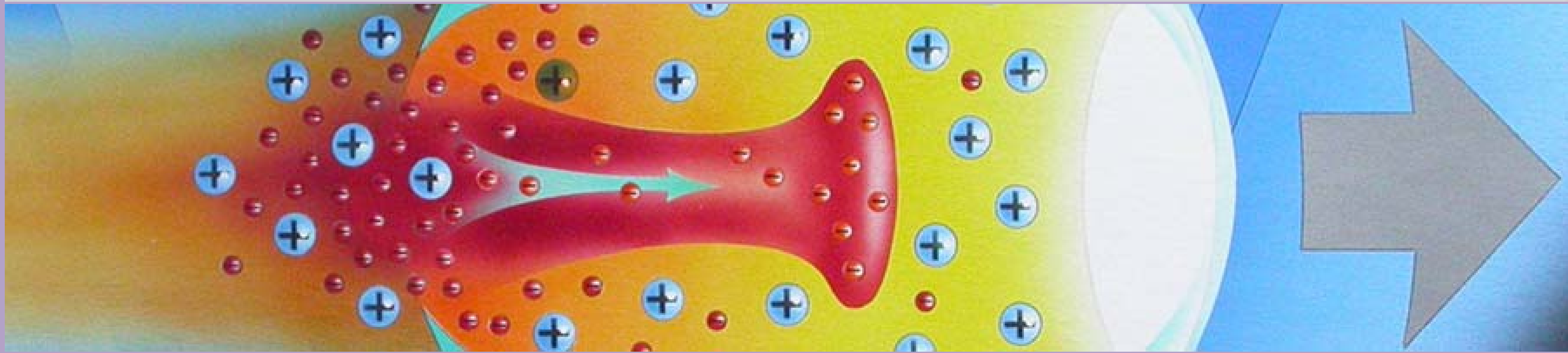

Fundamental QED processes in ultra-intense laser field

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Synchrotron radiation (Ivanenko, Pomeranchuk, Schwinger, et. al.)

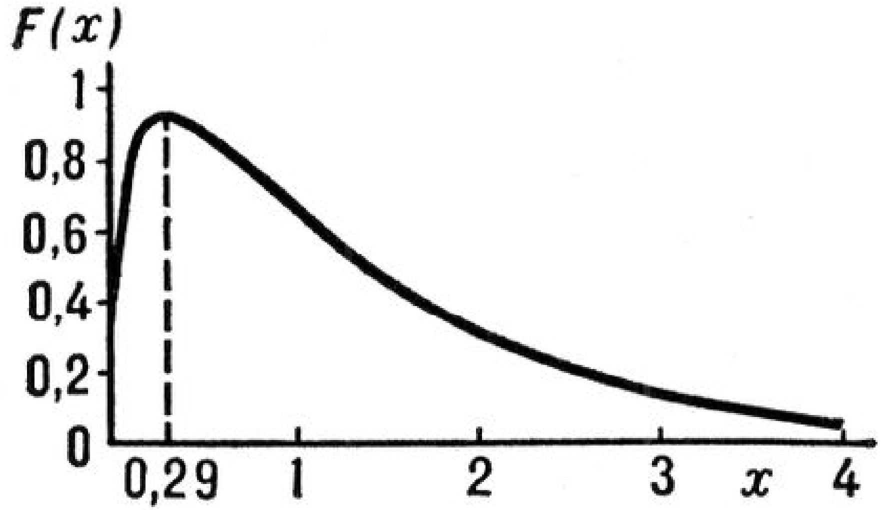
$$dI_{rad} = \frac{\sqrt{3}}{2\pi}\frac{e^3 H}{mc^2} F\left(\frac{\omega}{\omega_c}\right)\, d\omega \qquad F(x) = x\int_x^\infty K_{5/3}(x)\, dx$$

•Specific features:

$$\theta \lesssim \gamma^{-1}$$

$$\omega_c = \frac{3eH\gamma^2}{2mc}$$

$$I_{rad} = \frac{2e^4H^2\gamma^2}{3m^2c^3}$$



•In the laser field, $\gamma \sim a_0 = \dfrac{eE}{mc\Omega}, \quad E \sim H$

•Classical Radiation Damping regime (LAD, LL etc.):

$$I_{rad} \times \frac{\lambda}{c} \sim \gamma m c^2$$

$$a_R \sim \left(\frac{3mc^3}{4\pi e^2 \Omega}\right)^{1/3} \sim 300$$

$$I_L \sim 10^{23} \mathrm{W/cm^2}$$

Radiation reaction should be taken into account, e.g., via the Landau-Lifshitz equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2e^3}{3m}\frac{\partial F^{\mu\nu}}{\partial x^\lambda}u_\nu u^\lambda - \frac{2e^4}{3m^2}F^{\mu\nu}F_{\lambda\nu}u^\lambda$$

$$+\frac{2e^4}{3m^2}(F_{\nu\lambda}u^\lambda)(F_{\nu\sigma}u^\sigma)u^\mu$$

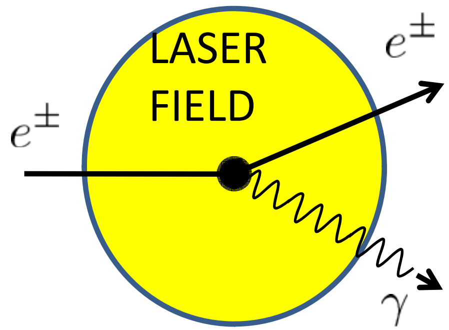
•Quantum radiation damping regime:

$$\hbar\omega_c \sim \gamma mc^2$$

$$a_Q \sim \sqrt{\frac{2mc^2}{3\hbar\Omega}} \sim 600$$

$$I_L \sim 10^{24}\mathrm{W/cm}^2$$

Intense field QED (IFQED) must be applied!



