Radiation and its back reaction on electrons in relativistically strong and QED-strong pulsed laser fields

Igor Sokolov,
University of Michigan,
Natalia Naumova and Gerard Mourou,
Institute de la Lumiere Extreme

Outline

- We consider effects in strong laser fields at $I \sim 10^{22} - 10^{24} W/cm^2$
- For the radiation-dominated regime we discuss the way along which the radiation and its back-reaction on electrons can be described theoretically and simulated numerically.
- Towards QED-strong fields
  - Action or no action? Lagrangian or no Lagrangian?
  - Retarding effect: what can be retarded in the point electron?
  - Transition to QED: frequency-of-what should be comparable with the Compton frequency (=c/Compton wavelength)?
  - Radiation back-reaction or radiation force?
  - Does radiation come from an accelerated electron or from an the field acting on the electron?
- Fundamentals of the numerical scheme:
  - Radiation force approximation.
  - Monte-Carlo approach.

Lagrangian, action and retarding effects

- It is not easy to treat the radiation effects in strong fields from the first principles, such as the principle of minimum action.
- This principle controls the motion within the time interval:

$$S = \int_{t_0}^{t_1} L\left(t, q_i, \frac{dq_i}{dt}\right) dt, \quad \delta S = 0$$

- The Lagrangian, L, is the INSTANTANNEOUS function of time and of the values of the generalized coordinates and velocities;
- The approach based on Lagrangian/action meets difficulties while applied to a system of charged particles:
  - Consider two charged particles at time t. To find the effect from the second particle on the first one: (a) One need not the second particle velocity in this time instant; (b) one needs this velocity in preceding time, $\frac{dq_2}{dt}(t - |q_1 - q_2|/c)$ found from the retarding principle
  - Retarding effect requires to involve the 'third order Lagrangian', which 'approximates' the retarded potentials via accelerations:

$$\vec{A}_1 = \frac{e\vec{v}_2(t - |q_1 - q_2|/c)}{cr} = \frac{1}{|q_1 - q_2|}\frac{dq_2}{dt}(t - |q_1 - q_2|/c) \approx \frac{1}{|q_1 - q_2|}\frac{dq_2}{dt} - \frac{d^2 q_2}{dt^2}$$

  - The high-order time derivatives pose a bunch of problems.

Retarding effect within a single point electron

- In classical electrodynamics particles should be considered as points. For electrons (positrons) this is a very good approximation, well agreeing with the experimental results.
- A charge (distribution) of the electron can be considered as a point, however its energy distribution cannot:

$$p^\mu = m_0 \frac{dx^\mu}{d\tau} + \oint T^{\mu\nu} dS_\nu$$

- Here m_0 is the mass of bare (point) electron. The energy-momentum of its electromagnetic field, T, should be integrated over some space-like infinite hypersurface.
- As the result of the retarding effect, the electromagnetic energy at each space-time point was emanated by electron, while it had the velocity, somewhat different from $\frac{dx^\mu}{d\tau}$. Therefore, the total momentum of the dressed electron and the velocity of the bare electron do not need to be aligned! Re-normalization gives:

$$p^\mu p_\mu = m^2c^2, \quad \frac{dx^\mu}{d\tau} = \frac{p^\mu}{m} + \left(\frac{2e^2}{3mc^3}\right)\frac{eF^{\mu\nu}p_\nu}{m^2c}$$ (Sokolov,2009)

Equation of motion for a radiated electron

$$\frac{dp^i}{d\tau} = \frac{e}{c} F^{ik} \frac{dx_k}{d\tau} - \frac{I_{QED} p^i}{mc^2}$$

$$\frac{dx^i}{d\tau} = \frac{p^i}{m} + \tau_0 \frac{I_{QED}}{I_{cl}} \frac{eF^{ik} p_k}{m^2 c}$$

Here : I_{QED} is used instead of I_{cl} and $\tau_0 = 2e^2/(3mc^3)$

The derivation of these equations, a way to solve them, and a means of integrating the emission are described in Refs. [1-3]

[1] I.V. Sokolov, JETP **109**, 207 (2009) (by re-normalizing the mass operator);

[2] I.V. Sokolov, *et al*, PoP **16**, 093115 (2009); from the conservation law for a single-photon emission: for the electron to acquire the extra energy from the classical field, an extra classical current must be generated

$$p^{\mu}_{final} = p^{\mu}_{initial} + \hbar k^{\mu}_{field} - \hbar k^{\mu}_{emission}, \qquad \frac{\partial T^{\mu\nu}_{field}}{\partial x^{\nu}} = F^{\mu\nu} j_{\nu}$$

[3] I.V. Sokolov, *et al*, PRE (2010) (from QED in the classical limit, for 1D wave).

