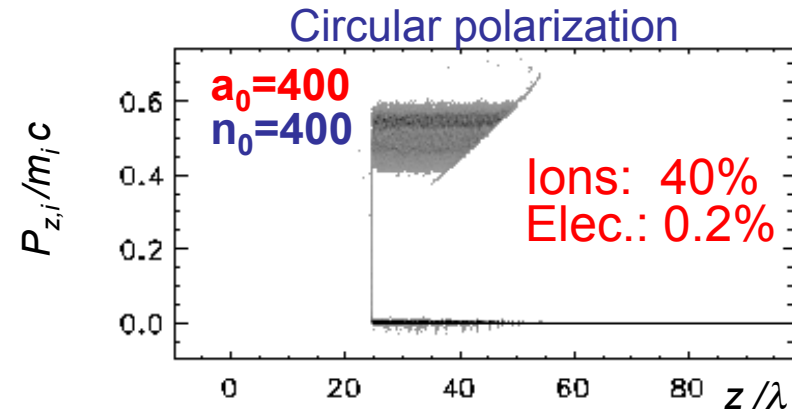
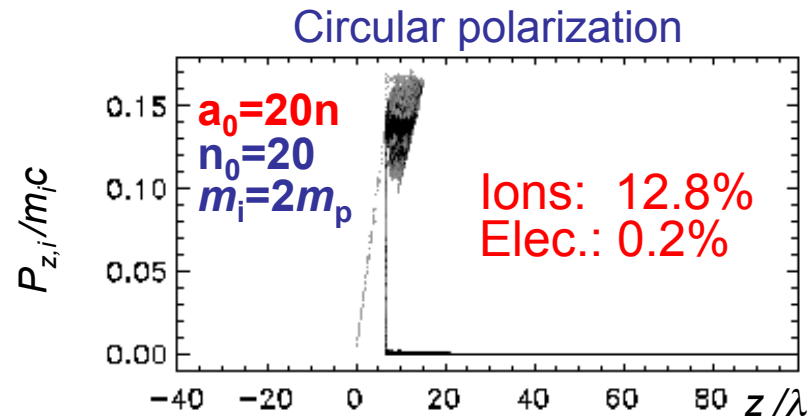
The radiation losses stabilize the piston

# Laser piston model: ion energy and efficiency vs intensity



1. Needed ion energy determines a ratio  $a_0^2/n_e$ .
2. Needed laser energy deposited does not depend on laser intensity  $E_{\text{laser}} \sim I_{\text{laser}} T_p$ , **but:**
3. Higher intensities allow to obtain more energetic ions as they allow to avoid **transparency** regime:  $n_e/a_0 n_c \sim a_0$ , **limiting** the applicability of the piston model.

# Ion acceleration in bulk targets for circular and linear polarization at various intensities



At super-high intensity the difference between linear and circular polarization is reduced in respect of conversion efficiency to ions.

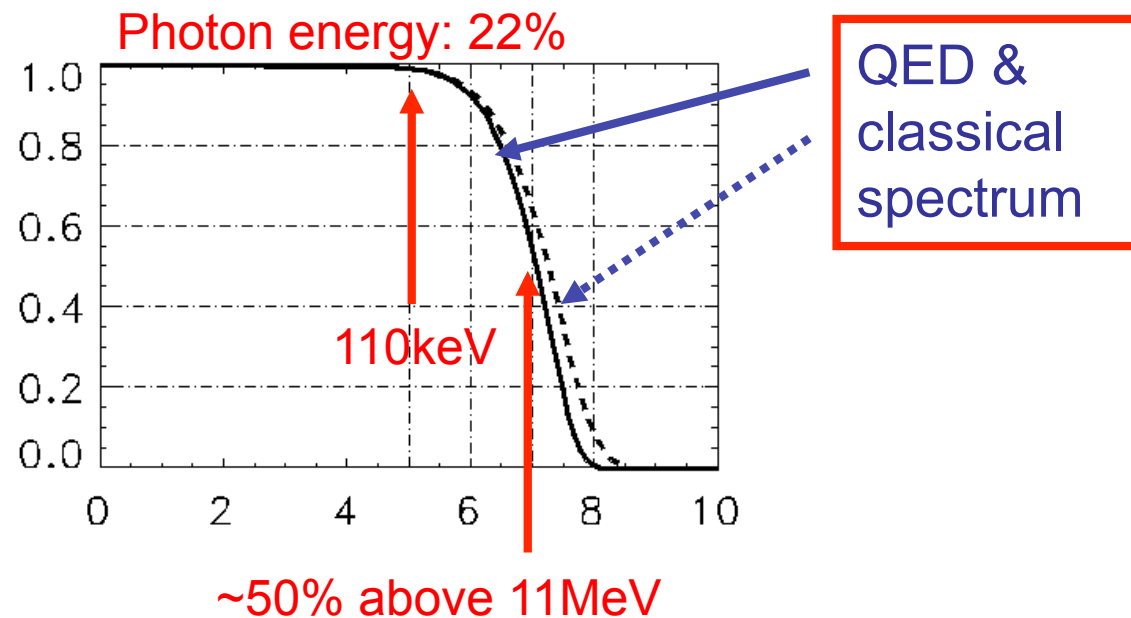
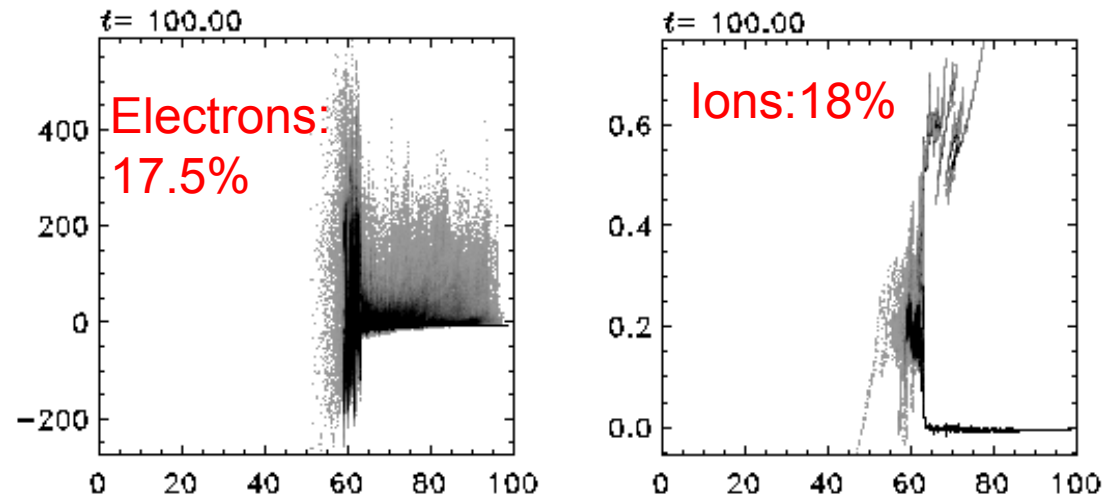
Nevertheless the radiation losses for linear polarization are high even for dense targets.