Sketch of IFQED:

Step I

Solve $$\{i\gamma^\mu[\partial_\mu - ieA_\mu^{(ext)}(x)] - m\}\Psi(x) = 0$$

Step II

Calculate amplitude, e.g.

$$A_{i\to f} = -ie\int d^4x\bar{\Psi}_f(x)\frac{(\gamma^\mu\epsilon_\mu^*)}{\sqrt{2\omega}}e^{ikx}\Psi_i(x)$$

Step III

Calculate probability, e.g.

$$dP_{i\to f} = |A_{i\to f}|^2\frac{d^3p'}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}$$

IFQED parameters:

$a_0 \gg 1$ -field can be considered ***constant***

$$-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = E^2 - H^2 \ll E_S^2$$
$$\frac{1}{8}\epsilon_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa} = \mathbf{E}\cdot\mathbf{H} \ll E_S^2$$

-field can be considered ***crossed*** $E = H,\ \mathbf{E} \perp \mathbf{H}$

Motion is quasiclassical!!!

$$E_S = \frac{m^2c^3}{e\hbar} = 1.32\cdot 10^{16}\frac{V}{cm}$$

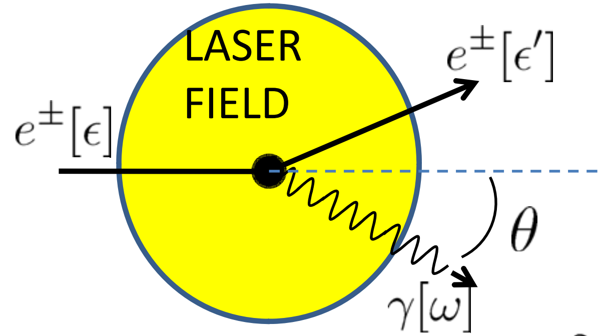
$$\Psi \sim e^{iS}$$

Dynamical quantum parameter:

$$\chi = \frac{e\hbar}{m^3}\sqrt{-(F_{\mu\nu}p^{\nu}_{ini})^2} = \frac{\gamma\sqrt{(\mathbf{E}+\mathbf{v}\times\mathbf{H})^2 - (\mathbf{E}\cdot\mathbf{v})^2}}{E_S}$$

$$= \frac{E_{proper\,frame}}{E_S} = \begin{array}{c}\text{proper acceleration}\\ \text{in Compton units}\end{array}$$

Photon emission:



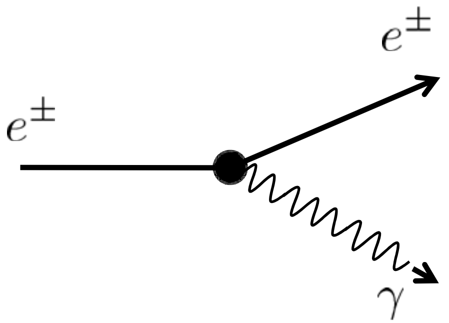
Total probability rate

$$W_{rad} = -\frac{\alpha m^2 \chi_e}{2\epsilon} \int_0^\infty \frac{dx}{\sqrt{x}} \frac{5 + 7\chi_e x^{3/2} + 5\chi_e^2 x^3}{(1 + \chi_e x^{3/2})^3} \mathrm{Ai}'(x)$$

$$= \begin{cases} 1.443 \dfrac{\alpha m^2 \chi_e}{\epsilon} \left(1 - 0.9238\chi_e + \ldots\right), & \chi_e \ll 1, \\ 1.461 \dfrac{\alpha m^2 \chi_e^{2/3}}{\epsilon}, & \chi_e \gg 1. \end{cases}$$

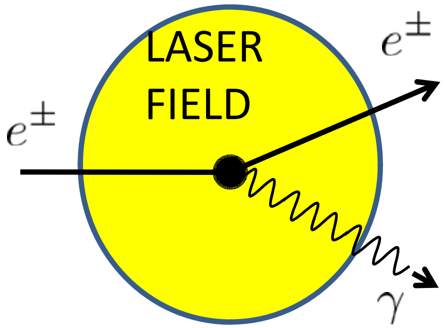
The concept of classical radiation force overestimates radiation damping in the quantum regime!

Formation length/time of a quantum process



Energy lack

$$\Delta\epsilon = \sqrt{(\mathbf{p} - \hbar\mathbf{k})^2c^2 + m^2c^4} + \hbar k - \sqrt{p^2c^2 + m^2c^4} \sim mc^2$$

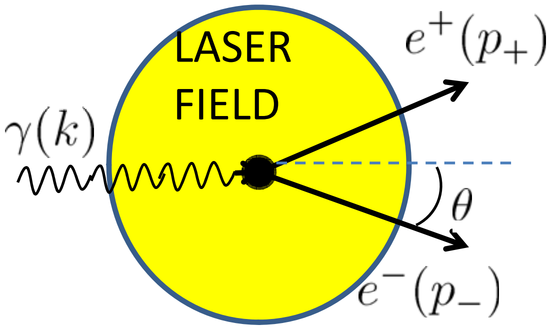


Work required from the laser field

$$A = eEl_{form} = \Delta\epsilon \sim mc^2$$

$$l_{form} \sim \frac{mc^2}{eE} \qquad t_{form} \sim \frac{mc}{eE}$$

Pair creation:



Quantum amplitude:

$$A_{i\to f} = -ie\int d^4x\bar{\Psi}_-(x)\frac{(\gamma^\mu\epsilon_\mu)}{\sqrt{2\omega}}e^{-ikx}\Psi_+(x)$$

Differential probability:

$$dP_{i\to f} = |A_{i\to f}|^2\frac{d^3p'}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}$$

$$W_{cr} = \begin{cases} 0.23\dfrac{\alpha m^2\chi_\gamma}{\omega}\exp\left(-\dfrac{8}{3\chi_\gamma}\right), & \chi_\gamma \ll 1, \\ 0.38\dfrac{\alpha m^2\chi_\gamma^{2/3}}{\omega}, & \chi_\gamma \gg 1. \end{cases}$$

Under optimal conditions (χ~1) the rates W_{rad} and W_{cr} are merely comparable!

$$\chi = \frac{E_{proper frame}}{E_S}$$

↓

$$E_{||lab} \sim E_{\perp lab}$$

→

Lorentz tranformations:

$$E_{||proper} \sim E_{||lab}$$

$$E_{\perp proper} \sim \gamma E_{\perp lab}$$

→

$$E_{proper} \sim \gamma E_{\perp lab}$$

↓

$$\chi = \frac{\gamma E_{\perp lab}}{E_S}$$

χ~1 either if

- $E \sim E_S$,

or even if

- $E \ll E_S$ but $\gamma \gg 1$!