Radiation in relativistically strong fields: where we are?

- **The suggested equation of motion conserves:**
  - **Total energy-momentum in the system including the external field, the radiating electron and the emitted radiation;**
  - **Generalized momentum of an emitting electron in symmetric fields;**
  - **Maintains the relativistic identity,** $p^{\mu} p_{\mu} = -p^2 + E^2/c^2 = m^2 c^2$ **;**
  - **This set of properties seems to be unique (in the LAD equation the conserving momentum does not satisfy the relativistic identity, the LL equation does not care on the generalized momentum);**
  - **Probably, the simplicity of the transition to QED-strong fields is also unique for this equation only.**
- **It is easy to implement in numerical modeling. Most of our results are provided in our paper in the proceedings of the ELI meeting in Romania. See also today's presentation by Dr. Natalia Naumova on more fresh studies.**

Effect of radiation self-force on the radiation spectrum

Normalized spectrum versus normalized frequency for laser pulses of durations:

(1) 5 fs,

(2) 36 fs,

(3) 220 fs,

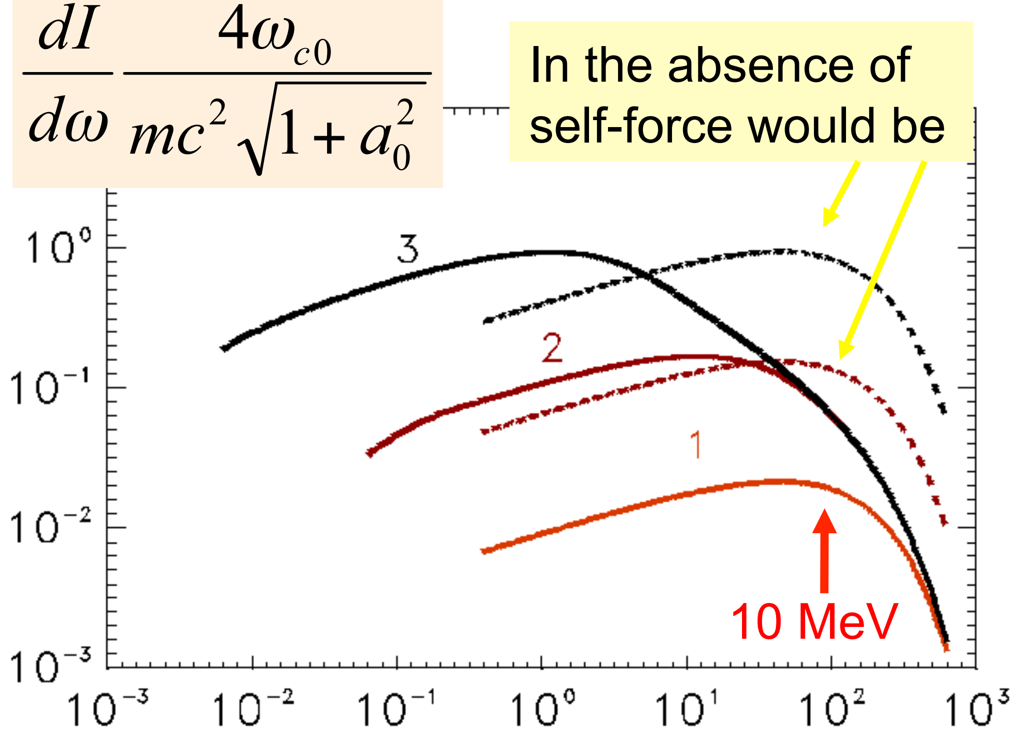
at an intensity of

and a counter-propagating 180-MeV e- beam

Here: $\lambda = 0.8 \mu m$

circular polarization

$$\frac{dI}{d\omega} \frac{4\omega_{c0}}{mc^2\sqrt{1+a_0^2}}$$



ω / ω_{c0}

QED-strong fields

- QED effects occur when the “Schwinger field” is reached. This can be achieved with a laser beam of 10^{29} W/cm2.
- The Schwinger field is determined in terms of the electron charge, e, electron rest mass energy, mc^2, and the Compton (=relativistic De Broglie) wavelength.
- An electron which experiences the Schwinger field:
  - is accelerated till the speed of light within τ_C;
  - emits the photon of the energy of 0.5 MeV within the time of $137\tau_C$, where 137=1/the fine structure constant;
  - this photon is absorbed by vacuum within the time of $137\tau_C$, with creating a pair