# Super-intense laser-plasma interaction: QED affects the radiation spectrum

$I_0 = 10^{23} \text{ W/cm}^2$   
 $a_0 = 220$   
Linear pol.  
 $n_0 = 100 n_c$   
 $m_i = 2 m_p$

## Estimates:

$1 - R = 0.33$   
 $\beta_f = 0.2$   
 $T_b = 40T$   
 $p_i/m_i c = 0.43$



# Conclusions

- Ultra-high intensities accessible with ELI are attractive due to high conversion efficiency of ion acceleration.
- Quasi-stationary laser piston model describes the basic features of ion acceleration.
- PIC simulations reveal non-stationary effects of laser-plasma interaction:
  - (1) oscillations of the piston velocity which leads to broadening of ion spectrum
  - (2) electron cooling which helps to maintain an efficient ion acceleration

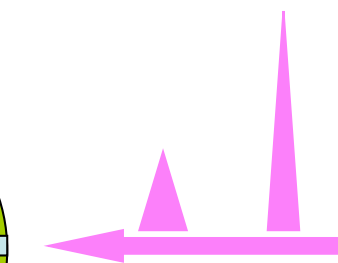
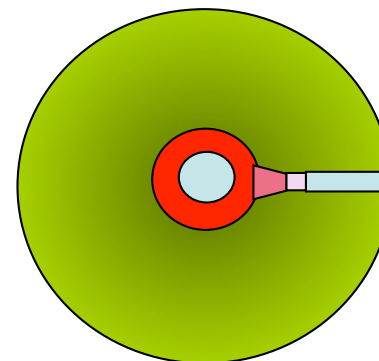
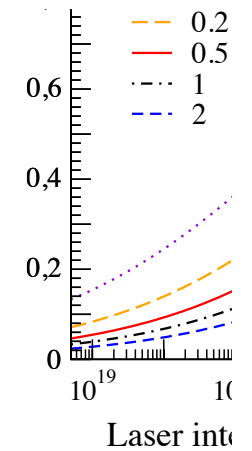
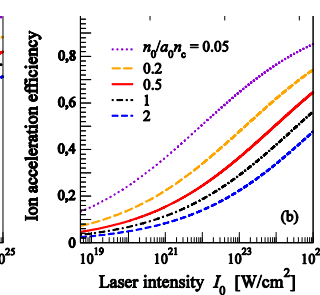
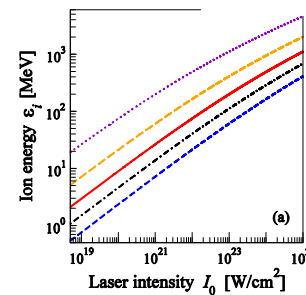
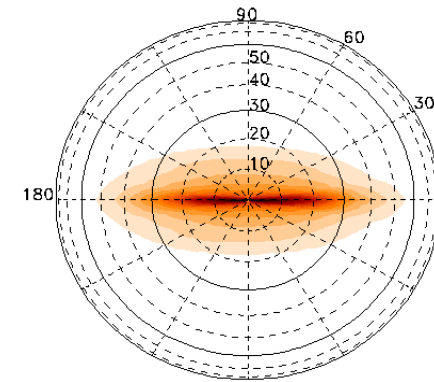
# Applications:

**Efficient sources** of gamma-rays can be designed by proper modelling of the radiation back-reaction

**The developed laser piston model** for bulk targets, allows to design ion beams

**A new competitive** fast ignitor scheme has been proposed, using

- hole boring by multiple pulses and
- acceleration of target ions driven by ultra-high intensity pulses



Collaboration **ELI & HiPER**

Poster : T.Schlegel,  
Tikhonchuk et al.(2010)