SLAC experiment

D.L.Burke, *et al.*, PRL, 79, 1626 (1997)
C.Bamber, *et al.*, PRD, 60, 092004(1999)

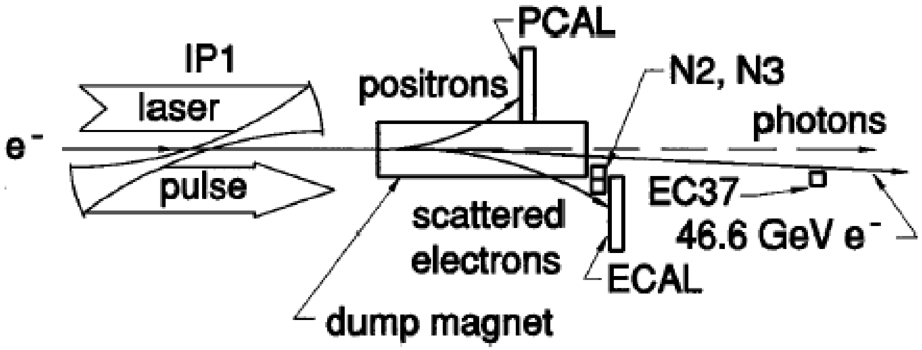


FIG. 1. Schematic layout of the experiment.

$$s\omega + e^- \to e^- \gamma$$

$$s\omega + \gamma(\theta = \pi) \to e^- e^+$$

$$\lambda = 0.527\mu m, I \approx 1.3 \times 10^{18} W/cm^2$$

Theory

(multiphoton regime)

A.I.Nikishov, V.I.Ritus, 1964

N.B.Narozhny, A.I.Nikishov, V.I.Ritus, 1964

maximal backscattered photon energy:

$$\varepsilon_\gamma = 29.2\, Gev$$

threashold:

$$s_{min} = 5 \qquad R_e \propto I^{s_{min}}$$

Experiment:

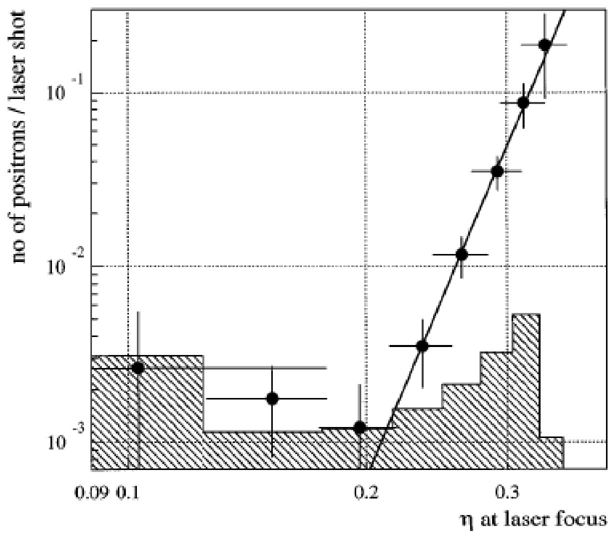


FIG. 4. Dependence of the positron rate per laser shot on the laser field-strength parameter η. The line shows a power law fit to the data. The shaded distribution is the 95% confidence limit on the residual background from showers of lost beam particles after subtracting the laser-off positron rate.

$$s = 5.1 \pm 0.2(stat)^{+0.5}_{-0.8}(syst)$$

CENTRAL POINT OF THE TALK:

$$\chi = \frac{\gamma E_\perp}{E_S}$$

For oscillatory motion in a laser fields on modern and perspective facilities,

$$\gamma \sim a_0 \gg 1!$$

But can particles be accelerated transversely to the field?

Pecularity of acceleration in general laser field: a toy model – uniformly rotating electric field

$$\frac{d\mathbf{p}(t)}{dt} = e\mathbf{E}(t), \quad \mathbf{p}(0) = 0$$

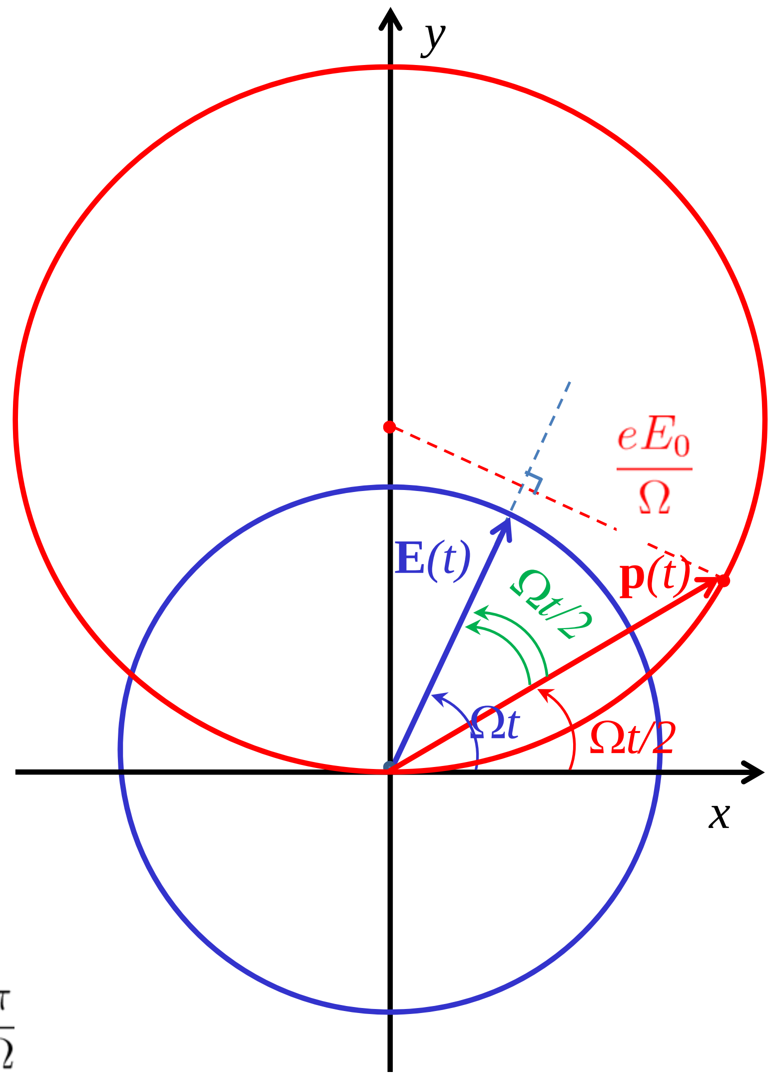
$\mathbf{E}(t)-$ uniformly rotating, Ω - rotation freq. (an analogue of laser frequency)