-Electric field strength exerted on electron matters, while the field in the laboratory frame does not;
-We do not consider the fields with large invariants, which are capable of breaking vacuum. For simplicity, take 1D Wave

MICHIGAN CRASH ATM

Electron in a super-Schwinger laser field

- A moving electron feels the Lorentz transformed field. Therefore, to generate that high field in the laboratory frame of reference may be either INSUFFICIENT or NOT NEEDED at all, it depends.

$$\chi \approx 0.7 \cdot \sqrt{\frac{I}{10^{23} W/cm^2} \frac{E - p_{\|}c}{10^3 mc^2}}, need\ \chi > 1.$$

- Here E,p are the particle energy and momentum. NOTE: the particle motion in the direction of wave REDUCES χ. Three scenarios:
  - (Traditional: $10^{29} W/cm^2$ is enough)
  - (Arrest electrons: $10^{24} W/cm^2$ is enough - Bell&Kirk,2008)

$$p_{\|} = 0, \quad \frac{E}{mc^2} = \sqrt{\left(\frac{eA}{mc^2}\right)^2 + 1} \sim 10^3 \sqrt{\frac{I}{10^{24} W/cm^2}}$$

Note that the forward motion of electrons may be arrested by ions, therefore, in the laser-plasma interaction at $10^{24} W/cm^2$:

electrons experience super-Schwinger field.

1 GeV electron counter-propagating in a strong laser field what we will see?

- **A moving electron feels the Lorentz transformed field**

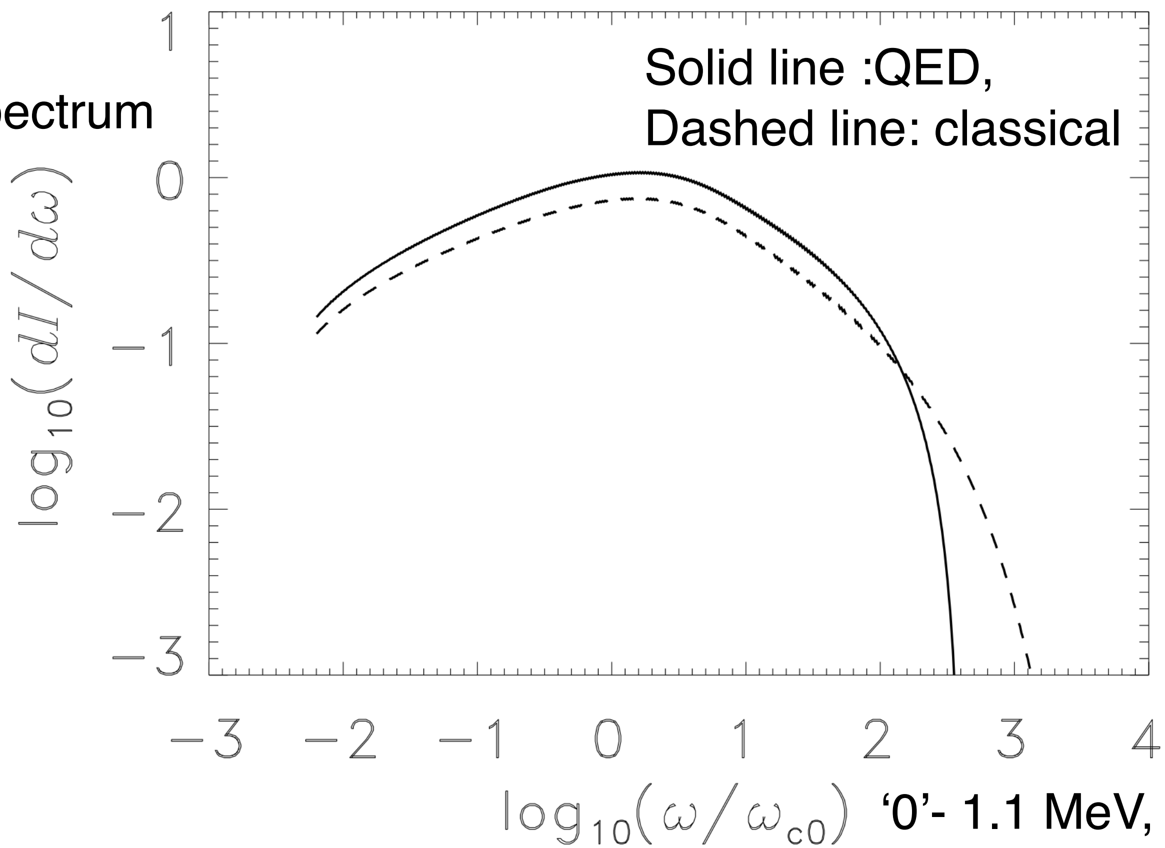
$$\chi \approx 1.4 \cdot \sqrt{\frac{I}{10^{23} W/cm^2}} \left(E[GeV] + | p_{\parallel} c | [GeV] \right)$$