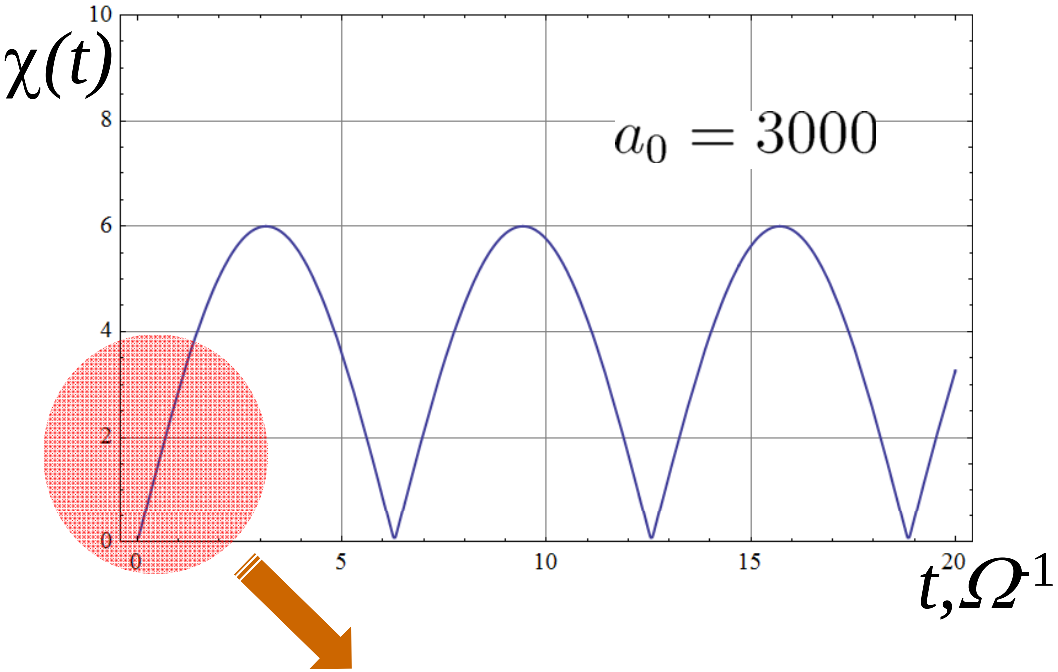
$$\epsilon(t) = mc^2\sqrt{1 + 4\left(\frac{eE_0}{\Omega mc}\right)^2 \sin^2\left(\frac{\Omega t}{2}\right)} \approx eE_0 t$$

$$\frac{mc}{eE_0} \ll t \ll \frac{\pi}{\Omega}$$

$$\chi(t) = \frac{E_0}{E_S}\sqrt{1 + 4\left(\frac{eE_0}{\Omega mc}\right)^2 \sin^4\left(\frac{\Omega t}{2}\right)} \approx \left(\frac{E_0}{E_S}\right)^2 \frac{mc^2\Omega t^2}{2\hbar}$$

$$\sqrt{\frac{E_S}{E_0}\frac{\hbar}{mc^2\Omega}} \ll t \ll \frac{\pi}{\Omega}$$



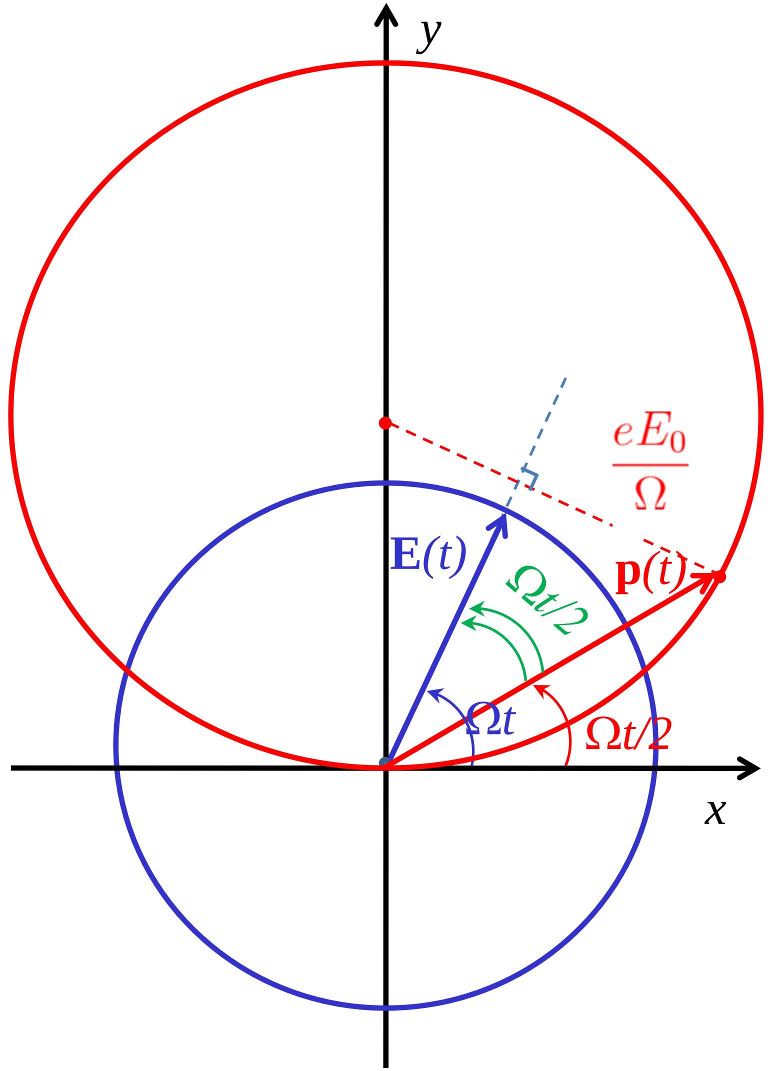


Initial segment of the curve:

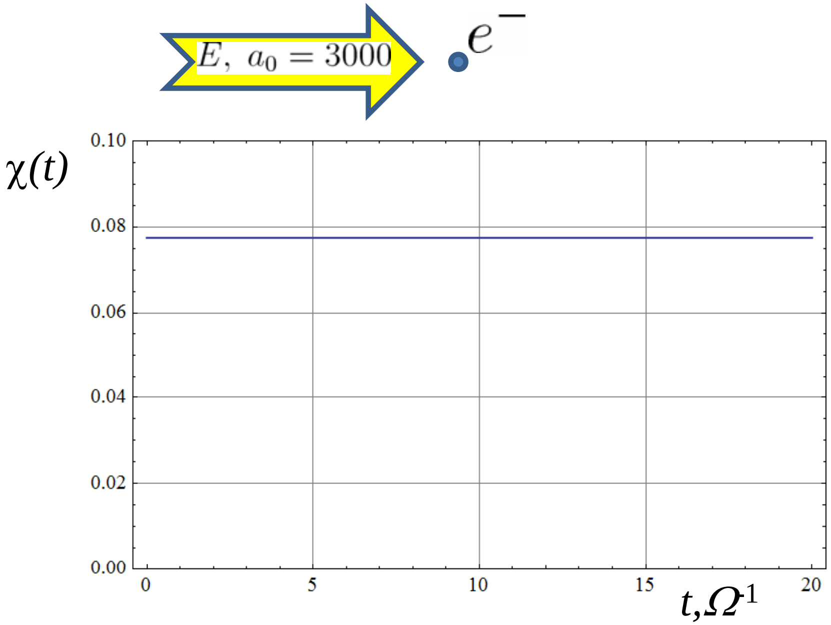
$$\chi_e(t) \sim \frac{E_\perp}{E_S}\frac{\epsilon}{mc^2} \approx \left(\frac{E_0\Omega t/2}{E_S}\right)\frac{eE_0 t}{mc^2} \sim 1$$

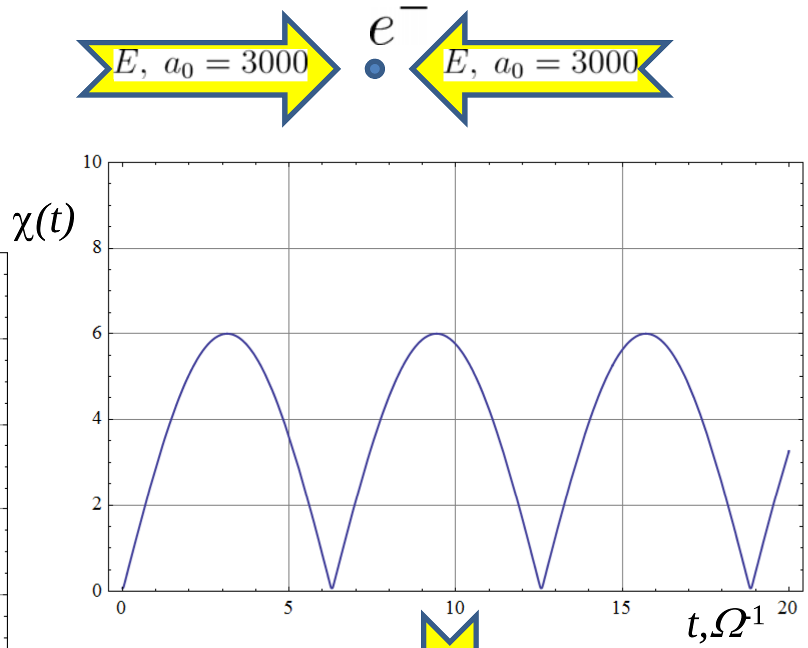
$$\text{at } t \sim t_{acc} = \frac{E_S}{E_0}\sqrt{\frac{\hbar}{mc^2\Omega}} \ll \Omega^{-1}$$

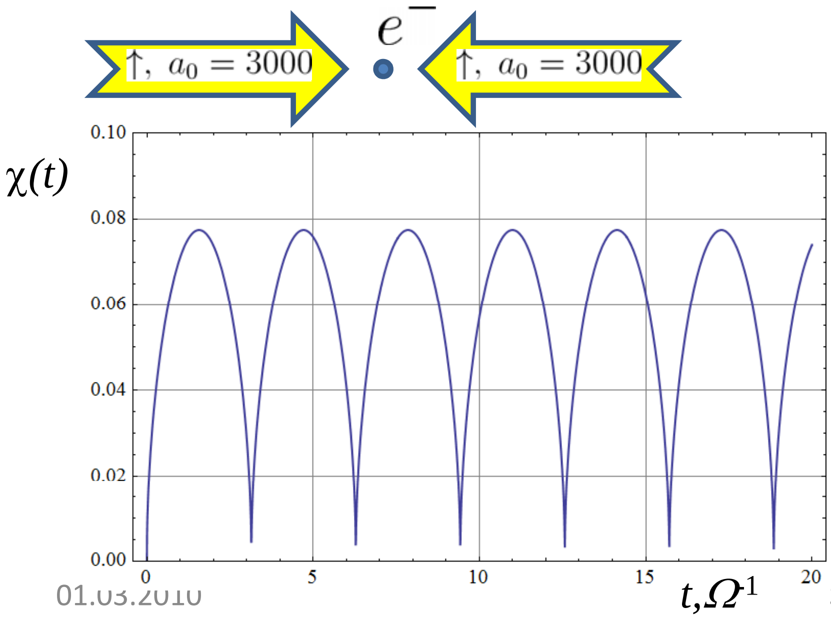
$$E_0 \gg E_S\sqrt{\frac{\hbar\Omega}{mc^2}} \sim 10^{-3}E_S$$

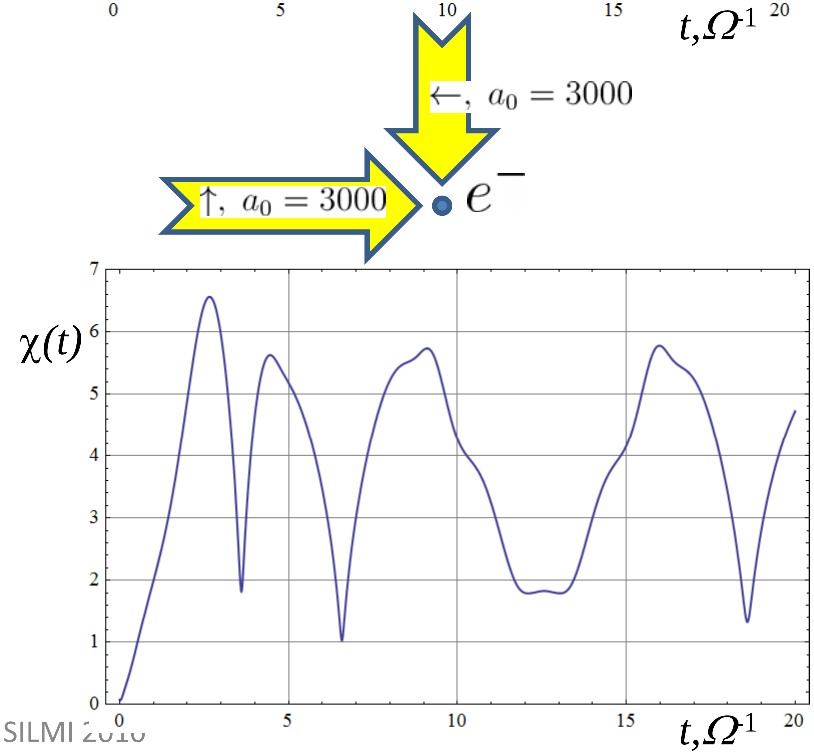


More examples:









Set of time scales in the problem

- ✓ formation time

$$t_{form} = \frac{mc}{eE}$$

- ✓ "acceleration" time

$$t_{acc} = \frac{E_S}{E_0}\sqrt{\frac{\hbar}{mc^2\Omega}}$$

} discussed above

- ✓ "free path" time with respect to quantum processes

$$\int_0^{t_{free}} W_{rad}(\chi(t))\,dt = 1$$

for slow electrons,

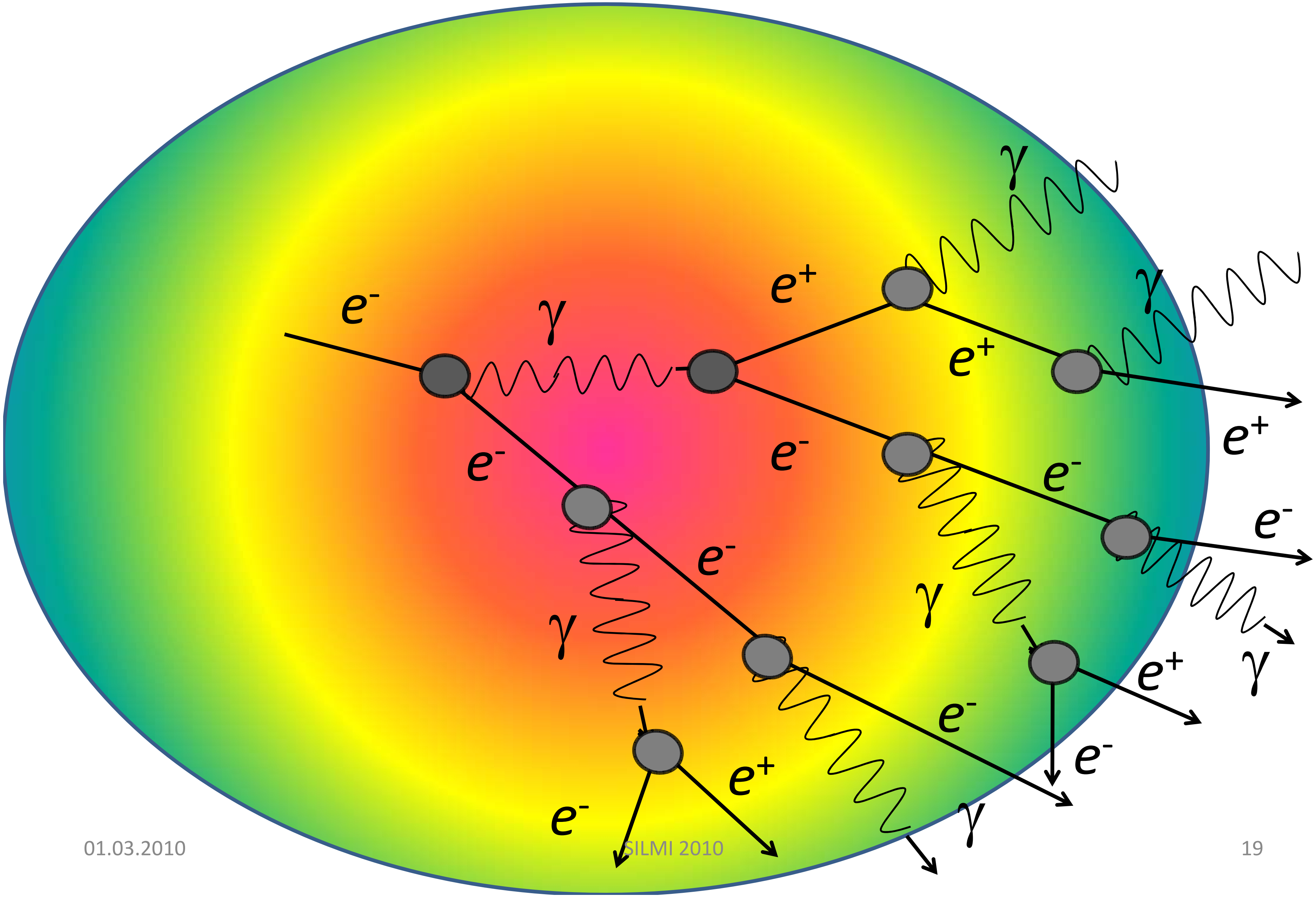
$$t_{free} \sim \sqrt{\frac{E_S}{\alpha E_0}\frac{\hbar}{mc^2\Omega}}$$

Expected hierarchy of the time scales in the problem

$(\hbar\Omega = 1eV)$

Time scale, sec.	**10^{20}W/cm²**	**10^{23}W/cm²**	**10^{24}W/cm²**	**10^{25}W/cm²**	**10^{26}W/cm²**
formation time t_{form}	$8.8\cdot10^{-17}$	$2.8\cdot10^{-18}$	$8.8\cdot10^{-19}$	$2.8\cdot10^{-19}$	$8.8\cdot10^{-20}$
acceleration time t_{acc}	$6.3\cdot10^{-14}$	$2.0\cdot10^{-15}$	$6.3\cdot10^{-16}$	$2.0\cdot10^{-16}$	$6.3\cdot10^{-17}$
free path time t_{free} ($e\to e\gamma$, for $\gamma\to e^+e^-$ about half-order greater)	$2.8\cdot10^{-15}$	$5.0\cdot10^{-16}$	$2.8\cdot10^{-16}$	$1.6\cdot10^{-16}$	$9.0\cdot10^{-17}$
laser half-period, π/Ω	$2.1\cdot10^{-15}$				
notes	$t_{acc}>\pi/\Omega$ $t_{free}< t_{acc}$	$t_{acc}\sim\pi/\Omega$ $t_{free}< t_{acc}$	$t_{free}\sim t_{acc}<<\pi/\Omega$		

QED cascade



Component of the cascade:	Description
Ultrarelativistic electrons	$f_-(\mathbf{r}, \mathbf{p}, t)$
Ultrarelativistic positrons	$f_+(\mathbf{r}, \mathbf{p}, t)$
Soft photons $(\omega \lesssim \omega_0)$	$A_\mu(x) \to \mathbf{E}(\mathbf{r}, t),\ \mathbf{H}(\mathbf{r}, t)$
Hard photons $(\omega \gtrsim \omega_0)$	$f_\gamma(\mathbf{r}, \mathbf{k}, t)$

In numerical simulations, it is reasonable to assume $\omega_0 \sim \dfrac{1}{\Delta t_{grid}}$

Conjecture for cascade equations (motivated by EAS theory)

$$\frac{\partial f_\pm(\mathbf{p},t)}{\partial t} + \frac{\mathbf{p}}{\epsilon}\cdot\nabla f_\pm(\mathbf{p},t) \pm e\left(\mathbf{E} + \frac{\mathbf{p}}{\epsilon}\times\mathbf{H}\right)\cdot\frac{\partial f_\pm(\mathbf{p},t)}{\partial \mathbf{p}}$$

$$= \int\limits_{\omega>\omega_0} w_{rad}(\mathbf{p}-\mathbf{k}\to\mathbf{k}) f_\pm(\mathbf{p}-\mathbf{k},t)\, d^3k - f_\pm(\mathbf{p},t)\int\limits_{\omega>\omega_0} w_{rad}(\mathbf{p}\to\mathbf{k})\, d^3k$$

$$+ \int\limits_{\omega>\omega_0} w_{cr}(\mathbf{k}\to\mathbf{p}) f_\gamma(\mathbf{k},t)\, d^3k.$$

$$\frac{\partial f_\gamma(\mathbf{k},t)}{\partial t} + \frac{\mathbf{k}}{\omega}\cdot\nabla f_\gamma(\mathbf{k},t) = \int w_{rad}(\mathbf{p}\to\mathbf{k})[f_+(\mathbf{p},t) + f_-(\mathbf{p},t)]\, d^3p$$

$$- f_\gamma(\mathbf{k},t)\int w_{cr}(\mathbf{k}\to\mathbf{p})\, d^3p.$$

$$A_\mu(x) = A_\mu^{ext}(x) + \int D_{reg}^{(\omega<\omega_0)}(x,x') j_\mu(x') d^4x'$$

e.g., Maxwell solver on the grid!