- **In terms of the parameter, χ, 1GeV electron in the field of 10^{23} W/cm^2 is equivalent to an arrested electron in Bell-Kirk scheme at 10^{24} W/cm^2**
- **Pair production may be of the order of 1 pair/electron**
- **Gamma photons generation at 100 MeV-1 GeV.**
- **New challenges in numerical modeling. Relevance of the experimental results at lower intensities to the future results at higher intensities (but at the same χ).**

An Illustration

Emission spectrum for 600 MeV electrons interacting with 30-fs laser pulse of intensity 2×10^{22} W/cm^2 .The above equation with QED-correction for the radiation force had been solved with exact QED emission probabilities.

Frequency spectrum



'0'- 1.1 MeV, '3'-1.1GeV

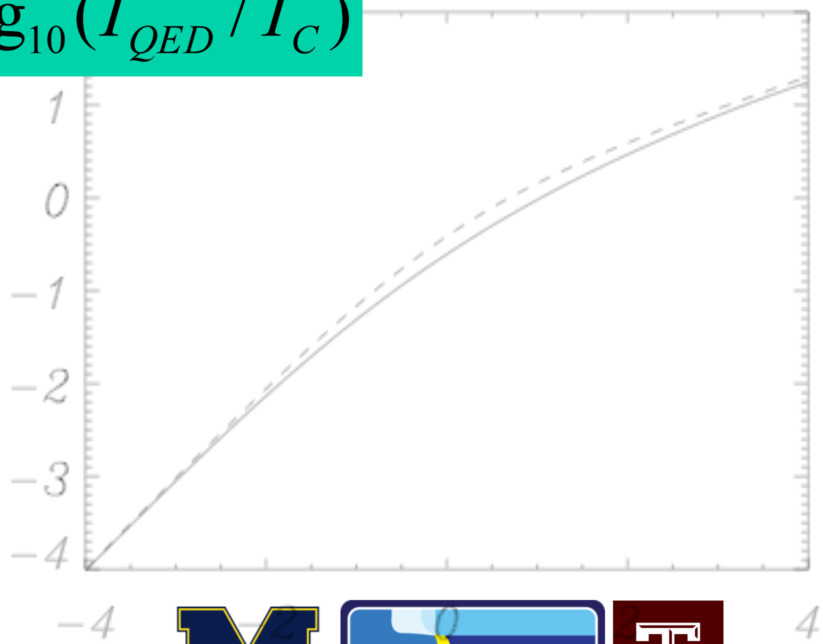
We see that the physically absurd prediction (see the dashed curve) that the maximum photon energy exceeds 1 GeV is eliminated by the QED effects.

Emitted radiated power

$$\frac{dI}{d\Omega dr_0} = I_{cl}\delta\left(\Omega - \frac{\vec{p}}{p}\right)\left(\frac{I_{QED}}{I_{cl}}\right)Q(r_0,\chi)$$

$$\frac{I_{QED}}{I_{cl}} = \frac{9\sqrt{3}}{8\pi}\int_0^\infty dr_0 r_0\left(\int_{r_\chi}^\infty \mathrm{K}_{\frac{5}{3}}(y)dy + r_0 r_\chi \chi^2 \mathrm{K}_{\frac{2}{3}}(r_\chi)\right)$$

$\log_{10}(I_{QED}/I_C)$



$\log_{10}(I_{cl}/I_C)$

Interpolation formula (dashed line):

$$I_{QED} = \frac{I_{cl}}{\left(1 + 1.04\sqrt{I_{cl}/I_C}\right)^{4/3}}$$

Emission spectra

$$Q(r_0,\chi)=\frac{r_0\left(\int_{r_\chi}^{\infty}K_{\frac{5}{3}}(y)dy+r_0r_\chi\chi^2K_{\frac{2}{3}}(r_\chi)\right)}{\int_0^{\infty}dr_0r_0\left(\int_{r_\chi}^{\infty}K_{\frac{5}{3}}(y)dy+r_0r_\chi\chi^2K_{\frac{2}{3}}(r_\chi)\right)}$$