$$j_\mu(x) = e\int \frac{d^3p}{\epsilon} p_\mu [f_+(\mathbf{p},x) - f_-(\mathbf{p},x)]$$

Quasi-1D approximation

$$f_{\pm}(\mathbf{r}, \mathbf{p}, t) \to f_{\pm}(\chi_e, t)$$

$$f_{\gamma}(\mathbf{r}, \mathbf{k}, t) \to f_{\gamma}(\chi_{\gamma}, t)$$

Master equations:

$$\text{LHS}\left[\frac{\partial f}{\partial t} + \ldots\right] = \text{RHS}\left[\text{"Gain-loss" terms}\right]$$

recomputed in terms of $\partial/\partial\chi$ by use of the toy model from above

small angle approximation: $\Theta_{decay} \sim \gamma^{-1} \ll \Omega t_{acc}$

- discretization: $t_{form} \ll \Delta t_{grid} \ll t_{acc}$, t_{free}

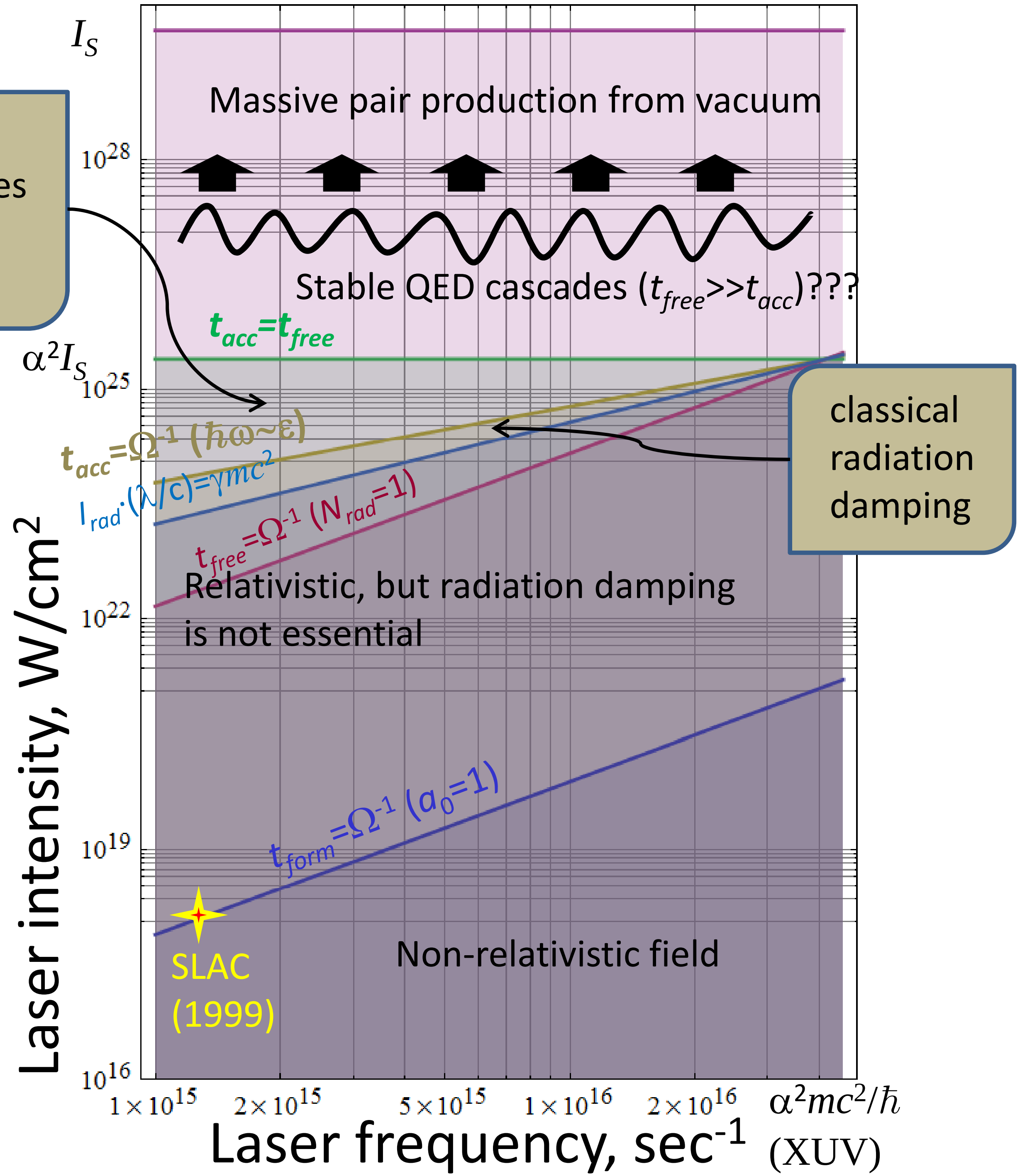
- neglection of soft photon emission *(better to take them into account by means of self-consistent classical field via PIC-like approach)*

Estimated multiplicity of electron-positron component in the cascade (quasi 1d)

I, W/cm^2	N_{ee}
$2.7 \cdot 10^{23}$	negligible and depends on initial conditions
$6.7 \cdot 10^{24}$	15
$2.7 \cdot 10^{25}$	7800
$1.1 \cdot 10^{26}$	$1.6 \cdot 10^{10}$
$6.7 \cdot 10^{26}$	$3.6 \cdot 10^{30}$

At larger intensities we expect complete depletion of a laser pulse due to spontaneous creation of a pair from vacuum and subsequent cascade development

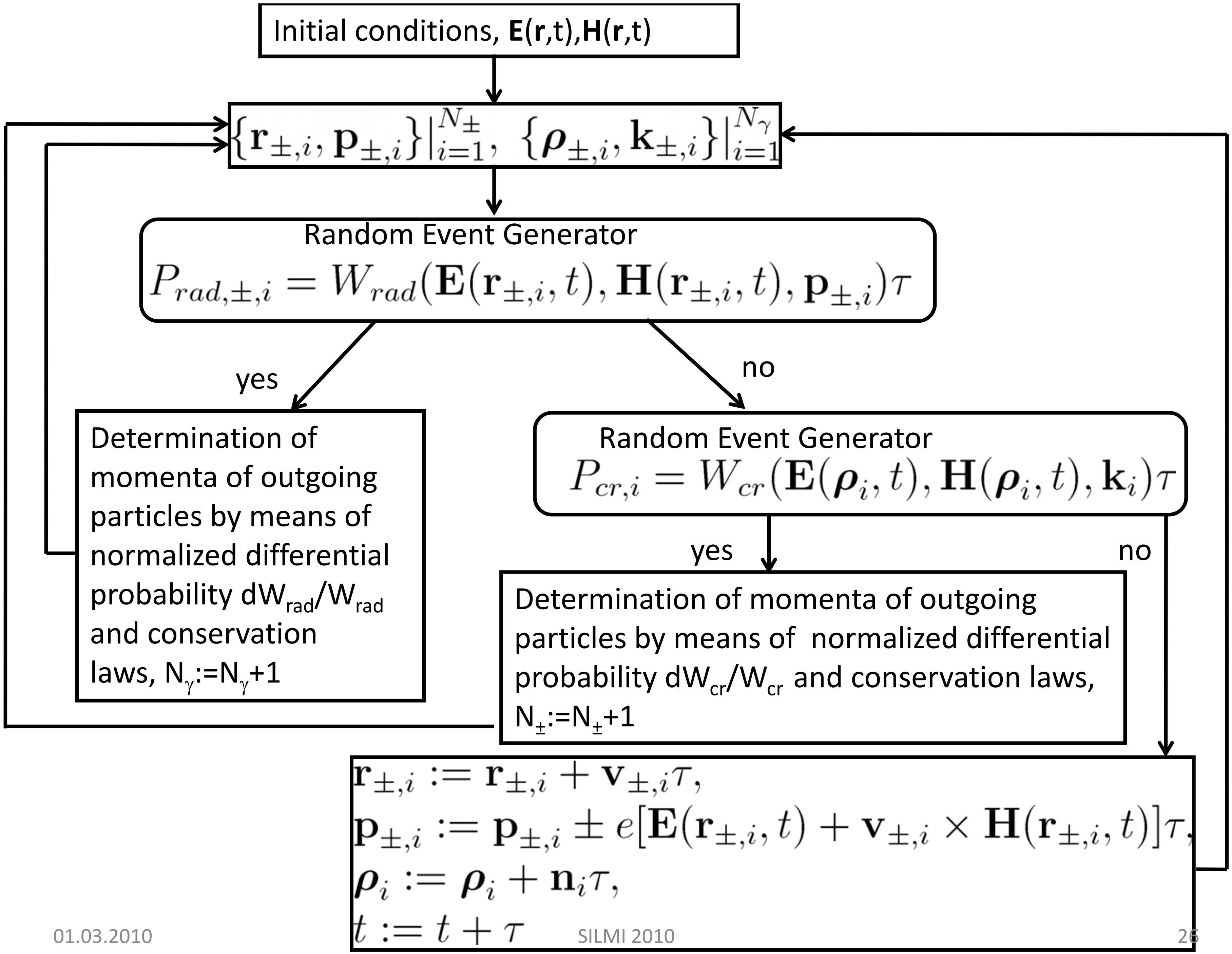
Lanscape of high-intensity laser-matter Interactions (fundamental point of view)



Summary

New physical regime of laser-matter interaction is expected at the intensity level 10^{24}-10^{26} W/cm^2 due to massive formation of plural laser-supported QED cascades. Formation of these cascades may be the leading mechanism for depletion of extremely intense laser beams focused in vacuum or on targets.

We are working on implementation of the laser-induced quantum processes into the approved 3d Monte-Carlo and PIC-codes.


Initial conditions, **E**(**r**,t),**H**(**r**,t)
$\{\mathbf{r}_{\pm,i}, \mathbf{p}_{\pm,i}\}|_{i=1}^{N_\pm}, \ \{\boldsymbol{\rho}_{\pm,i}, \mathbf{k}_{\pm,i}\}|_{i=1}^{N_\gamma}$
Random Event Generator
$P_{rad,\pm,i} = W_{rad}(\mathbf{E}(\mathbf{r}_{\pm,i}, t), \mathbf{H}(\mathbf{r}_{\pm,i}, t), \mathbf{p}_{\pm,i})\tau$
yes
no
Determination of momenta of outgoing particles by means of normalized differential probability dW_{rad}/W_{rad} and conservation laws, $N_\gamma := N_\gamma + 1$
Random Event Generator
$P_{cr,i} = W_{cr}(\mathbf{E}(\boldsymbol{\rho}_i, t), \mathbf{H}(\boldsymbol{\rho}_i, t), \mathbf{k}_i)\tau$
yes
no
Determination of momenta of outgoing particles by means of normalized differential probability dW_{cr}/W_{cr} and conservation laws, $N_\pm := N_\pm + 1$
$\mathbf{r}_{\pm,i} := \mathbf{r}_{\pm,i} + \mathbf{v}_{\pm,i}\tau,$
$\mathbf{p}_{\pm,i} := \mathbf{p}_{\pm,i} \pm e[\mathbf{E}(\mathbf{r}_{\pm,i}, t) + \mathbf{v}_{\pm,i} \times \mathbf{H}(\mathbf{r}_{\pm,i}, t)]\tau,$
$\boldsymbol{\rho}_i := \boldsymbol{\rho}_i + \mathbf{n}_i\tau,$
$t := t + \tau$

Main distinctions from BKA [1,2] treatment

1. "Problem of injection of initial electron": high-intensity fields can be created in near future just as tightly ($R_f \sim \lambda$) focused ultra-short laser pulses. Under these conditions, ponderomotive potential prevents penetration of charged particles inside the focal area (where the intensity is high enough). Can be resolved only within realistic treatment of focused field and **just for certain scenarios**.

2. Cascade approach (enables studies far above the threshold of cascade initiation).

3. Accurate account for quantum nature of radiation reaction as a discrete probabilistic event (important because acceleration mechanism is very sensitive to initial conditions).

4. Partially: qualitative analysis based on length scale hierarchy and the toy model.

[1] A.R. Bell, J.G. Kirk "Possibility of prolific pair production with high-power lasers" Phys. *Rev. Lett. 101, 200403 (2008).*
[2] J.G. Kirk, A.R. Bell, I. Arka "Pair production in counter-propagating laser beams" Plasma Physics and Controlled Fusion 51, 085008 (2009).

Thank you for attention