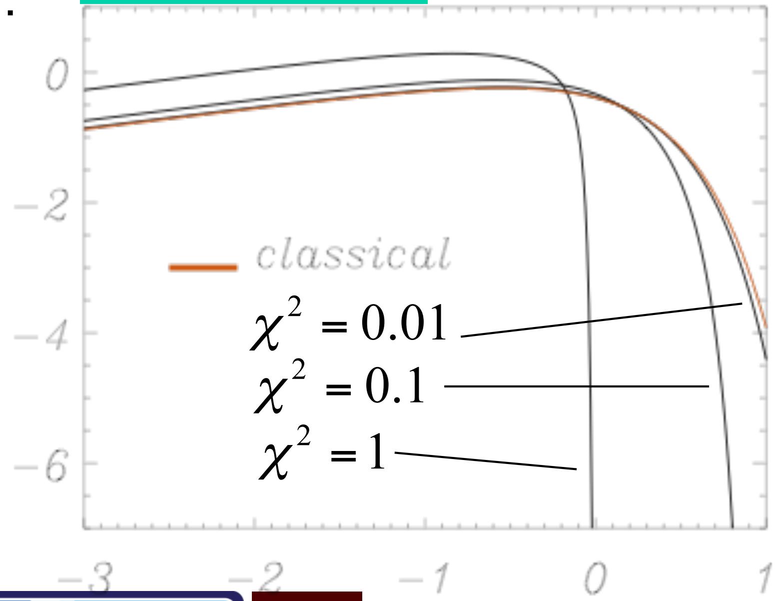
For various values of χ:

$$\chi=\sqrt{\frac{I_{cl}}{I_C}}$$

$$I_C=\frac{8}{27}\frac{e^2c}{\lambda_C^2}$$

$$\omega_c=\varepsilon c\chi/\lambda_C$$

$\log_{10}(I/I_C)$



$\log_{10}(\omega/\omega_c)$

State of art with QED effects in PIC simulations

- **Unless the parameter χ approaches ~10 (even 3 is OK) the best way to incorporate QED effects is the use of correction coefficients, I_{QED}/I_{cl}, within the radiation force approximation.**
- **As comparing to the scheme to account for the classical radiation theory effects, as described above, the use of the correct QED emission probability is:**
  - **More strict;**
  - **Less restrictive (allows us to simulate higher intensities);**
  - **Eliminates some unphysical predictions, such as the generation of the high-energy photons from lower energy electrons;**
  - **Not more cumbersome as compared to the classical emission theory.**

Future work: Monte-Carlo approach.

- **Radiation back-reaction or radiation force:**
  - ‘Newton’s law’ for a force: within the infinitesimal time interval the increment of the momentum of the radiating particle is infinitesimal: $\Delta p^{\mu} = f^{\mu}\Delta\tau$
  - At large χ for short time intervals infinitesimal is the probability of emission, but the change in momentum is finite.
- **The use of the emission probability with the Dirac function approximation for the angular spectrum:**

$$\frac{dW_{fi}}{d\Omega d(\hbar\omega/\chi\gamma mc^2)} = \delta\left(\Omega - \frac{\vec{p}}{p}\right)\left(\frac{I_{QED}\Delta\tau}{mc^2}\right)\frac{Q(r_0,\chi)}{r_0\chi}, \quad \frac{W_{fi}}{\Delta\tau} = \left(\frac{I_{QED}}{mc^2}\right)\int_0^{\infty}\frac{Q(r_0,\chi)}{r_0\chi}dr_0,$$

- **Total probability of emission is expressed in terms of the complete integral of probability. Time interval should be sufficiently short, to have the total probability of emission less than unity. Using the incomplete integral of probability, one can gamble the ratio, $\hbar\omega/\gamma mc^2$, in terms of a sole random scalar (to be published soon).**

Conclusions

- Gamma-photons dominate (as the loss mechanism, as the most abundant sort of particles, as the main effect in the electron motion) for laser intensities at $I \sim 10^{23} - 10^{24} W/cm^2$
- Electrons are in QED-strong field.
- All reactions with incoming or outgoing electrons have modified probabilities.
- 10^{23} W/cm2 combined with the oppositely propagating 1GeV electron beam (a finite angle is OK) could allow us to foresee what will occur at 10^{24} W/cm2
- Diagnostics for >100 MeV gamma-photons.
- Challenges in modeling:
  - Gamma-emission spectrum, radiation back-reaction, QED corrections
  - Radiation transport, gamma-to-pair absorption.